



Influence of the multiple scattering of light on shower images in UHE fluorescence experiments

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Abstract: The paper concerns the images in the fluorescence light of EAS of ultra-high energies. The effect of the multiple scattering of the light in the atmosphere on the way from the shower to the observer is investigated. We show what are the relevant parameters of the geometry for describing this effect. We also show that when analysing the scattered light not delayed too much (with respect to the non-scattered light) the inhomogeneity of the atmosphere can be taken into account quite easily. The calculations are partly analytical, partly numerical, so that some scaling of the effect can be deduced. This is possible when treating the scattered light as a sum of consequent generations (light scattered only once is the first generation, twice - the second, and so on.) The results show that the main contribution to the scattered light gives the first generation (at least for the experimental conditions such as in the Pierre Auger Observatory.) These considerations are necessary then reconstructing shower parameters from the images in the telescopes.

Introduction

A distant shower can be treated as a point source of the isotropic fluorescence light moving with light velocity c along the shower track. At the camera of the fluorescence detector, measuring the angular and temporal distribution of the arriving light, it causes an elongated track of hit pixels (PMTs), each having a small angular field of view. Usually it is assumed that it is only the light emitted in the pixel's field of view which arrives, after some attenuation, at the pixel. However, the attenuation consists also in the scattering of photons produced in the field of view of neighbouring pixels arriving in the pixel considered.

The angular and time distributions of photons scattered in the Rayleigh process in a homogeneous medium are presented in our earlier paper [1]. Here we continue by calculating what is the impact on EAS images when both molecular (Rayleigh) and aerosol (Mie) scattering take place in homogeneous and real atmosphere.

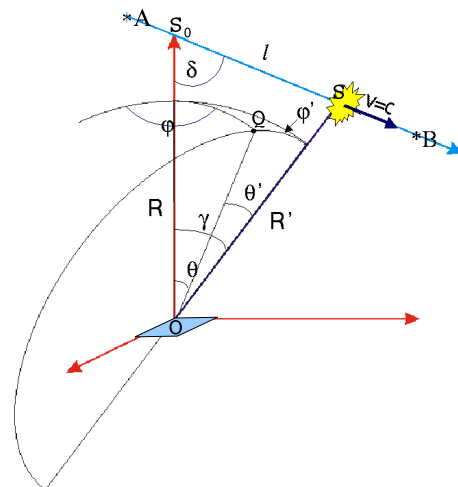


Figure 1: Geometry for a moving point source of light. Detector is at point O . Light from the source at S is scattered at Q .

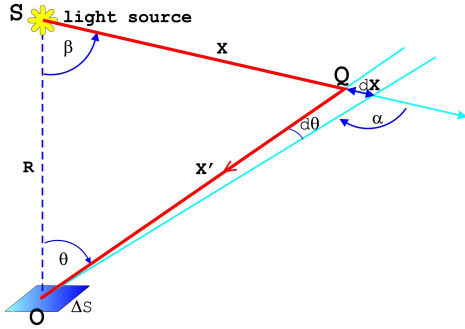


Figure 2: Geometry of the first scattering

The method

The scattered light is a sum of the number of photons n_1 scattered once, n_2 twice, and so on. We calculate analytically the angular and time distributions of consequent generations n_i . We apply our calculations to the fluorescence detector camera in the Auger experiment (a pixel field of view is 1.5°).

Let us consider a moving, isotropic point source. At a distance R from a chosen point S_0 on the track, there is a unit surface, perpendicular to R (Fig.1). The normal to the surface determines $\theta = 0$. Time $t = 0$ corresponds to the moment when the moving source is at point S_0 . Let $J(\theta, \varphi; R, t)d\Omega dt$ be the number of photons arriving in the solid angle $d\Omega(\theta, \varphi)$ in the time interval $(t, t + dt)$ at the unit surface Then:

$$J(\theta, \varphi, t; R) = \frac{1}{2\pi} \int_A^B \frac{d^2 n(\theta', t')}{d\theta' dt'} \cdot \frac{C dl}{\sin\theta'} \quad (1)$$

where $\frac{d^2 n(\theta, t)}{d\theta dt}$ is the total light (all generations) distribution from a point ($\theta = 0$), instantaneous source ($t = 0$); $t' = t - l/c$.

A and B are the limiting points of the shower track and $C dl$ is the number of photons produced along the track element dl . To calculate how many scattered photons arrive at a particular pixel of the detector camera, one has to integrate $J(\theta, \varphi; R, t)$ over its field of view.

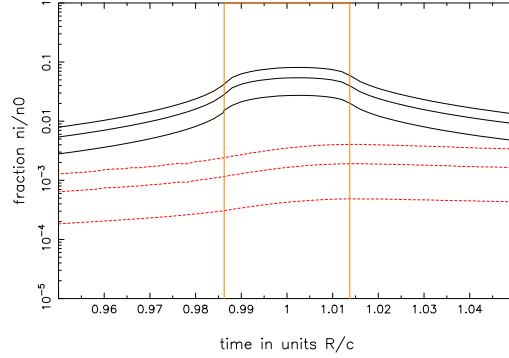


Figure 3: Time distributions of light scattered once (solid lines) and twice (dashed lines) collected by 1.5° camera pixel as a fraction to the direct light. Shower track is perpendicular to the line of sight. Yellow rectangle describes the unscattered component n_0 . Curves are for distances $k = R/\lambda = 3, 2, 1$ from top to bottom. Uniform medium

Uniform atmosphere

We have shown in [1] that if the light is produced by an instant, isotropic point source then the first generation has the following distribution:

$$\frac{d^2 n_1}{d\theta dt} = \frac{c}{\lambda R^2} e^{-ct/\lambda} \cdot f(\alpha) \frac{\cos^2 \frac{\beta+\theta}{2}}{\sin\theta} \quad (2)$$

(see Fig.2), where λ is the total (Rayleigh and Mie) mean scattering path length; $f(\alpha) = \lambda(f_R/\lambda_R + f_M/\lambda_M)$ and f_R and f_M are angular distributions of the scattered (R and M) light [1], [2].

Any next generation n_{i+1} can be calculated using the previous one n_i :

$$\frac{d^2 n_{i+1}(\theta, t)}{d\theta dt} = \frac{2}{\lambda R} \int_0^{\beta_{max}} e^{-x'/\lambda} \cdot r^2 d\beta \cdot \int_0^\pi \frac{dn_i(r, \theta_i, t' - x'/c)}{d\theta_i dt_i} d\theta_i \int_0^\pi f(\alpha) d\varphi \quad (3)$$

Inserting (2) and (3) into (1) we have obtained the time dependence of the signals in a pixel due to first and second generation (third generation is negligible) [3]. Figure 3 shows the time distributions of the scattered photons (ones and twice) collected by a single 1.5° camera pixel for three distances $R/\lambda = k = 1, 2, 3$. The scattered light is presented as a fraction of the direct (unscattered) light. The track of the constant light source is perpendicular

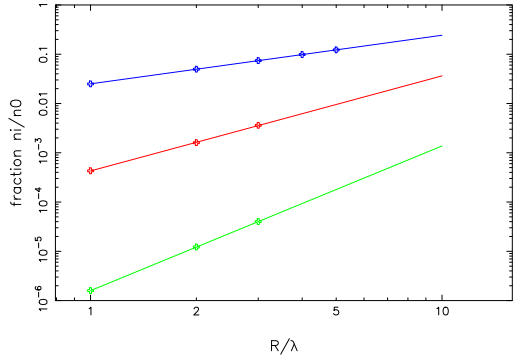


Figure 4: Fraction of the scattered light (one, two and three times) collected by 1.5° camera pixel as a function of $k = R/\lambda$, in time window specified by the direct light arrival. Shower track is perpendicular to the line of sight. Uniform medium

to the line of sight.

In Figure 4 we present the scattered light fraction integrated within time window of direct photons, on the distance R/λ . The fraction of the scattered light scales as $k^i = (R/\lambda)^i$ for the i -th generation. It can be seen that the first generation dominates within distances of detection interest.

Real atmosphere

In the non homogeneous atmosphere we have to include the exponential increase of the mean scattering path length λ with different scale height for Rayleigh and Mie scatterings.

$$\lambda_{R,M}(h) = \lambda_{R,M}(0)e^{h/H_{R,M}} \quad (4)$$

Here, we have adopted the following values: $\lambda_R(0) = 18 \text{ km}$ and $\lambda_M(0) = 15 \text{ km}$; Scale heights: $H_R = 9 \text{ km}$ and $H_M = 1.2 \text{ km}$. Thus, the equation (2) changes to:

$$\frac{d^2 n_1}{d\theta dt} = \frac{c \cdot f_Q(\alpha)}{\lambda_Q R^2} \frac{\cos^2 \frac{\beta+\theta}{2}}{\sin \theta} e^{-\left(\frac{x}{\lambda_{SQ}} + \frac{x'}{\lambda_{QO}}\right)} \quad (5)$$

with total effective mean scattering path lengths:

$$\begin{aligned} \lambda_{SQ} &= \lambda_S \frac{Z_Q - Z_S}{1 - e^{-Z_Q - Z_S}} \\ \lambda_{QO} &= \lambda_O \frac{Z_Q}{1 - e^{-Z_Q}} \end{aligned} \quad (6)$$

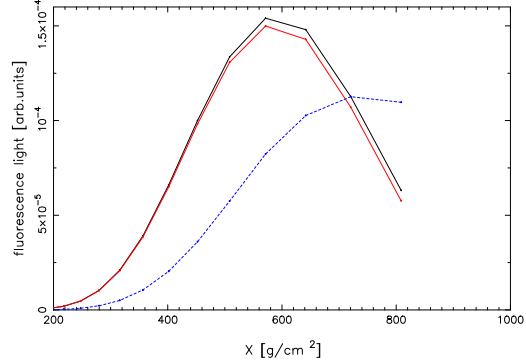


Figure 5: Scattering effect on fluorescence EAS profile. The vertical shower with $X_{max} = 750 \text{ g/cm}^2$ (which corresponds to proton 10^{19} eV), core position distant at $k = 4$. The upper black curve is total light, the red curve is direct light flux when scattered photons are subtracted. Blue dashed curve in the difference multiplied by 20.

where $Z_P = h_P/H$ and h_P is the height of the point P $f_Q(\alpha)$ is the angular distribution of the scattering taking place at point Q . In our calculations we consider only the scattered light which is delayed only a little with respect to the travel time of the unscattered light. Thus, we could assume the azimuthal symmetry of the former, around the latter, what simplifies greatly the calculations of the first generation. We expect, that the contributions of the further generations is small (as is the case of a homogeneous medium) and we have not calculated those.

Figure 5 shows an example illustrating how scattering of the fluorescence photons changes the reconstruction of the detected light profile of an air shower. Here we apply a longitudinal profile (Gaiser-Hillas) of fluorescence photons produced along extensive air shower vertical track with the maximum at $X_{max} = 750 \text{ g/cm}^2$ (primary proton with energy 10^{19} eV), the core position at $k = 4$. The upper black curve represents light collected by the camera, the lower red curve represents direct light flux when one scattering is taken into account. Figure 6 shows time distributions of the light scattered once (as a fraction of the direct light) from the previous vertical shower seen at four elevation angles (0° ; 15° ; 30° and 45°). Signal time range is two times wider than the arrival time window of

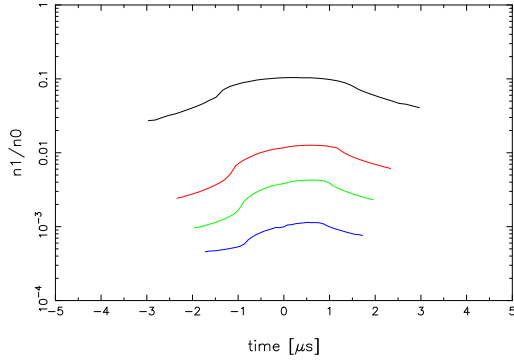


Figure 6: Time distributions of the signal from scattered photons as a fraction to direct light. Curves are for track seen in elevation angles $0^\circ, 15^\circ, 30^\circ, 45^\circ$ from top to bottom.

the direct light. It is easy to see the increase of the scattered light flux within time window of the direct light and change of the time profile due to different pixel - track geometries.

Comparison with Monte-Carlo work

Roberts [2] used a Monte-Carlo approach to describe effects of multiply scattered fluorescence photons from EAS in the real atmosphere. He introduced the following parametrisation of the fraction κ (scattered to total light):

$$\kappa(\%) = 77.4 \cdot (OD \cdot \alpha \cdot R^{1/2} \cdot \zeta^{1.1})^{0.68} \quad (7)$$

where OD is the optical depth and R is the distance in meters, both are between the detector and the source, α is the total scattering coefficient at the source in m^{-1} , ζ is the half angle acceptance in degrees (FOV/2 of the detector pixel).

In Figure 7 we compare his MC results with ours. The points are from our calculations made for 4 cases: vertical (diamonds) and inclined (zenith= 60° ; stars) tracks observed by 1.5° (blue) and 4° (red) field of view.

It can be seen that our results agree quite well with the parametrisation curve. However, there is noticeable dependence of the results on the geometry of the shower with respect to the detector and on the pixel field of view. Thus, these effects should be taken into account.

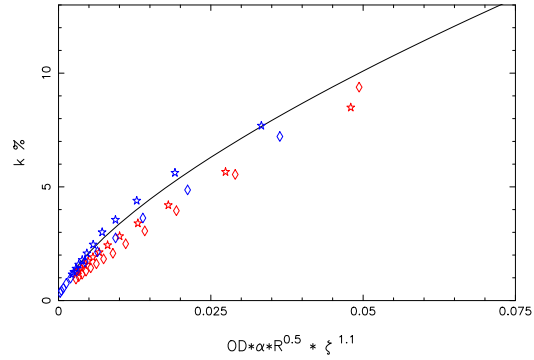


Figure 7: Fraction of the scattered to total light parametrised through equation [7]. Curve from MC work, points are our calculations (see text for details).

Conclusions

We have shown that it is possible to calculate the multiple scattering of light analytically, with some numerical help. Although, the number of scattered photons is not large in case of fluorescence photons from extensive air showers (but it can be $\simeq 10\%$ of the total light for distant showers) our formulas and calculations may be applied to longitudinal profile reconstruction and energy estimation of the showers observed by fluorescence detectors.

Acknowledgement

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References

- [1] Giller M., Smialkowski A., 2005, Proceedings of the 29th International Cosmic Rays Conference, Pune, India
- [2] Roberts M. D., 2005, J. Phys. G: Nucl. Phys. 31, 1291
- [3] Giller M., Smialkowski A., 2006, Proceedings of the 20th European Cosmic Ray Symposium, Lisbon, Portugal