THE $\omega-\rho-\pi$ COUPLING AND THE INFLUENCE OF HEAVIER RESONANCES

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Motivation
- The $\omega - \rho - \pi$ mesons coupling ($g_{\omega\rho\pi}$) in precision measurements.

The hadronic vertex in several processes
- VMD
- Radiative decays
- Low energy theorems and VMD
  - $\omega \rightarrow \pi \pi \pi$
  - $\rho'$ as an effective contact term
  - $e^+ e^- \rightarrow \pi \pi \pi$
- Global results

Conclusions
The $\omega-\rho-\pi$ mesons coupling ($g_{\omega\rho\pi}$) in precision measurements.

**Motivation**

- $W$: $I=1 \& V,A$
- CVC: $I=1 \& V$
- $\gamma$: $I=0,1 \& V$
- $\tau$, $W$, $\nu_\tau$, hadrons, $e^+$, $e^-$, hadrons
muon g-2

QCD

Hadrons

...another new physics?
The importance of knowing hadrons

The most important channel is $e^+ e^- \rightarrow \pi \pi$
The hadronic vertex in several processes

VMD

The VMD Lagrangian including the $\rho$, $\pi$ and $\omega$ mesons can be set as:

$$
\mathcal{L} = g_{\rho\pi\pi} \epsilon_{abc} \rho_a^a \pi^b \partial^\mu \pi^c + g_{\omega\rho\pi} \epsilon^{\mu\nu\lambda\sigma} \partial_\mu \omega_\nu \partial_\lambda \rho_a^a \pi^b \\
+ g_3 \pi \epsilon_{abc} \epsilon^{\mu\nu\lambda\sigma} \omega_\mu \partial_\nu \pi^a \partial_\lambda \pi^b \partial_\sigma \pi^c + \frac{e m^2}{g_V} V_\mu A^\mu,
$$

Chiral Anomaly: Wess-Zumino-Witten
The $g_{\omega\rho\pi}$ coupling can be obtained, within VMD, from radiative decays using the following relations:

$$g_{\omega\rho\pi} = g_{\rho\pi\gamma} g_{\omega,\rho} / e$$

The coupling is obtained using

$$\Gamma(V \to \pi\gamma) = g_{V\pi\gamma}^2 \frac{m_{V}^2 - m_{\pi}^2}{96 \pi m_{V}^3}$$

<table>
<thead>
<tr>
<th>Decay</th>
<th>$g_{\omega\rho\pi}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho \to \pi\gamma$</td>
<td>$12.3 \pm 0.9$</td>
</tr>
<tr>
<td>$\rho \to \pi\gamma$</td>
<td>$14.2 \pm 1.0$</td>
</tr>
<tr>
<td>$\omega \to \pi\gamma$</td>
<td>$11.4 \pm 0.2$</td>
</tr>
<tr>
<td>$\pi \to \gamma\gamma$</td>
<td>$11.6 \pm 0.6$</td>
</tr>
<tr>
<td>Weighted average</td>
<td>$11.6 \pm 0.2$</td>
</tr>
</tbody>
</table>
The $g_{\omega\rho\pi}$ coupling can be obtained, linking VMD and low energy theorems in the $\pi \rightarrow \gamma\gamma$ decay

$$g_{\omega\rho\pi} = \frac{g_\rho \ g_\omega}{8\pi^2 f_\pi}$$

using different approaches for the involved parameters, we get

<table>
<thead>
<tr>
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<th>$g_{\omega\rho\pi}$</th>
</tr>
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<tbody>
<tr>
<td>$g_\rho = g_{\rho\pi\pi}$</td>
<td>11.5</td>
</tr>
<tr>
<td>$g_\rho = g_{\rho\pi\pi}$ : $g_\rho = 3g_\omega$ (SU3)</td>
<td>14.4</td>
</tr>
<tr>
<td>$g_{\rho\pi\pi} = m_\rho/(2f_\pi)^{1/2}$ (KSFR)</td>
<td>14.2</td>
</tr>
</tbody>
</table>
The amplitude is given by

\[ M_D = \varepsilon_{\mu \alpha \beta \gamma} \eta^\mu p_1^\alpha p_2^\beta p_3^\gamma A, \]

\[ A = g_{3\pi} + 2g_{\omega \rho \pi} g_{\rho \pi \pi} (D^{-1}[\rho^0, p1 + p2] + D^{-1}[\rho^+, p1 + p3] + D^{-1}[\rho^-, p2 + p3]) \]

Using \( g_{\omega \rho \pi} \) the average from rad. decays and no contact term misses the experimental width by 45%

In order to reach the 100% of the experimental width, we can either increase \( g_{\omega \rho \pi} \) up to 15.7 or keep its value from radiative decays and add the contact term. A blind inclusion requires \( g_{3\pi} = -65 \text{ pm} \) 7 and \( 406 \text{ pm} \) 10 GeV\(^{-3}\).

The contact term can be seen as due to the presence of a heavier meson in the intermediate state and the chiral anomaly have a truly contact term. Therefore the information in an effective contact term have many contributions.

Linking VMD and the low energy theorems requires

\[
\frac{\alpha}{\pi f_{\pi}^3} = e^2 \frac{6(g_3^\pi + g_{\omega\rho\pi} g_{\rho\pi\pi}/m_{\rho}^2)}{g_\omega} \\
\rightarrow g_3^\pi = g_{\rho\pi\pi}/16\pi^2 f_{\pi}^3 = 47 \text{ GeV}^{-3}.
\]
Using the VMD ideas, the contact term can be seen as a pinched diagram due to the presence of a heavier meson in the intermediate state. In this case the natural candidate is the \( \rho' (1450) \), \( m' = 1465 \text{ MeV} \) and \( \Gamma' = 400 \text{ MeV} \).

The heavy mass of the \( \rho' \) allows to make the following approximation:

we assume that \( \frac{g_{\omega \rho' \pi}}{g_{\rho' \pi \pi}} = \frac{g_{\omega \rho \pi \pi}}{g_{\rho \pi \pi}} = 2 \) and \( g_{\omega \rho' \pi} \approx 10 - 18 \text{ GeV}^{-1} \)

\[ |g_{3\pi}| \approx \frac{g_{\omega \rho' \pi}}{g_{\rho' \pi \pi}} \frac{g_{\rho \pi \pi}}{m_{\rho'}^2} \]

|          | \( |g_{3\pi}| \)   |
|----------|-------------------|
| Rudaz    | 47                |
| Dominguez| 29 pm 3           |
| Kuraev   | 123               |
| Kaymakcalan | 37            |
| **This work from \( \Gamma(\omega \rightarrow 3\pi) \)** | 65 pm 7 |
| **This work from \( \rho' \)**            | 49 pm 24 |

We can write the amplitude for the $\omega$ channel as follows:

$$\mathcal{M} = \frac{e^2 m_\omega^2}{g_\omega} \frac{\bar{v}\gamma^{\mu} u \epsilon_{\mu\alpha\beta\gamma} p_1^{\alpha} p_2^{\beta} p_3^{\gamma}}{q^2 (q^2 - m_\omega + i m_\omega \Gamma_\omega)} A$$
## Global Results

<table>
<thead>
<tr>
<th>Process</th>
<th>Without contact $\text{GeV}^{-1}$</th>
<th>With contact $\text{GeV}^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radiative processes</td>
<td>$11.6 \pm 0.2$</td>
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</tr>
<tr>
<td>$\Gamma(\omega \rightarrow 3\pi)$</td>
<td>$15.7 \pm 0.1$</td>
<td>$12.6 \pm 1.3$</td>
</tr>
<tr>
<td>$\sigma (e^+ e^- \rightarrow 3\pi)$</td>
<td>$13.1 \pm 0.1$</td>
<td>$9.8 \pm 1.4$</td>
</tr>
<tr>
<td>Weigthed average</td>
<td>$13.8 \pm 0.1$</td>
<td>$11.6 \pm 0.2$</td>
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</table>
We have determined the coupling for the omega-rho-pi mesons from several processes.

The value obtained for $g_{\omega\rho\pi}$ is sensible to the inclusion of the contact term.

We have taken the contact term as produced by a heavier resonance.

The combined values are useful to make more solid estimates of hadronic contributions in precision measurements.