Probing QCD at high energy via correlations

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Abstract. A hadron or nucleus at high energy or small x_{Bj} contains many gluons and may be described as a Color Glass Condensate. Angular and rapidity correlations of two particles produced in high energy hadron-hadron collisions is a sensitive probe of high gluon density regime of QCD. Evolution equations which describe rapidity dependence of these correlation functions are derived from a QCD effective action.

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INTRODUCTION

Electron-proton Deeply Inelastic Scattering (DIS) experiments at HERA has revealed very interesting and unexpected results on the growth of parton distribution functions of a proton at small x where x is the fraction of the proton energy carried by a parton. For $Q^2 \ll M_Z^2$, the dominant process is the exchange of a photon with virtuality Q^2 which couples to the quarks in the proton. Variation of the quark structure functions with virtuality of the photon leads to the gluon distribution function. One of the most exciting results from HERA experiments was the rather fast growth of gluon distribution function with decreasing x which led to the development of the field of small x or high energy QCD ($x \equiv \frac{Q^2}{S}$ where S is the virtual photon-hadron center of mass energy squared). A central question in high energy QCD is the behavior of hadronic cross section at high energy where one expects high gluon density effects to unitarize the cross section (at fixed impact parameter) [1, 2]. Roughly speaking when the gluon distribution function is large enough so that $\frac{\alpha_s\, xG(x,Q^2)}{b_t^2\, Q^2} \sim 1$, then a proton or nucleus can be thought of as a dense system of gluons. This relation is satisfied at scale Q_s^2, called the saturation scale [3]. The saturation scale provides a semi-hard scale which allows one to apply weak coupling methods ($\alpha_s(Q_s^2) << 1$) even though it is a non-perturbative problem due to high gluon density effects. This enables one to compute, from first principle, quantities such as number and energy density of gluon produced in a high energy collision, which can not be computed from the standard collinear factorization based pQCD. Therefore, the CGC approach is ideally suited for studying soft or semi-hard processes in high energy QCD as long as the relevant saturation scale is large enough. The CGC approach is based on an effective action formalism [3] which uses a Wilsonian renormalization group to re-sum large quantum corrections to the classical solution of the effective action. To do this, one introduced sources of color charge, ρ, which represent the large x degrees of freedom and couple to the gluon field. To compute an observable, one solves the classical equations of motion at fixed color charge ρ and then averages over all color charges using a weight functional $W[\rho(x_t)]$. This weight functional depends on the rapidity or x and satisfies a non-linear evolution equation called the JIMWLK equation [4].

STRUCTURE FUNCTIONS IN DIS

In small x limit, the DIS total cross section or the structure function F_2 (as well as F_L) can be thought of as a two stage process; first the virtual photon fluctuates into a quark anti-quark pair (a dipole in fundamental representation) which then scatters on the target proton or nucleus. This cross section can symbolically written as

$$F_2(x,Q^2) \sim |\Psi(z,r_t,Q^2)|^2 \otimes T(x,r_t,b_t) \tag{1}$$

where $|\Psi(z,r_t,Q^2)|^2$ is squared of the virtual photon wave function, i.e. the probability for the photon to split into a quark anti-quark pair which are separated in transverse coordinate space by distance r_t, and T is the probability for the dipole with size r_t to scatter on the target at an impact parameter of b_t whereas z is the fraction of the photon energy carried away by the quark. There is a convolution over the dipole size r_t and quark momentum fraction z as well as an integral over the impact parameter b_t. The dipole scattering probability is defines as

$$T(x,r_t,b_t) \equiv \frac{1}{N_c} < \mathrm{tr}\,[1 - V^\dagger(x_t)\,V(y_t)] > \tag{2}$$

where $r_t = x_t - y_t$ and $b_t = \frac{1}{2}(x_t + y_t)$. The x (or energy) dependence of the T matrix comes from inclusion of quantum loop effects which give rise to powers of $\alpha_s\, log 1/x$ and are re-summed by the JIMWLK equation which gives the rapidity evolution of any observable O as

$$\frac{d}{dy}\langle O\rangle = \frac{1}{2}\left\langle \int d^2x\, d^2y\, \frac{\delta}{\delta\alpha_x^b}\,\eta_{xy}^{bd}\,\frac{\delta}{\delta\alpha_y^d}\,O \right\rangle\,, \tag{3}$$

where

$$\eta_{xy}^{bd} = \frac{1}{\pi}\int \frac{d^2z}{(2\pi)^2}\frac{(x-z)\cdot(y-z)}{(x-z)^2(y-z)^2}\left[1 + U_x^\dagger U_y - U_x^\dagger U_z - U_z^\dagger U_y\right]^{bd} \tag{4}$$

This equation can be used to derive an evolution equation for the dipole scattering probability and gives

$$\frac{d}{dy}\langle \mathrm{tr}\, V_r^\dagger\, V_s\rangle = -\frac{N_c\,\alpha_s}{2\pi^2}\int d^2z \frac{(r-s)^2}{(r-z)^2(s-z)^2}\left\langle \mathrm{tr}\, V_r^\dagger\, V_s - \frac{1}{N_c}\mathrm{tr}\, V_r^\dagger\, V_z\ \mathrm{tr}\, V_s\, V_z^\dagger \right\rangle \tag{5}$$

where $y \equiv log 1/x$ and r,s are transverse coordinates of the original quark anti-quark dipole and z is the transverse coordinate of the radiated gluon after one step in evolution. This equation is not a closed form equation in the sense that it couples the evolution of expectation value of two Wilson lines to the expectation value of a higher number of Wilson lines. It can and has been solved numerically using lattice gauge theory techniques. Nevertheless, to gain some insight, it is useful to make a large N_c and mean field approximation in order to reduce it to a closed form equation which is known as the Balitsky-Kovchegov (BK) equation [5]:

$$\frac{d}{dy}S(r-s) = -\frac{N_c\,\alpha_s}{2\pi^2}\int d^2z\,\frac{(r-s)^2}{(r-z)^2(s-z)^2}\left[S(r-s)-S(r-z)\,S(z-s)\right] \tag{6}$$

where $S(r-s)$ is defined as

$$S(r-s) \equiv \frac{1}{N_c} < \mathrm{tr}\,V_r^\dagger\,V_s > \tag{7}$$

The S matrix is the basic building block that appears in many processes computed in the CGC formalism, such as the inclusive structure functions in DIS and single particle production in proton-nucleus collisions in the forward rapidity region. For instance, single hadron production cross section in pA collisions can be written as [6, 7]

$$\begin{aligned}\frac{d\sigma^{pA\to hX}}{dY\,d^2P_t\,d^2b} \;=\; & \frac{1}{(2\pi)^2}\int_{x_F}^1 dx\,\frac{x}{x_F}\Big\{f_{q/p}(x,Q^2)\;S[\frac{x}{x_F}P_t,b]\;D_{h/q}(\frac{x_F}{x},Q^2)+\\ & f_{g/p}(x,Q^2)\;S_A[\frac{x}{x_F}P_t,b]\;D_{g/h}(\frac{x_F}{x},Q^2)\Big\}\end{aligned} \tag{8}$$

where S and S_A are the fundamental and adjoint dipoles and f and D are the parton distribution functions of the incoming proton and fragmentation functions of the outgoing partons. Quite recently, Next to Leading Order corrections to the BK equation have recently been computed [8]. The biggest contribution is due to the running coupling constant which changes the evolution kernel. The solution to the BK equation with running coupling has been used to fit the single hadron production in deuteron-nucleus collisions at RHIC at rapidities of $y \geq 2.2$ and can be taken as a strong indication for the validity of application of CGC formalism to small x processes [9]. The main theoretical uncertainty is the role of large x (from the projectile) partons scattering from the target and possibly losing energy, usually referred to as cold matter energy loss [10]. It will be helpful to compare the CGC predictions with those of other approaches for electromagnetic processes [11] since in the CGC approach electromagnetic processes can be related to single hadron production in pA collisions [12]. For more details, we refer the reader to some recent reviews on CGC and its applications [13].

Di-jet production in DIS and proton-nucleus collisions

While fully inclusive quantities such as structure functions, and single inclusive particle production in proton-nucleus (pA) collisions involve dipoles as the basic building blocks of the cross section, di-jet production in either DIS or pA collisions involve higher point correlators of Wilson lines. For instance, quark anti-quark as well as two gluon production in DIS involve correlators of at least four Wilson lines, knows as a quadrupole [12, 14]. Here we focus on the structure of the higher point correlators on

Wilson lines that appear in quark-gluon production in pA collisions [14, 15]. The production cross section involves the following correlators, which we call O_4 and O_6

$$O_4(r,\bar{r}:s) \equiv \mathrm{tr}\, V_r^\dagger\, t^a\, V_{\bar{r}}\, t^b\, [U_s]^{ab} = \frac{1}{2}\left[\mathrm{tr}\, V_r^\dagger\, V_s\;\; \mathrm{tr}\, V_{\bar{r}}\, V_s^\dagger - \frac{1}{N_c}\mathrm{tr}\, V_r^\dagger\, V_{\bar{r}}\right] \tag{9}$$

and

$$O_6(r,\bar{r}:s,\bar{s}) \equiv \mathrm{tr}\, V_r\, V_{\bar{r}}^\dagger\, t^a\, t^b\, [U_s\, U_{\bar{s}}^\dagger]^{ba} = \frac{1}{2}\left[\mathrm{tr}\, V_r\, V_{\bar{r}}^\dagger\, V_{\bar{s}} V_s^\dagger\;\; \mathrm{tr}\, V_s\, V_{\bar{s}}^\dagger - \frac{1}{N_c}\mathrm{tr}\, V_r\, V_{\bar{r}}^\dagger\right] \tag{10}$$

where the following identity between adjoint and fundamental matrices is used

$$U^{ab}\, t^b = V^\dagger\, t^a\, V. \tag{11}$$

In order to determine the x dependence of these higher point correlators, one needs to solve the JIMWLK equation for the weight functional $W[\rho\,(x_t)]$ which can then be used to evaluate these correlators at the rapidity of interest. However, this has not been done yet. Therefore, here we write down the explicit evolution equations that these correlators satisfy [16]:

$$\begin{aligned}
\frac{d}{dy}\langle O_4(r,\bar{r}:s)\rangle = &-\frac{N_c\,\alpha_s}{(2\pi)^2}\int d^2z \Bigg\langle 2\left[\frac{(r-s)^2}{(r-z)^2(s-z)^2} + \frac{(\bar{r}-s)^2}{(\bar{r}-z)^2(s-z)^2}\right] O_4(r,\bar{r}:s) \\
- \quad & \frac{1}{N_c}\Bigg[\frac{(r-s)^2}{(r-z)^2(s-z)^2}\,\mathrm{tr}\, V_r^\dagger\, V_z\;\; \mathrm{tr}\, V_s^\dagger\, V_{\bar{r}}\;\; \mathrm{tr}\, V_z^\dagger\, V_s + \\
& \frac{(\bar{r}-s)^2}{(\bar{r}-z)^2(s-z)^2}\mathrm{tr}\, V_r^\dagger\, V_s\;\; \mathrm{tr}\, V_z^\dagger\, V_{\bar{r}}\;\; \mathrm{tr}\, V_s^\dagger\, V_z\; - \\
& \frac{1}{2}\left[\frac{(r-s)^2}{(r-z)^2(s-z)^2} + \frac{(\bar{r}-s)^2}{(\bar{r}-z)^2(s-z)^2} - \frac{(r-\bar{r})^2}{(r-z)^2(\bar{r}-z)^2}\right] \\
& \left[\mathrm{tr}\, V_r^\dagger\, V_z\, V_s^\dagger\, V_{\bar{r}}\, V_z^\dagger\, V_s + \mathrm{tr}\, V_r^\dagger\, V_s\, V_z^\dagger\, V_{\bar{r}}\, V_s^\dagger\, V_z\right]\Bigg] \\
+ \quad & \frac{1}{N_c^2}\,\frac{(r-\bar{r})^2}{(r-z)^2(\bar{r}-z)^2}\;\mathrm{tr}\, V_r^\dagger\, V_z\;\; \mathrm{tr}\, V_z^\dagger\, V_{\bar{r}}\Bigg\rangle
\end{aligned} \tag{12}$$

We note that the leading N_c part of the above evolution equation involves only products of dipoles (when properly normalized) whereas the sub-leading N_c parts involve correlators of larger number of Wilson lines. Since O_4 appears explicitly in the evolution equation for the dipole scattering probability, *one concludes that with leading N_c accuracy, structure functions in DIS and single particle production are sensitive only to dipoles which satisfy the BK evolution equation*. We now consider the evolution equation for O_6 which also appears in di-jet production cross section, in both DIS and proton-nucleus collisions. It is given by

$$\frac{d}{dy}\langle O_6(r,\bar{r}:s,\bar{s})\rangle = -\frac{N_c\,\alpha_s}{2(2\pi)^2}\int d^2z \Big\langle 2\,\Big[\frac{(r-s)^2}{(r-z)^2(s-z)^2}+\frac{(r-\bar{r})^2}{(r-z)^2(\bar{r}-z)^2}+$$

$$\frac{(\bar{r}-\bar{s})^2}{(\bar{r}-z)^2(\bar{s}-z)^2}+3\frac{(s-\bar{s})^2}{(s-z)^2(\bar{s}-z)^2}\Big]\,O_6(r,\bar{r}:s,\bar{s})-\frac{1}{N_c}\Bigg[$$

$$\Big[\frac{(r-\bar{r})^2}{(r-z)^2(\bar{r}-z)^2}+\frac{(r-s)^2}{(r-z)^2(s-z)^2}-\frac{(s-\bar{r})^2}{(s-z)^2(\bar{r}-z)^2}\Big]\,\mathrm{tr}\,V_z V_{\bar{r}}^\dagger V_{\bar{s}} V_s^\dagger\;\,\mathrm{tr}\,V_r V_z^\dagger\;\,\mathrm{tr}\,V_s V_{\bar{s}}^\dagger$$

$$+\Big[\frac{(r-\bar{r})^2}{(r-z)^2(\bar{r}-z)^2}+\frac{(\bar{r}-\bar{s})^2}{(\bar{r}-z)^2(\bar{s}-z)^2}-\frac{(r-\bar{s})^2}{(r-z)^2(\bar{s}-z)^2}\Big]\,\mathrm{tr}\,V_r V_z^\dagger V_{\bar{s}} V_s^\dagger\;\,\mathrm{tr}\,V_z V_{\bar{r}}^\dagger\;\,\mathrm{tr}\,V_s V_{\bar{s}}^\dagger$$

$$+\Big[\frac{(r-s)^2}{(r-z)^2(s-z)^2}+\frac{(s-\bar{s})^2}{(s-z)^2(\bar{s}-z)^2}-\frac{(r-\bar{s})^2}{(r-z)^2(\bar{s}-z)^2}\Big]\,\mathrm{tr}\,V_r V_{\bar{r}}^\dagger V_{\bar{s}} V_z^\dagger\;\,\mathrm{tr}\,V_z V_s^\dagger\;\,\mathrm{tr}\,V_s V_{\bar{s}}^\dagger$$

$$+\Big[\frac{(\bar{r}-\bar{s})^2}{(\bar{r}-z)^2(\bar{s}-z)^2}+\frac{(s-\bar{s})^2}{(s-z)^2(\bar{s}-z)^2}-\frac{(\bar{r}-s)^2}{(\bar{r}-z)^2(s-z)^2}\Big]\,\mathrm{tr}\,V_r V_{\bar{r}}^\dagger V_z V_s^\dagger\;\,\mathrm{tr}\,V_{\bar{s}} V_z^\dagger\;\,\mathrm{tr}\,V_s V_{\bar{s}}^\dagger$$

$$+2\frac{(s-\bar{s})^2}{(s-z)^2(\bar{s}-z)^2}\,\mathrm{tr}\,V_r V_{\bar{r}}^\dagger V_{\bar{s}} V_s^\dagger\;\,\mathrm{tr}\,V_s V_z^\dagger\,\mathrm{tr}\,V_z V_{\bar{s}}^\dagger$$

$$+\Big[\frac{(\bar{r}-s)^2}{(\bar{r}-z)^2(s-z)^2}-\frac{(\bar{r}-\bar{s})^2}{(\bar{r}-z)^2(\bar{s}-z)^2}-\frac{(r-s)^2}{(r-z)^2(s-z)^2}+\frac{(r-\bar{s})^2}{(r-z)^2(\bar{s}-z)^2}\Big]$$

$$\mathrm{tr}\,V_r V_s^\dagger\;\,\mathrm{tr}\,V_{\bar{r}}^\dagger V_{\bar{s}}\;\,\mathrm{tr}\,V_s V_{\bar{s}}^\dagger$$

$$-\Big[-\frac{(\bar{r}-s)^2}{(\bar{r}-z)^2(s-z)^2}-\frac{(r-\bar{s})^2}{(r-z)^2(\bar{s}-z)^2}+\frac{(r-\bar{r})^2}{(r-z)^2(\bar{r}-z)^2}+\frac{(s-\bar{s})^2}{(s-z)^2(\bar{s}-z)^2}\Big]$$

$$\mathrm{tr}\,V_r V_{\bar{r}}^\dagger\,\mathrm{tr}\,V_{\bar{s}} V_s^\dagger\;\,\mathrm{tr}\,V_s V_{\bar{s}}^\dagger$$

$$-\Big[\frac{(r-s)^2}{(r-z)^2(s-z)^2}-\frac{(r-\bar{s})^2}{(r-z)^2(\bar{s}-z)^2}+\frac{(s-\bar{s})^2}{(s-z)^2(\bar{s}-z)^2}\Big]\,\mathrm{tr}\,V_r V_{\bar{r}}^\dagger V_{\bar{s}} V_s^\dagger V_z V_{\bar{s}}^\dagger V_s V_z^\dagger$$

$$+\Big[\frac{(\bar{r}-s)^2}{(\bar{r}-z)^2(s-z)^2}-\frac{(\bar{r}-\bar{s})^2}{(\bar{r}-z)^2(\bar{s}-z)^2}-\frac{(s-\bar{s})^2}{(s-z)^2(\bar{s}-z)^2}\Big]\,\mathrm{tr}\,V_r V_{\bar{r}}^\dagger V_z V_{\bar{s}}^\dagger V_s V_z^\dagger V_{\bar{s}} V_s^\dagger$$

$$+\Big[\frac{(\bar{r}-s)^2}{(\bar{r}-z)^2(s-z)^2}-\frac{(\bar{r}-\bar{s})^2}{(\bar{r}-z)^2(\bar{s}-z)^2}-\frac{(r-s)^2}{(r-z)^2(s-z)^2}+\frac{(r-\bar{s})^2}{(r-z)^2(\bar{s}-z)^2}\Big]$$

$$\mathrm{tr}\,V_r V_z^\dagger V_s V_{\bar{s}}^\dagger V_z V_{\bar{r}}^\dagger V_{\bar{s}} V_s^\dagger - 2\frac{(s-\bar{s})^2}{(s-z)^2(\bar{s}-z)^2}\,\mathrm{tr}\,V_r V_{\bar{r}}^\dagger V_{\bar{s}} V_z^\dagger V_s V_{\bar{s}}^\dagger V_z V_s^\dagger$$

$$-\Big[\frac{(\bar{r}-s)^2}{(\bar{r}-z)^2(s-z)^2}-\frac{(\bar{r}-\bar{s})^2}{(\bar{r}-z)^2(\bar{s}-z)^2}-\frac{(r-s)^2}{(r-z)^2(s-z)^2}+\frac{(r-\bar{s})^2}{(r-z)^2(\bar{s}-z)^2}\Big]$$

$$\mathrm{tr}\,V_r V_{\bar{s}}^\dagger V_s V_{\bar{r}}^\dagger V_{\bar{s}} V_s^\dagger$$

$$-\Big[\frac{(\bar{r}-s)^2}{(\bar{r}-z)^2(s-z)^2}+\frac{(r-\bar{s})^2}{(r-z)^2(\bar{s}-z)^2}-\frac{(s-\bar{s})^2}{(s-z)^2(\bar{s}-z)^2}\Big]\,\mathrm{tr}\,V_r V_{\bar{r}}^\dagger\Bigg]$$

$$+ \quad \frac{1}{N_c^2}\frac{(r-\bar{r})^2}{(r-z)^2(\bar{r}-z)^2}\,\mathrm{tr}\,V_r^\dagger V_z\ \mathrm{tr}\,V_z^\dagger V_{\bar{r}}\Big\rangle - \frac{1}{4N_c}\frac{d}{dy} < \mathrm{tr}\,V_r^\dagger V_{\bar{r}} > . \tag{13}$$

We note the appearance of the quadrupole terms involving four Wilson lines of the form, $\mathrm{tr}\,V_z V_{\bar{r}}^\dagger V_{\bar{s}} V_s^\dagger$ for the first time. These terms can not be written as products of two dipoles [17] and as is clear from above, would evolve with energy differently differently (for an interesting limit where these expression simplify, see [18]). We also note that there are a large number of sub-leading N_c terms appearing on the right hand side of the equation. This may lead to significant violation of large N_c approximation for di-jet production.

It is also worth noting appearance of the dipole scattering probability, proportional to O_2 on the right hand side of the equation. This is intriguing since in the evolution of all other correlators of Wilson lines only higher point correlators of Wilson lines appear on the right hand side of the evolution equation. This has some resemblance to the phenomenon of "pomeron loops" [19] where in the evolution equation for the two point function square root of the two point function appears on the right hand side. Here, the origin of this term seems to be purely kinematic.

The most interesting aspect of these higher point correlators may be their energy dependence [20]. If one assumes, as naively expected, that a four point functions would evolve faster with rapidity than a two point function (but slower than square of two point function), then the N_c suppression of these terms can eventually be compensated by the stronger energy dependence. In passing, we note that photon-hadron correlations in pA offer a unique observable where one can investigate the angular dependence of the production and that the production cross section involves only dipoles [21].

Two hadron production in DIS or proton-nucleus collisions at high energy therefore offer a unique opportunity to go beyond the dipole approximation and to probe the correlators of Wilson lines, the effective degrees of freedom in CGC. Since there is new and exciting data on di-jet correlations in the forward rapidity region at RHIC, there is a need to solve the JIMWLK equation in order to make a quantitative comparison between the predictions of CGC and the experimental data. The LHC will also soon measure these correlations at even smaller x which would further probe the dynamics of gluon saturation and CGC.

Another interesting phenomenon where higher point correlators of Wilson lines appear is in the long range rapidity correlations measured in heavy ion and proton-proton collisions. This correlation must be generated very early after the collision due to causality [22] and survives the subsequent final state re-scatterings present in the Quark-Gluon Plasma generated in heavy ion collisions. Even though calculation of gluon production in heavy ion collisions can not be carried out analytically, one can consider the high p_t region (just above Q_s) where a k_t factorized form of the cross section may be written down. This cross section will also involve higher point functions of the produced gluon fields [23] which can not be written as products of two-point functions. Very recent observation of this long range rapidity correlation, also known as the ridge, by the CMS collaboration at the LHC [24] has generated much excitement. While a quantitative understanding of the data requires detailed modeling, qualitative features of the data agree with the CGC calculations [25].

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