Conjecture about the 2-Flavour QCD Phase Diagram: preliminary results

M. A. Nava Blanco[1] W. Bietenholz[2] A. Fernández Téllez[1]

[1]FCFM-BUAP

[2]ICN-UNAM

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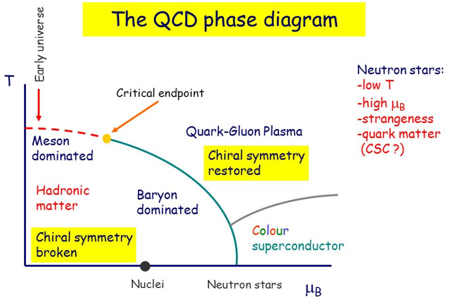
Motivation



Hypothetical QCD phase diagram.

- The QCD phase diagram is one of the most prominent outstanding mysteries within the Standard Model of particle physics.
- Quarks do have masses, but two flavours are very light compared to the intrinsic scale $\Lambda_{QCD} = 341(12)$MeV. (Bruno et al., '16)

3d O(4) non-linear σ-model

- In QCD with 2 flavors, chiral symmetry breaks spontaneously as $SU(2)_L \times SU(2)_R \to SU(2)_{L=R}$. This is isomorphic to $O(4) \to O(3)$, and high T induces dimensional reduction. Therefore, 2-flavor QCD at the chiral phase transition is assumed to belong to the universality class of the 3d O(4) model ("Rajagopal/Wilczek '93"), whose Hamilton function is

$$\mathcal{H} = -\sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j , \qquad \vec{S}_i \in S^3 ,$$

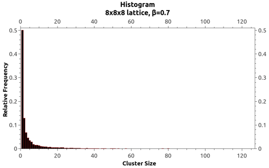
where $\langle i, j \rangle$ means the nearest-neighbor sites on a 3d cubic lattice.

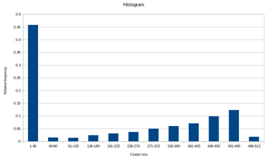
3d O(4) model on an $L \times L \times L$ lattice

Goal: generate field configurations $[\vec{S}] \propto \exp(-\beta \mathcal{H}[\vec{S}])$ for the importance sampling of the functional integral.

Wolff cluster algorithm: choose a random 3d reflection subspace in the 4d spin space. Choose randomly a site i in the lattice. Visit all neighbour sites j of i. Their spins join the cluster with a certain probability. Repeat until the cluster does not grow anymore. Apply the reflection to all spins in the cluster.

Cluster size distributions on a $8 \times 8 \times 8$ lattice at $\beta = 0.7$ and $\beta = 1.5$; the latter leads to larger clusters.

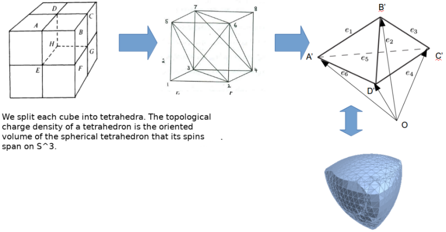




Topological charge

The winding number of a field configuration on the sphere S^3 is its topological charge $Q \in Z$. In our low energy effective theory, it represents the baryon number.

Sketch of 3d O(4) non-linear sigma model



We split each cube into tetrahedra. The topological charge density of a tetrahedron is the oriented volume of the spherical tetrahedron that its spins span on S^3.

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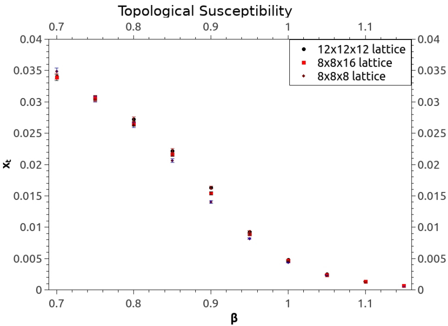
Topological susceptibility.

The definition of topological susceptibility is

$$\chi_t = \frac{\langle Q^2 \rangle}{V} \, ,$$

where V is the volume. At large β, there are only few top. windings, which are exponentially suppressed, as one expects for a dilute instanton gas. Below $\beta_c \simeq 0.936$ ... the dependence ceases to be exponential - here the top. windings are quite dense, i.e. not dilute anymore.

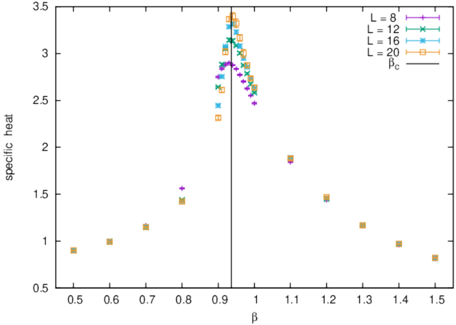


Specific heat

The definition of the specific heat is

$$c = \frac{\beta^2}{V}(\langle \mathcal{H}^2 \rangle - \langle \mathcal{H} \rangle^2)$$

We confirm a peak between $\beta = 0.9$ and 0.95, where simulations indicate a phase transition of second order at $\beta_c = 0.93590$ (Engels et al., '03).



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Magnetic susceptibility

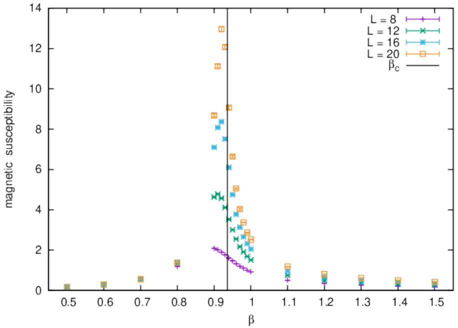
The definition of magnetization is

$$\vec{M} = \sum_x \vec{S_x}$$

from where one defines the magnetic susceptibility

$$\chi_m = \frac{\beta}{V}(\langle M^2 \rangle - \langle |M| \rangle^2).$$

We can see the expected divergence in χ_m increases. In the limit $V \to \infty$ this peak turns into a divergence due to the phase transition between $\beta = 0.9$ and 0.95. The peak is enhanced when the volume increases. Thus as one approaches to the limit $V \to \infty$, this peak turns into a divergence.



Correlation lengths

The correlation function $C(r)$ is given by

$$C(r) = \langle \vec{S}_{x3} \cdot \vec{S}_{x3+r} \rangle \propto \cosh\left(\frac{r - L/2}{\xi}\right),$$

where ξ is the correlation length and $\vec{S}_{x3} = \frac{1}{L^2} \sum_{x1,x2} \vec{S}_x$.

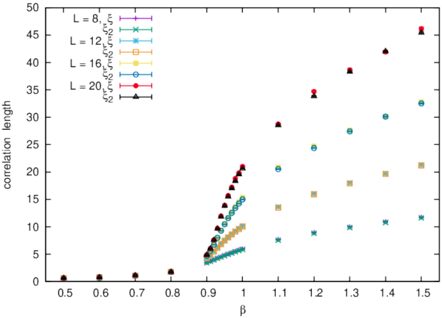
The second moment correlation length is

$$\xi_2 = \left(\frac{(\chi_m/\mathcal{F}) - 1}{4\sin^2(\pi/L)}\right)^{1/2},$$

where

$$\mathcal{F} = \frac{1}{V}\sum_{x,y} \langle \vec{e}_x \cdot \vec{e}_y \rangle \cos\left(\frac{2\pi(x_1 - y_1)}{L}\right).$$

The physical size is L/ξ. Therefore these data points are affected by strong finite-size effects, except for the data at the lowest β-values. This is why the divergence of ξ at β_c is not yet visible - it will show up in larger volumes.



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3d O(4) model with baryon density

The 3d O(4) model has been simulated successfully; the results agree with the expected 2nd order phase transition, and the topological charge was measured.
Next the baryonic chemical potential μ_B will be included,

$$\mathcal{H} = -\sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j + \mu_B Q[\vec{S}]$$

which will reveal the phase diagram of the effective theory at finite baryon density.

Sign problem: the inclusion of μ_B attaches an imaginary part to the Euclidean action S_{QCD}

Baryonic density in Monte Carlo simulation

1. Build the single cluster using the Wolff algorithm without considering the new term.
2. Compute the charge Q for the present configuration, $Q[\vec{S}]$, and for the configuration which would emerge when this flip is performed, $Q[\vec{S}']$.
3. Perform a *Metropolis* accept/reject step to decide if flip the cluster or not assuming $\mu_B > 0$.

Metropolis step:

- If the flip decreases Q, flip the cluster.
- If the flip increases Q, flip the cluster with probability $p = \exp(-\mu_B(Q[\vec{S}'] - Q[\vec{S}]))$

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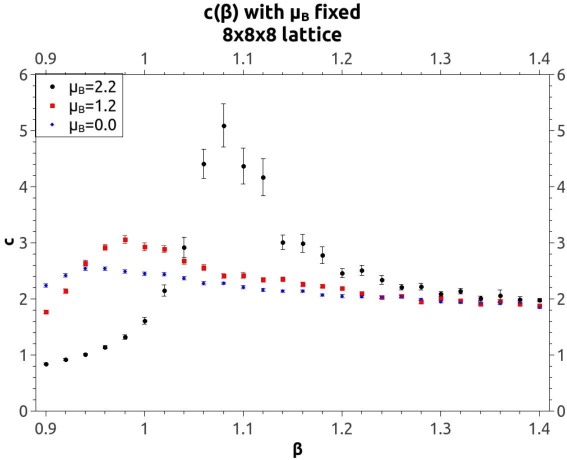
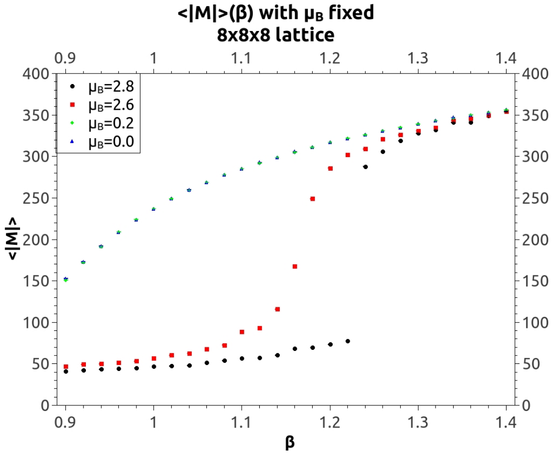
Preliminar results I



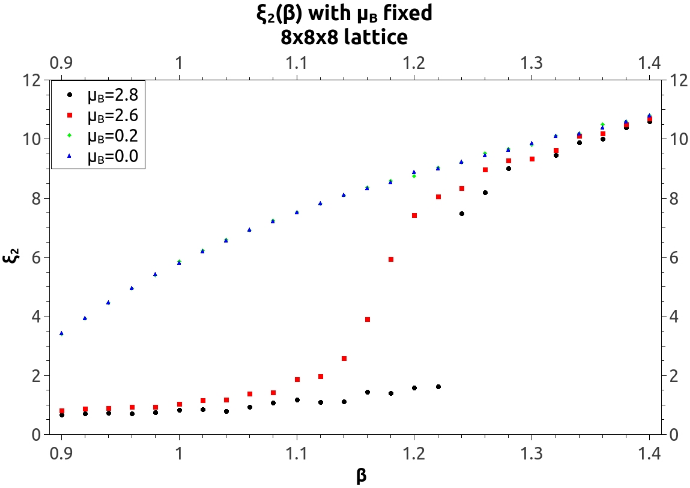
Observe the change in β_c as we rise μ_B.

Preliminar results II



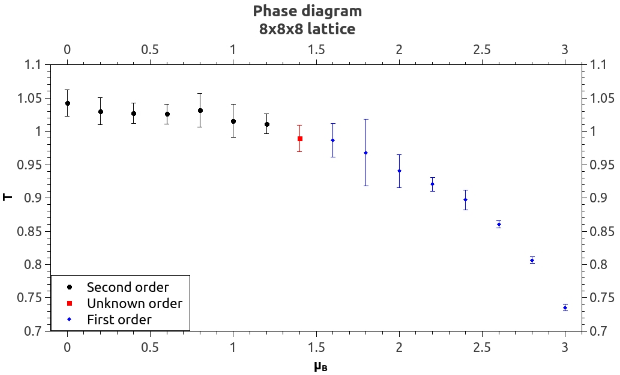
Observe the change in behavior of the plots with different μ_B.

Preliminar results III



Behaviour of ξ_2. Observe the likeness in behaviour with the parameter order.

Preliminar results IV



Phase diagram for the σ non-linear $3d\ O(4)$ model

Conclusions and Outlook

A preliminary result for the phase diagram of the $3d\ O(4)$ model has been obtained. To confirm this result L will be risen.
In a future work, the masses of the quarks will be included in this model.

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THANK YOU