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The misleading nature of the leaky box models in cosmic ray physics

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Abstract: Many experimental results around and above the energies where the solar modulation affects cosmic ion fluxes were quantified, conceptualized and debated using leaky box models. These models exploit the notion of equilibrium between creation and destruction processes of cosmic ions in an undifferentiated arbitrary volume representing the Galaxy, ignoring the galactic magnetic field, the size of the Galaxy, the position of the solar cavity, the spatial distribution of the sources, the space variation of the interstellar matter and other pertinent observations. The progress in the measurements of the quoted observational parameters makes obsolete the use of the leaky box models. Specific examples substantiating the inadequacy of the leaky box models are analyzed like the conversion of the boron-to-carbon flux ratio into grammage and the residence times of cosmic ions in the Galaxy. The unphysical and misleading nature of the leaky box models is ascertained and illustrated at very high energy.

Introduction

Many experimental results on the primary cosmic radiation at very low energy gathered in the last five decades by balloon-borne instruments and satellite experiments have been elaborated using simple theories of galactic cosmic rays referred to as Leaky Box Models. Two basic physical quantities namely the gas column encountered by galactic cosmic rays ($grammage$) and the related residence time, T, were believed to correctly interpret a number of measurements. Ritual fittings of the grammage above an adjustable energy E_0 (or the rigidity R_0) based on numerous measurements have the form:

$$g = Gr^{-\delta} \quad (1)$$

where G is a constant grammage at the rigidity R_0, δ a constant and r= R/R_0 being R the ion rigidity. Numerical values in classical interpolations are, for example: G=10.8 g/cm^2, δ=0.6 and R_0=4 GV [1], or G=24.0 g/cm^2, δ=0.65 and R_0=5.5 GV [2].

Grammage is converted into a residence time by: T = $g/\rho v$ where v the ion velocity and ρ the matter density. The extrapolation at very high energy of these residence times caused fictitious problems as pointed out by Hillas [3, 4]. Leaky Box Models relate somehow the B/C flux ratio to the grammage. Figure 1 purposely reports in a linear energy scale the B/C flux ratio in the energy band 10-200 GeV/u. In this range there is no compelling evidence for a grammage decrease of the form $Gr^{-\delta}$ as established in the region 1-4 GeV/u. The recent measurements of the B/C flux ratio at 700 GeV/u of the Runjob Collaboration [5] along with the data shown in figure 1 exclude the functional form (1) above 20 GeV/u. The data in figure 1 and a computed B/C flux ratio [6] vividly testify the inadequacy of the grammage fitting via $r^{-\delta}$ to physical reality in an area believed in past decades to be the realm of the Leaky Box Models.

The unjustified form of the cosmic-ray source power versus energy

In a suitable galactic container, at a given energy E, two processes are at work to maintain cosmic-ray intensity at a constant value: escape from the container and nuclear interactions inside. The simplified equation describing these processes are:

$$dQ/dE = k(dN/dE)(1/\lambda + 1/f) \quad (2)$$

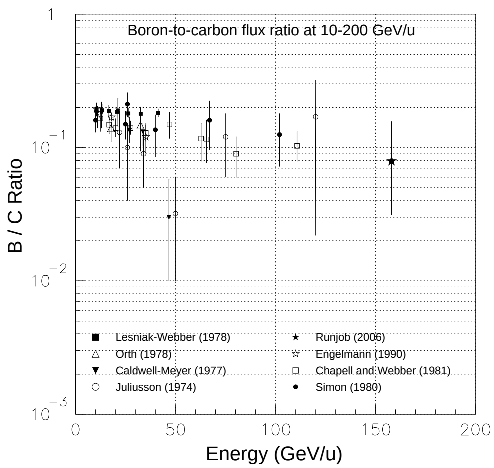

Figure 1: Measurements of the boron-to-carbon flux ratios above 10 GeV/u up to the energy of 200 GeV/u displayed in a linear scale of energy. No decreasing trend of the B/C flux ratio is evident. Data below 10 GeV/u may be found elsewhere [7].

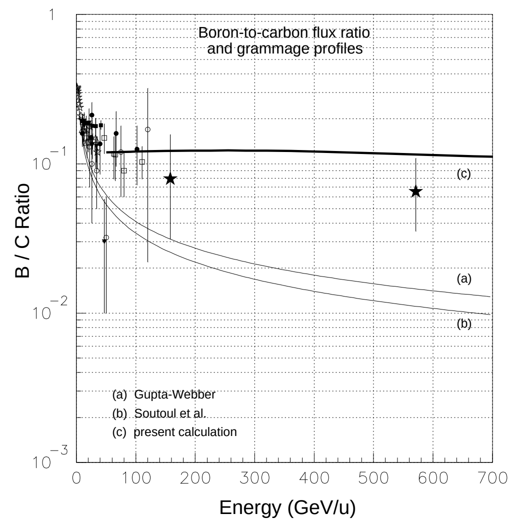

Figure 2: Interpolated grammage via G $r^{-\delta}$ (thin lines a and b) for the two sets of constants mentioned in the Introduction. It is evident that the slope δ tuned to the B/C flux ratio in the range 1-5 GeV/u is not in accord with the experimental data above 10 GeV/u. The computed grammage (c) refers only to a tiny energy fraction of that shown in figure 3.

where dQ/dE is the source power, dN/dE is the differential intensity in a given location of the disc, λ is the nuclear collision length, f is the average escape length and k a suitable normalization constant. It is an established result that the differential intensity at Earth, dN/dE, measured in many energy bands, obeys a power law i.e. dN/dE = a$E^{-\gamma}$ where γ is the spectral index and a a constant.

Whenever one of the two terms (1/λ) and (1/f) dominates, the equation (1) simplifies further. At sufficient high energy, especially for light ions, the term 1/λ is negligible compared to 1/f. Using the $ad\,hoc$ hypothesis that the escape time has the form $E^{-\delta}$ it follows that dQ/dE is proportional to E^{-s} where s= γ - δ. Therefore from the assumption $E^{-\delta}$ (disproved by the B/C data above 10 GeV/u) the unknown form of the source power versus energy of the cosmic radiation automatically transforms into a power law with a constant index s. The splitting of γ in two constant parts, s and δ, in a large energy band remains unjustified taking into account, not only the B/C flux ratio at 160 and 700 GeV/u measured by the Runjob experiment [5], but also the classical data shown in figure 1.

The unphysical grammage extrapolated at high energy

From simple elaborations of the equation (1) follows that the grammage, g, is related to the secondary-to-primary flux ratio of cosmic rays (for example, B/C but also $^3He/^4He$, subFe/Fe, Nitrogen/Oxygen and others) by the equation:

$$\frac{g}{m} = \frac{1}{\sigma_{sp}\frac{N_p}{N_s} - \sigma_s} \qquad (3)$$

where the N_s/N_p is the flux ratio observable by experiments at a given energy, m the mean mass of the interstellar atom, σ_s is the inelastic nuclear cross section of the secondary with the interstellar matter and σ_{sp} secondary production rate. Equation (3) applies to a single parent nucleus generating a unique secondary while primary-to-secondary ratios resulting from many tributaries have more articulated formulae.

In the limited energy range, 1-5 GeV/u, there is empirical evidence that N_s/N_p ratio (i.e. the B/C ratio) is decreasing with energy. Consequently, since σ_{sp} and σ_s are smooth functions, the grammage has a decreasing trend with the energy, often interpolated by $Gr^{-\delta}$ with a constant δ. On the contrary, in the energy band 10-200 GeV/u, the classical measurements of the B/C flux ratio [8, 9, 10, 11, 12, 13, 14] do not conform to the same δ extracted at lower energies, as apparent from the data shown in figure 2 (with or without the Runjob data). The functional form $r^{-\delta}$ fails just above 20 GeV/u, exhibiting a large discrepancy at 700 GeV/u. The B/C flux ratios measured by the Runjob experiment [5] reconfirm the failure of the formula (1) with a unique δ in the interval 1-700 GeV/u.

It is surprising that most of the data in the region 10-100 GeV/u shown in figure 1 are simply omitted in the comparison with the formula $g = Gr^{-\delta}$ (see, for example, fig.1 [15] or fig.1 [16]) or with direct calculations (fig.2 [6]).

The $grammage$, the gas column encountered by galactic cosmic rays, g, can be calculated using directly the trajectory length L_D, via g=m n L_D, where n the number density of the interstellar atom. Figure 3 gives the He and Fe grammage versus energy in the interval 10^{10}-10^{18} eV. Details of the calculation are given elsewhere [18]. The computed He grammage diminishes with a gentle, increasing slope up to 10^{15} eV beyond which a steep descend sets on, finally attaining a minimum value, at 10^{18} eV (rectilinear propagation regime). The dependence of the He and Fe grammage $versus$ energy is explained elsewhere [18, 17] by the notion of galactic basin.

The grammage computed by the direct formula, g=m n_H L_D, is certainly different, by definition, from that evaluated by the transport equation used in Leaky Box Models, which claim to relate the grammage to the B/C flux ratio. For instance, the spatial distribution of the secondaries should differ from that of the primaries and this difference is not incorporated in Leaky Box Models. The carbon grammage versus energy evaluated via g=m n_H L_D, is shown in figure 2 (thick line) arbitrarily normalized at 50 GeV/u to the B/C ratio of 0.125. The grammage versus energy in the band 50-700 GeV/u is consistent with the data. Note that at lower energy, below 10 GeV/u, the grammage for the stable ^7Be (quite similar to C or B grammages) has been evaluated with the same method resulting in a much steeper slope [19].

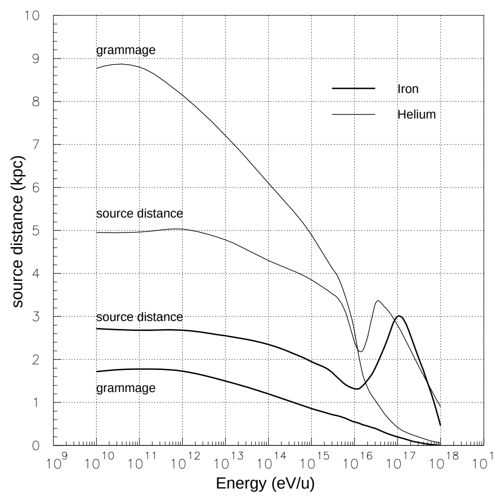

Figure 3: The average distance of the sources from the Earth in the energy range 10^{10}-10^{18} eV and the related grammage of galactic cosmic rays (He and Fe) intercepting the Earth, expressed in g/cm^2 in the same vertical axis. This calculation, unlike those based on Leaky Box Models, exploits numerous astronomical and radioastronomical observations (see Section 2 in [17]).

Also the computed grammage [20] for the stable ^9Be results in a decrease of 30 per cent in the range 1-10 GeV/u both for the spiral and the circular galactic magnetic fields. Note that the mass and the charge of ^9Be are similar to those of B and C, and consequently, the grammage profiles with energy are analogous. For comparison, the measured B/C flux ratio decreases from 0.32 at 1 GeV/u to 0.20 at 10 GeV/u, exactly the same decrease of the computed ^9Be grammage (see fig.2 [20]).

The unphysical walls reflecting back cosmic rays in the disc

The $ad\ hoc$ assumption that the residence time of cosmic rays is compatible with a single value (for example, $15 \times 10^6\ years$) is unphysical because light cosmic rays (proton, He) originated in the Bulge reside longer than those populating the disc periphery, at $15\ kpc$ from the galactic center. The volume where cosmic rays propagate is not specified and therefore the residence volume is undefined. The physical motion (migration, diffusion, convection, trapping or combinations of these classes of displacements) of cosmic rays is not defined. Since the magnetic field does not exist in Leaky Box Models and the ion motion is not specified, cosmic rays should travel freely in the undefined containment volume. But this free motion, a silent element of any variants of the Leaky Box Model, implies that a physical, real process, at some boundary of the disc, reverses the ion motion (reflection). Without the reflection at some boundary of the disc the grammage cannot accumulate to high values, because the free traversal of the disc without reflection entails a grammage of a few milligrams per cm^2, some 4 orders of magnitude below the standard $10\ g/cm^2$. What is the physical mechanism accomplishing this operation? To date (2007), it remains unknown.

Notice further that the matter density in the disc, adopted in Leaky Box Models for intrinsic calculation procedure to determine nuclear spallation rates, is also inconsistent with the matter density necessary to determine residence time of cosmic rays using radioactive clock measurements (^{10}Be/Be and others). A tangible sign of this embarassing feature is that the mean gas density in the disc turns out to be in the range 0.25-0.35 $atom/cm^3$ (see, for example, [21] for the data and [22] for the calculation procedure), a factor 3-4 below the average, observed value of $1\ atom/cm^3$.

A paradox is encountered by extrapolating the residence time at energies above 10^{17} adopting $Gr^{-\delta}$ with δ=0.65 [2]. The high energy galactic sources would populate a small volume in the disc, concentric to the Earth, and they all would reside in the solar system at energies above $10^{23}\ eV$.

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