



## A kinetic approach to non resonant modes and growth rates of streaming instability: consequences for shock acceleration

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**Abstract:** We show here that a purely kinetic approach to the excitation of waves by cosmic rays in the vicinity of a shock front leads to predict the appearance of a non-alfvenic fastly growing mode which is the same that was found by Bell (2004) by treating the plasma in the MHD approximation. The kinetic approach we present is more powerful in that it allows us to investigate different models for the compensation of the cosmic ray current in the background upstream plasma.

### Introduction

There are different perspectives from which we can look at the role of supernova remnants (SNRs) as sources of the bulk of cosmic rays: from one, they represent the only class of sources which are energetically viable in terms of hosting the necessary energy, provided about 10 – 20% of the kinetic energy of the expanding shell is converted into accelerated particles. On the other hand it has been known for a long time [1, 2] that the standard model needs to be pushed to its limits and beyond in order to explain the maximum energies which are observed in cosmic rays (about  $10^6$  GeV for protons). Observationally there have been recent claims of detection of strong magnetic fields, as measured from the thickness of the X-ray brightness profiles of several SNRs. These fields would make the acceleration process certainly easier if they rearrange topologically in a way to scatter the particles approximately at the Bohm rate. Both the issues of efficient acceleration and nonlinear amplification of the magnetic field lead to the need to develop a nonlinear theory of particle acceleration at SNR shocks (see [3], these proceedings).

From the theoretical point of view, the most important aspect consists of providing a fundamental explanation of the mechanisms that relate cosmic rays as accelerated particles to the magnetic field amplification which has been observed. The nat-

ural channel seems to be streaming instability, but other mechanisms are also possible.

Here we discuss a kinetic approach to waves excited by the streaming of cosmic rays upstream of the shock and show that there are modes which may be responsible for magnetic field amplification by a factor of order  $\sim 100 - 1000$  for typical parameters of a SNR. A similar conclusion was also obtained by using a different approach in [4].

### Streaming instability and compensating currents

In the reference frame of the shock, cosmic rays are approximately stationary and roughly isotropic. The upstream background plasma moves with a velocity  $v_s$  towards the shock and is made of protons and electrons. The charge of cosmic rays, assumed to be all protons (positive charges) is compensated by processes which depend on the microphysics and need to be investigated accurately. For instance, if  $n_i$  and  $n_e$  are the proton and electron density in the background plasma respectively, then one can compensate the cosmic ray charge  $N_{CRE}$  by either requiring that  $n_i + N_{CR} = n_e$ , thereby assuming that the electrons and protons have different densities (and different velocity with respect to the shock), or we can require that protons and electrons in the background gas have the

same density and velocity and that there is a fourth population of particles, which are cold electrons, at rest in the shock frame which drift together with cosmic rays and cancel their positive charge. The first avenue was adopted in [5], while the second was used in [6, 7]. The latter option resembles more closely the underlying physical approach of [4] where the background gas was however treated in the MHD approximation, and as such described in terms of a single density ( $n_i = n_e$ ). In the reference frame in which cosmic rays are isotropic (namely  $\partial f_{CR}/\partial\mu = 0$ , where  $f_{CR}$  is the distribution function of cosmic rays) the distribution functions of the background ions, electrons and cold electrons are respectively

$$\begin{aligned} f_i(p) &= \frac{n_i}{p^2} \delta(p - m_i v_s) \delta(\mu + 1) \\ f_e(p) &= \frac{n_e}{p^2} \delta(p - m_e v_s) \delta(\mu + 1) \\ f_e^{cold}(p) &= \frac{N_{CR}}{2p^2} \delta(p). \end{aligned}$$

The cosmic ray distribution can also be written as

$$f_{CR}(p) = \frac{N_{CR}}{2} g(p), \quad (1)$$

where  $g(p)$  is a function normalized so that  $\int_0^\infty dp p^2 g(p) = 1$ . The dispersion relation of waves in this composite plasma can be written as [8]:

$$\begin{aligned} \frac{c^2 k^2}{\omega^2} &= 1 + \sum_{\alpha} \frac{2\pi q_{\alpha}^2}{\omega} \int_0^\infty dp \int_{-1}^{+1} d\mu \\ &\frac{p^2 v(p)(1 - \mu^2)}{\omega - kv(p)\mu \pm \Omega_{\alpha}} \left[ \frac{\partial f_{\alpha}}{\partial p} + \left( \frac{kv}{\omega} - \omega \right) \frac{1}{p} \frac{\partial f_{\alpha}}{\partial \mu} \right], \end{aligned}$$

where  $\alpha$  runs over the particle species in the plasma,  $\omega$  is the wave frequency corresponding to the wavenumber  $k$  and  $\Omega_{\alpha}$  is the relativistic gyrofrequency of the particles of type  $\alpha$ , which in terms of the cyclotron frequency and the Lorentz factor is  $\Omega_{\alpha} = \Omega_{\alpha}^*/\gamma$ . For the background plasma and for the cold electrons compensating the cosmic ray charge one has  $\Omega_{\alpha} \approx \Omega_{\alpha}^*$ .

Carrying out the usual algebraic handling of the different terms appearing in Eq. 2 for the four components of the plasma, one obtains the following dispersion relation, as written in the shock frame:

$$\begin{aligned} \frac{c^2 k^2}{\omega^2} &= 1 - \frac{4\pi e^2 n_i}{\omega^2} (\omega + kv_s) \times \\ &\left[ \frac{1}{m_p} \frac{1}{\omega + kv_s \pm \Omega_i^*} + \frac{1}{m_e} \frac{1}{\omega + kv_s \pm \Omega_e^*} \right] \\ &- \frac{4\pi e^2 N_{CR}}{\omega m_e} \frac{1}{\omega \pm \Omega_e^*} + \frac{4\pi e^2 N_{CR}}{\omega} \times \\ &\left\{ \frac{1}{4k} \int_0^\infty \frac{dg}{dp} \left[ \pm 2p_k p + (p_k^2 - p^2) \ln \left| \frac{1 \mp p/p_k}{1 \pm p/p_k} \right| \right] - \right. \\ &\left. - \frac{i\pi}{4k} \int_{Max(p_k, p_0)}^\infty dp \frac{dg}{dp} (p^2 - p_k^2) \right\} \end{aligned}$$

Here we have introduced the momentum  $p_k = m_p \Omega_i^*/k$  which is the minimum momentum of the particles which can resonate with a wave with wavenumber  $k$ . The momentum  $p_0$  is the minimum momentum of the accelerated particles. The above dispersion relation is considerably simplified in the limit of frequency  $\omega$  small compared with  $\Omega_i^*$ . In particular we shall assume that  $\omega + kv_s \ll \Omega_i^* \ll |\Omega_e^*|$ . Moreover we will neglect the displacement current (the unity), so that the dispersion equation reads

$$\begin{aligned} v_A^2 k^2 &= (\omega + kv_s)^2 \pm \omega \Omega_i^* \frac{N_{CR}}{n_i} \left\{ 1 \pm \right. \\ &\left. \frac{p_k}{4} \int_{p_0}^\infty dp \frac{dg}{dp} \left[ \pm 2p_k p + (p_k^2 - p^2) \ln \left| \frac{1 \mp p/p_k}{1 \pm p/p_k} \right| \right] \right. \\ &\left. \mp \frac{i\pi}{4} p_k \int_{Max(p_k, p_0)}^\infty dp \frac{dg}{dp} (p^2 - p_k^2) \right\} \end{aligned}$$

The dispersion relation in the frame of the upstream plasma is obtained by the change  $\omega \rightarrow \omega - kv_s$ . Moreover, neglecting again  $\omega$  with respect to  $kv_s$  we get

$$v_A^2 k^2 = \omega^2 \mp \frac{kv_s \Omega_i^* N_{CR}}{n_i} [1 \pm I_1(k) \pm iI_2(k)]. \quad (2)$$

This dispersion relation is basically the same as that obtained in [4] and has therefore the same implications. The terms proportional to  $I_2$  are those describing the resonant interaction of the waves and particles. The non resonant part is proportional to  $1 + I_1$ .

In the following we investigate these implications in the case of a power law spectrum of cosmic rays  $g(p) \propto (p/p_0)^{-4}$ . In order to have the correct normalization of  $g(p)$  we must require  $g(p) = (1/p_0^3)(p/p_0)^{-4}$ . It follows that the energy density

in the form of accelerated particles is

$$U_{CR} = \frac{N_{CR}}{2} \int_{p_0}^{p_{max}} dp p^3 c g(p) = \frac{N_{CR}}{2} p_0 c \ln R \quad (3)$$

where  $R = p_{max}/p_0$ , and  $p_{max}$  is the maximum momentum of the accelerated particles. The cosmic ray number density can then be written in terms of energy density as

$$N_{CR} = \frac{2U_{CR}}{p_0 c \ln R}. \quad (4)$$

The constant in front of the square brackets in Eq. 2 can therefore be written as

$$\frac{k v_s \Omega_i^* N_{CR}}{n_i} = \zeta \frac{k}{r_{L,0}} v_s^2 \equiv \sigma(k), \quad (5)$$

where  $\zeta = 2\eta(v_s/c)(1/\ln R)$  and  $\eta = U_{CR}/(\rho u^2)$  is the fraction of the kinetic energy that is transformed into cosmic rays at the shock.  $r_{L,0}$  is the gyration radius of protons with momentum  $p_0$ .

The solution for the imaginary part of the frequency from the dispersion relation Eq. 2 can be easily found to be:

$$\omega_I \equiv \text{Im}[\omega] = -\frac{1}{2} (v_A^2 k^2 \pm \sigma(1 \pm I_1)) \pm \frac{1}{2} \sqrt{(v_A^2 k^2 \pm \sigma(1 \pm I_1))^2 + \sigma^2 I_2^2}$$

In Eq. 2, for  $k \sim 1/r_{L,0}$ , the two functions  $I_1(k)$  and  $I_2(k)$  are of order unity. Therefore it is interesting to investigate what happens in this situation when  $\sigma \ll k^2 v_A^2$  and  $\sigma \gg k^2 v_A^2$ .

When  $\sigma \ll k^2 v_A^2$  we can expand Eq. 6 in the ratio  $\sigma/(v_A^2 k^2) \ll 1$  and we get:

$$(\omega_I)_{res} = \frac{\sigma I_2}{2v_A k}. \quad (6)$$

This is the standard result already obtained in previous literature, which implies the (resonant) growth of Alfvén waves weakly driven by the streaming of cosmic rays. It is however instructive to see what the condition  $\sigma \ll k^2 v_A^2$  corresponds to for typical parameters of a supernova. Let us adopt  $B_0 = 1\mu G$ ,  $v_s = 5000$  km/s,  $n = 1$  cm<sup>-3</sup>,  $p_{max} = 10^6$  GeV/c,  $p_0 = 1$  GeV/c. The condition therefore leads to  $\eta \ll 10^{-6}$ . This is a very

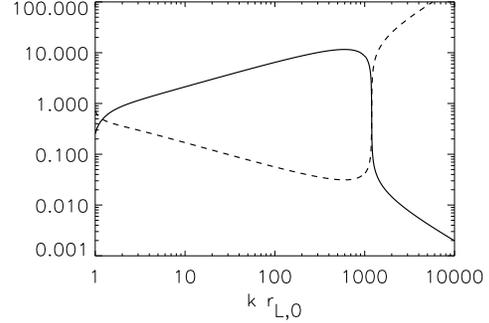


Figure 1: Imaginary part (solid curve) and real part (dashed curve) of the frequency.

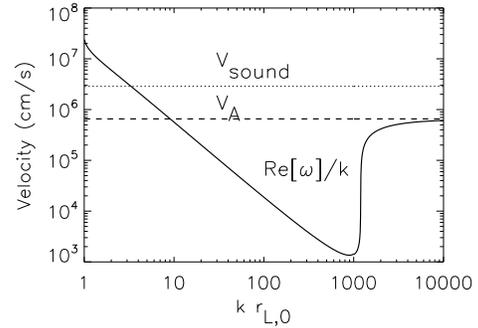


Figure 2: Phase speed of the waves compared with the Alfvén speed and the sound speed.

stringent condition, requiring that only a millionth of the SNR energy is converted into cosmic rays. If this were the case, SNRs would have a negligible role as sources of galactic cosmic rays. It is possible to change the values of the parameters and bring the limit on  $\eta$  up to  $\sim 10^{-3} - 10^{-4}$ , but still it appears that the condition necessary for the growth to be considered as weakly driven by the cosmic ray streaming is too stringent. In other words, the request that  $\eta \sim 0.1$  typically formulated, naturally leads to the conclusion that the wave growth takes place in the opposite regime,  $\sigma \gg k^2 v_A^2$ .

In this case it is straightforward to show that  $\omega_I \approx \sqrt{\sigma}$ .

There are two interesting aspects of this growing mode that deserve some more attention. As al-

ready pointed out in [4], the fact that  $\sigma \gg k^2 v_A^2$  implies that the mode does not resemble an Alfvén wave growing under the effect of cosmic rays. Another point is that the growth takes place also due to the non-resonant interaction of the wave with the streaming cosmic rays.

A more detailed analysis of the modes requires to keep track of the wavenumber  $k$ . The imaginary and real parts of the frequency are shown in Fig. 1 for  $B_0 = 3\mu G$ ,  $V_s = 10^9 \text{cm s}^{-1}$ ,  $\eta = 0.1$ ,  $n_i = 1 \text{cm}^{-3}$ . On the x-axis we plotted  $kr_{L,0}$ , while the y-axis is in units of  $v_s^2/(cr_{L,0})$ . The plot is limited to the most interesting region  $kr_{L,0} > 1$ . The peak in  $Im[\omega]$  corresponds to the maximum growth rate. Note that the highest  $k$  part of the plot corresponds to the case of resonant scattering, and in fact for lower values of the shock speed (already for  $v_s = 10^8 \text{cm s}^{-1}$ ) this is the only mode left. This suggests that at different stages in the evolution of the remnant the non-resonant or the resonant growth will be relevant. Note also that the maximum growth occurs at  $k \gg 1/r_{L,0}$ , namely on scales much smaller than the Larmor radius of the particles. The effect of these magnetic oscillations on the motion of the particles need to be carefully investigated.

One final note is on the phase velocity of the waves: in Fig. 2 we show the phase velocity  $Re[\omega]/k$  as compared with the Alfvén speed and the sound speed (for a temperature of the upstream gas  $T = 10^5 K$ ). We can see that there is a range of values of  $k$  (not the ones where the maximum growth occurs) for which the phase speed is larger than the Alfvén speed and for small  $k$ 's even larger than the sound speed. These supersonic modes are likely to develop shocks in the precursor that might inhibit the acceleration process, also due to the fact that the effective speed of the scattering centers,  $v_s - Re[\omega]/k$  becomes small enough that the acceleration may be substantially suppressed.

## Conclusions

We showed that the growing mode found in [4] also results from a kinetic approach. The dispersion relation leading to the identification of the resonant and non-resonant modes is basically the same in the two approaches. It is also shown that for typ-

ical parameters of a supernova the magnetic field is likely to be substantially amplified in the shock vicinity as due to non-resonant growth of a mode that does not resemble an Alfvén wave weakly modified by cosmic ray streaming. The growth of this mode is faster than the standard resonant growth obtained in the limit of small efficiency of acceleration and may make particle acceleration to high energies easier.

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