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Suppression of particle drifts by magnetic turbulence

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Abstract: We present results from direct numerical simulations showing the suppression of the large-scale drift motion of an ensemble of charged particles in a non-uniform turbulent magnetic field. We find that when scattering is negligible, the ensemble drifts in the direction predicted by the usual guiding center theory. When scattering is very strong, we find that all large-scale drift motions vanish. For an intermediate amount of scattering we find that the drift velocity is typically suppressed by a larger amount than the drift coefficient.

Introduction

It has long been known that large-scale drift plays an important role in the heliospheric modulation of cosmic rays [1]. However, with the advent of more realistic numerical models of cosmic ray modulation [2, 3, 4], it soon became apparent that drift effects for low to intermediate energy cosmic rays should be suppressed to properly account for cosmic ray observations [5].

From a simple hard-sphere scattering approach [6] or using a velocity correlation function analysis [7] one can see that the drift coefficient should be suppressed in the presence of scattering. Various numerical studies have aimed to quantify this suppression of the drift coefficient [8, 9]. However, all previous numerical studies used models with a uniform large-scale magnetic field and scattering, making it impossible to relate the drift coefficient to the divergence of the anti-symmetric diffusion tensor.

In order to relate the drift velocity $pv/(3q)\boldsymbol{\nabla}\times\boldsymbol{B}/B^2$, to the drift coefficient $pv/(3qB)$, it is *required* that there be gradients in the system, otherwise the drift velocity is zero and any relation between the drift velocity and the drift coefficient is arbitrary.

In the present study we use a non-uniform large-scale magnetic field in our numerical simulations to ensure that the large-scale drift velocity of the ensemble of particles is non-zero. We are therefore able for the first time to directly test the well known relation between the large-scale drift velocity and large-scale drift coefficient.

Preliminary results are presented in [10] and a detailed investigation into this phenomenon of the suppression of drift is presented in [11].

Drift Theory

The drift velocity of an ensemble of particles is related to the off-diagonal element of the diffusion tensor through [1, 11]

$$\boldsymbol{v}_D = \boldsymbol{\nabla}\times\kappa_A\boldsymbol{e}_B\,, \tag{1}$$

with κ_A the anti-symmetric entry in the diffusion tensor that also includes the effects of scattering, and $\boldsymbol{e}_B$ is the unit vector in the direction of the background magnetic field.

Investigations of charged particle transport based on isotropic scattering over some time scale τ [6], quasi-linear treatment of particle streaming perpendicular to a large-scale magnetic field [12], or de-correlation of the particle velocities after some time scale τ [7] all lead to the same functional form for the drift coefficient, namely

$$\kappa_A = \kappa_A^{ws} \frac{(\Omega\tau)^2}{1+(\Omega\tau)^2}\,, \qquad (2)$$

with $\kappa_A^{ws} = vr_M/3$ the weak-scattering value of the drift coefficient, r_M the maximal Larmor radius (90° pitch-angle particles) and Ω the cyclotron frequency of the particles. Regardless of the model that is used to derive this drift coefficient, the result is a quantity with a maximum value of $vr_M/3$ when $\Omega\tau \to \infty$ in the weak-scattering limit, and a minimum value of 0 when $\Omega\tau \to 0$ in the strong-scattering limit.

Following the analysis of [13] we use equation (2) in equation (1) to obtain

$$\boldsymbol{v}_D = f_s \boldsymbol{v}_D^{ws} + \boldsymbol{\nabla} f_s \times \kappa_A^{ws} \boldsymbol{e}_B\,, \qquad (3)$$

with $\boldsymbol{v}_D^{ws} = \boldsymbol{\nabla}\times\kappa_A^{ws}\boldsymbol{e}_B$ the weak-scattering drift velocity determined by the weak-scattering drift coefficient κ_A^{ws}, and $f_s = (\Omega\tau)^2/[1+(\Omega\tau)^2]$ the drift coefficient suppression factor from equation (2). It is apparent that the weak-scattering drift coefficient and drift velocity will only be suppressed by the same amount (f_s) if the second term in equation (3) is negligible.

The second term on the right hand side of equation (3) can be neglected when scattering is either very strong or very weak. To see this, we write

$$\boldsymbol{\nabla} f_s = \boldsymbol{\nabla}\left(\frac{(\Omega\tau)^2}{1+(\Omega\tau)^2}\right) = \frac{2\Omega\tau\boldsymbol{\nabla}(\Omega\tau)}{[1+(\Omega\tau)^2]^2}\,. \qquad (4)$$

In the strong-scattering limit ($\Omega\tau \to 0$) we find

$$\lim_{\Omega\tau\to 0} \frac{\Omega\tau}{[1+(\Omega\tau)^2]^2} = \lim_{\Omega\tau\to 0} \Omega\tau = 0\,, \qquad (5)$$

and in the weak-scattering limit ($\Omega\tau \to \infty$)

$$\lim_{\Omega\tau\to\infty} \frac{\Omega\tau}{[1+(\Omega\tau)^2]^2} = \lim_{\Omega\tau\to\infty} \frac{1}{(\Omega\tau)^3} = 0\,. \qquad (6)$$

For an intermediate amount of scattering we therefore see that the drift coefficient (eq. [2]) and drift velocity (eq. [3]) do not have the same dependence on the suppression factor f_s, implying that these quantities might be influenced differently by the occurrence of scattering. This behavior is found in the simulations, as discussed below.

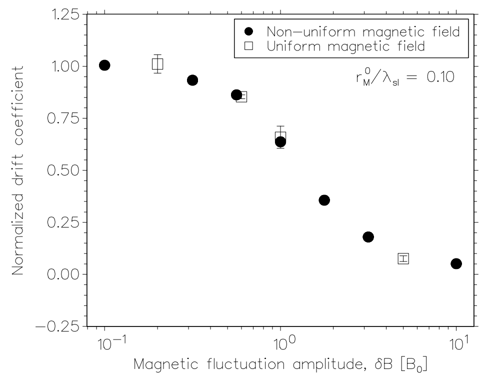

Figure 1: Simulated drift coefficient κ_A normalized to the weak scattering value $vr_M/3$ as a function of the magnetic fluctuation amplitude δB, in units of the magnitude of the background magnetic field B_0.

Numerical Experiment Design

A detailed discussion of the numerical experiment design is presented in [11]. In summary, a non-uniform large scale magnetic field is used to ensure that the ensemble of particles will drift in the absence of scattering at the velocity given by the first term on the right-hand-side of equation (3), with $f_s \to 1$, since $\Omega\tau \to \infty$ for weak-scattering. Then a composite of slab and 2D turbulence is superposed onto this large scale magnetic field, with a distribution of 20% slab and 80% 2D turbulence, characteristic of solar wind conditions near Earth [14]. The amplitude of this turbulence is allowed to vary and the reduction of the drift velocity and coefficient as a function of the magnetic fluctuation amplitude is investigated.

Results

In Figure 1 we show the simulated drift coefficient κ_A normalized to the weak-scattering value $vr_M/3$ as a function of magnetic fluctuation amplitude δB, which is normalized to the magnitude of the background magnetic field. Two data sets are presented. One data set is from [10] in which the

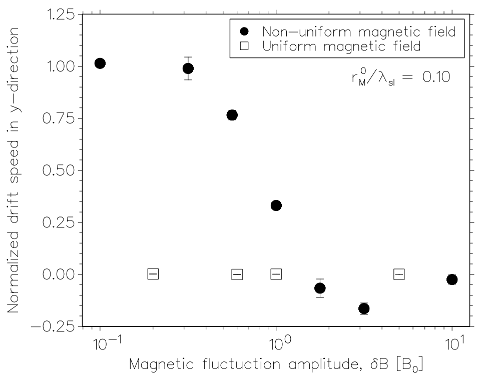

Figure 2: Simulated drift speed v_D normalized to the weak scattering value v_D^{ws} as a function of the magnetic fluctuation amplitude δB, in units of the magnitude of the background magnetic field B_0.

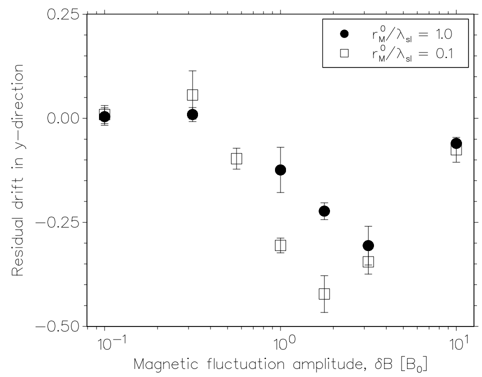

Figure 3: Residual drift as a function of the magnetic fluctuation amplitude δB in units of the magnitude of the background magnetic field B_0, defined to be the difference between the normalized drift coefficient and the normalized drift speed.

large scale magnetic field was taken to be uniform (open symbols), and another data set from more recent simulations [11] in which a non-uniform large-scale magnetic field was used (filled symbols). The particle rigidity was chosen such that the maximal Larmor radii $r_M = 0.1\lambda_{sl}$, where λ_{sl} is the bend-over scale of the slab-turbulence power spectrum. A notable feature from both data sets is that the value of the drift coefficient is independent of the topology of the large scale magnetic field, but only depends on the value of the magnetic field where the coefficients are evaluated. Furthermore, the values for the anti-symmetric drift coefficient lie between the weak-scattering value and zero as equation (2) predicts, becoming small when scattering becomes very strong at large δB.

In Figure 2 we show the simulated drift velocity $\boldsymbol{v}_D$ normalized to the weak-scattering value $\boldsymbol{v}_D^{ws}$. Due to the design of the experiment, the drift was only in the y-direction, and therefore the drift velocity is referred to as drift speed. The sources for the data in this figure are the same as for the data in Figure 1. Here we see that for the case of a uniform large scale magnetic field, the simulated drift speed is zero. This is expected, since there are no gradients in the system that can drive the large scale drift motion of the ensemble. However, for the case of a non-uniform large scale magnetic field the ensemble drifts at the weak-scattering drift speed when the amplitude of the magnetic fluctuations is sufficiently small. At intermediate magnetic fluctuation amplitudes the drift speed is reduced with increasing magnetic fluctuation amplitude, and at some point even changes direction. This change in drift direction is the result of the second term in equation (3) becoming larger than the first term and has been discussed in detail in [11], and has been alluded to by [13]. At large magnetic fluctuation amplitude the drift speed also approaches zero, similar to the drift coefficient, albeit from a negative value.

In Figure 3 we show the difference between the normalized drift coefficient and the normalized drift speed as a function of magnetic fluctuation amplitude. Note that it is valid to take this difference, because both are normalized, dimensionless quantities. We interpret this quantity to be nothing other than the second term on the right hand side of equation (3). The simulations then also seem to support the theoretical assessments of equations (5) and (6). It is also noteworthy that the residual drift appears to be smaller for higher rigidity particles. Extrapolating this feature, there could be an arbitrarily large rigidity at which this residual drift is zero and therefore the drift coefficient and drift velocity will be suppressed by the same amount, when they are non-zero of course.

Conclusion

In conclusion, we note that the effects discussed here are essential to our understanding of the ab initio modulation of cosmic rays in the heliosphere. The way in which large scale drift is included, and subsequently suppressed in modulation models might need revision. It is typical in modulation models to suppress the drift coefficient and drift speed by the same amount, but as shown by the simulation results presented here, this might not be the most realistic thing to do.

Acknowledgements

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