Phase transition dynamics and gravitational waves

Ariel Mégevand

Universidad Nacional de Mar del Plata
Argentina

XIII MEXICAN SCHOOL OF PARTICLES AND FIELDS
October 2008
Outline

Motivation
  First-order phase transitions
  Gravitational waves

Phase transition dynamics
  Thermodynamics
  Bubble nucleation
  Bubble growth

Gravitational waves from a first-order phase transition
  Turbulence in a first-order phase transition
  Gravitational waves from turbulence

Phase transition dynamics and gravitational waves
  GWs from detonations and deflagrations
  Global treatment of deflagration bubbles
  Results
Motivation

Phase transition dynamics

Gravitational waves from a first-order phase transition

Phase transition dynamics and gravitational waves
First-order phase transitions

Phase transition dynamics

- supercooling
- nucleation and expansion of bubbles
- bubble collisions
- departure form equilibrium

Possible consequences

- topological defects, magnetic fields
- baryogenesis, inhomogeneities
- cosmological constant
- gravitational waves (GWs)
Gravitational waves
from first-order phase transitions

- Since GWs propagate freely, they may provide a direct source of information about the early Universe.

The spectrum

- The characteristic wavelength of the gravitational radiation is determined by the characteristic length of the source.
- The characteristic length is the size of bubbles, which depends on the phase transition dynamics and the Hubble length $H^{-1}$.
- For the electroweak phase transition, the characteristic frequency, redshifted to today, is $\sim$ milli-Hertz.
- This is within the sensitivity range of the planned Laser Interferometer Space Antenna (LISA).
Motivation

Phase transition dynamics

Gravitational waves from a first-order phase transition

Phase transition dynamics and gravitational waves
Thermodynamics

The free energy

Thermodynamic quantities ($\rho$, $p$, $s$, ...) are derived from the free energy density (finite-temperature effective potential).

Example:

A theory with a Higgs field and particle masses $m_i(\phi)$

$$\mathcal{F}(\phi, T) = V_0(\phi) + V_{1\text{-loop}}(\phi, T),$$

$V_0(\phi) =$ tree-level potential

$V_{1\text{-loop}}(\phi, T) =$ zero-temperature corrections $+$ finite-temperature corrections
The effective potential

\[ \mathcal{F}(\phi, T) = V_0(\phi) + V_{1\text{-loop}}(\phi, T), \]

where

\[ V_0(\phi) = -\frac{1}{2} \lambda \nu^2 \phi^2 + \frac{1}{4} \lambda \phi^4 \]

\[ V_{1\text{-loop}}(\phi) = \sum \frac{\pm g_i}{64 \pi^2} \left[ m_i^4(\phi) \left( \log \left( \frac{m_i^2(\phi)}{m_i^2(\nu)} \right) - \frac{3}{2} \right) + 2 m_i^2(\phi) m_i^2(\nu) \right] \]

\[ + \sum \frac{g_i T^4}{2 \pi^2} I_\mp \left[ \frac{m_i(\phi)}{T} \right] \]

with

\[ I_\mp(x) = \pm \int_0^\infty dy \ y^2 \log \left( 1 \mp e^{-\sqrt{y^2+x^2}} \right) \]
First-order phase transition

Figure: The free energy $\mathcal{F}(\phi, T)$ around the critical temperature

High $T$: $\phi = 0$ \textit{(false vacuum)}
Low $T$: $\phi = \phi_m(T)$ \textit{(true vacuum)}

$T_c =$ critical temperature
First-order phase transition

Thermodynamic quantities are different in each phase

\[ T > T_c: \quad \Rightarrow \quad \mathcal{F}(\phi = 0, T) \equiv \mathcal{F}_+(T) \quad \Rightarrow \quad \rho_+, s_+, p_+, \ldots \]
\[ T < T_c: \quad \Rightarrow \quad \mathcal{F}(\phi_m(T), T) \equiv \mathcal{F}_-(T) \quad \Rightarrow \quad \rho_-, s_-, p_-, \ldots \]

High-temperature phase \( \phi = 0 \)

- Energy density: \( \rho_+(T) = \rho_\Lambda + g_* \pi^2 T^4 / 30 \)
  \[ = \text{false vacuum + radiation} \]

Low-temperature phase \( \phi = \phi_m(T) \)

- \( \rho_-(T) \) depends on the effective potential
First-order phase transition

Discontinuities at $T = T_c$

- At the critical temperature, $\mathcal{F}_+(T_c) = \mathcal{F}_-(T_c)$, but $\rho_+(T_c) > \rho_-(T_c)$.
- $L \equiv \rho_+(T_c) - \rho_-(T_c) =$ latent heat.

The latent heat

- $L$ is released during bubble expansion.
- Should not be confused with $\rho_\Lambda$ or $\Delta \mathcal{F}$. 
During the adiabatic cooling of the Universe, the temperature $T_c$ is reached.

The system is in the $\phi = 0$ phase [i.e., $\phi(x) \equiv 0$].

At $T < T_c$ bubbles of the stable phase [i.e., with $\phi = \phi_m$ inside] begin to nucleate in the supercooled $\phi = 0$ phase.

At $T = T_0$ the barrier disappears. ($T_0 \sim T_c$.)
Bubble nucleation

Nucleation rate

Thermal tunneling probability per unit volume per unit time:

\[ \Gamma \sim T^4 e^{-S_3(T)/T} \]

\( S_3(T) \) = three-dimensional instanton action

= free energy of the critical bubble

\( \Gamma \) is extremely sensitive to temperature:

- At \( T = T_c \), \( \Gamma = 0 \) \((S_3 = \infty)\)
- At \( T = T_0 \), \( \Gamma \sim T^4 \) \((S_3 = 0)\)
  - Nucleation becomes important as soon as \( \Gamma \sim H^4 \), and
  - \( H^4 \sim (T^2/M_{\text{Planck}})^4 \ll T^4 \).
Bubble growth

- Once nucleated, bubbles expand until they fill all space.
- The velocity of bubble walls depends on several parameters.
  - **Pressure difference** $\Delta p = p_- - p_+$
    Depends on supercooling. (At $T = T_c$, $p_- = p_+$).
  - **Friction** of bubble wall with plasma
    Depends on microphysics (particles-Higgs interactions).
  - **Latent heat** $L = \rho_+ - \rho_-$ injected into the plasma.
    Causes reheating and fluid motions.
- Hydrodynamics allows two propagation modes: detonations and deflagrations.
Hydrodynamics

Detonations

- The phase transition front (bubble wall) moves faster than the speed of sound: \( v_w > c_s \).
- No signal precedes the wall.
  It is followed by a rarefaction wave.
- A bubble wall does not influence other bubbles, except in the collision regions.

Deflagrations

- The deflagration front is subsonic \( (v_w < c_s) \).
- The wall is preceded by a shock wave which moves at a velocity \( v_{sh} \approx c_s \).
- Thus, it will influence other bubbles.
Motivation

Phase transition dynamics

Gravitational waves from a first-order phase transition

Phase transition dynamics and gravitational waves
Possible mechanisms

Bubble collisions
- The walls of expanding bubbles provide thin energy concentrations that move rapidly.

Turbulence
- In the early Universe, the Reynolds number is large enough to produce turbulence when energy is injected.

Magnetohydrodynamics (turbulence in a magnetized plasma)
- It develops in an electrically conducting fluid, in the presence of magnetic fields.
Cosmological turbulence

Kolmogoroff-type turbulence

- Energy is injected by a **stirring source** at a length scale $L_S$.
- Eddies of each size $L$ break into smaller ones.
- When turbulence is fully developed, a **cascade** of energy is established from larger to smaller length scales.
- The cascade begins at the **stirring scale** $L_S$ and stops at the **dissipation scale** $L_D \ll L_S$.
- Energy in the cascade is transmitted with a **constant rate** $\varepsilon$.
- For stationary turbulence, the **dissipation rate** $\varepsilon$ equals the power that is **injected by the source**.

A. Mégevand (Mar del Plata, Argentina)  
Phase transition dynamics and GWs  
San Carlos 2008
Cosmological turbulence

The energy spectrum

- Consider the velocity correlation tensor \( \langle v_i(x) v_j(y) \rangle \), where
  - \( v(x) = \) velocity of the fluid,
  - \( \langle \cdots \rangle = \) statistical average.
- For stationary, homogeneous, isotropic turbulence, we have for the Fourier transform of \( v_i \):
  \[
  \langle v_i(k) v_j^*(q) \rangle \propto \delta^3(k - q) \frac{E(k)}{k^2} \left( \delta_{ij} - \frac{k_i k_j}{k^2} \right),
  \]
  - \( E(k) = \) turbulent energy density spectrum.
- For Kolmogoroff turbulence, \( E(k) \propto \varepsilon^{2/3} k^{-5/3} \) for \( L_D < L < L_S \) (with \( k = 2\pi/L \)).
Gravitational waves from turbulence

- The source for the tensor metric perturbation $h_{ij}$ is the transverse and traceless piece of the stress-energy tensor $T_{ij}$.

- The relevant part of the stress-energy tensor for the relativistic fluid is

$$ T_{ij}(x) \propto v_i(x) v_j(x). $$

- The energy density in GWs is

$$ \rho_{GW} \sim \langle T_{ij} T_{ij} \rangle \sim \langle v_i v_j v_i v_j \rangle. $$

- The spectrum can be related to $\langle v_i v_j \rangle \sim E(k)$ (Kolmogoroff).
The expansion of the Universe

- Can be neglected in the *production* of GWs
- Once produced, their wavelength scales with the scale factor $a$ and their amplitude decays like $a^{-1}$. 
The GW spectrum

- The spectrum is characterized by

\[
\Omega_{GW} (f) = \frac{1}{\rho_c} \frac{d\rho_{GW}}{d \log f},
\]

where \(\rho_c = \text{critical density}\).

- Peak frequency:

\[
f_p = 1.6 \times 10^{-5} \, \text{Hz} \frac{T_*}{100 \, \text{GeV}} \left( \frac{g_*}{100} \right)^{1/6} \frac{L_S}{H_*}.
\]

- Peak amplitude:

\[
\Omega_{GW}(f_p) \approx \Omega_R \left( \frac{L_S}{H_*^{-1}} \right)^{10/3} \left( \frac{\varepsilon}{H_*} \right)^{4/3}
\]

where \(\Omega_R = \text{radiation}\).

[Caprini & Durrer, PRD 74, 063521 (2006)]
GWs and phase transition dynamics

The spectrum $\Omega_{GW}(f)$ depends on:

- The temperature $T \sim T_c =$ critical temperature
- The stirring scale $L_s \sim$ size of bubbles
- The dissipation rate $\varepsilon =$ Injected power
  $\sim$ latent heat $\times$ bubble wall velocity

- The parameters depend on hydrodynamics
  (How bubble walls propagate in the fluid).
Motivation

Phase transition dynamics

Gravitational waves from a first-order phase transition

Phase transition dynamics and gravitational waves
GWs from detonations and deflagrations

Detonations (supersonic walls)

- The injected energy is concentrated in a thin region near the bubble wall. (Simpler calculations).
- The wall velocity is $v_w = v_w(\alpha)$, where $\alpha = L/\rho_{th} = (latent\ heat)/(thermal\ energy)$
- The nucleation rate $\Gamma = e^{-S_3(T)/T}$ increases as temperature decreases with time.
- A Taylor expansion of the exponent gives $\Gamma = \Gamma_0 e^{\beta t}$.
- $\beta^{-1}$ is the only time scale in the problem.
- It determines the duration of the phase transition $\Delta t \sim \beta^{-1}$ and the bubble size $d \sim v_w \beta^{-1}$.
- As a consequence, the spectrum of gravitational waves depends only on two parameters, $\alpha$ and $\beta$. 
GWs from detonations and deflagrations

Deflagrations (subsonic walls)

- Calculations are more difficult:
- $v_w \sim \Delta p/\eta$
  - $\eta =$ friction coefficient
  - $\Delta p =$ pressure difference (depends on supercooling)
- Shock waves distribute the latent heat, causing reheating and bulk motions of the fluid far from the wall.
- The phase transition should be treated *globally.*
GWs from detonations and deflagrations

- Due to the difficulties of the deflagration case, calculations of the GW spectrum in specific models often assume that bubble walls propagate as detonations.
- The formulas for the detonation case (which depend only on $\alpha, \beta$) are used.
  - For instance, to investigate GWs in the electroweak phase transition for different extensions of the Standard Model.
- However, the bubbles expand in general as deflagrations.
  - It is known that in the electroweak phase transition, $v_w \sim 10^{-2} - 10^{-1}$,
  - i.e., the walls are deflagrations ($v_w < c_s \approx 0.6$).
Global treatment of deflagration bubbles

Approximation for slow bubble walls:

- If \( v_w \ll c_s \), the quick distribution of latent heat causes a *homogeneous* reheating (\( T \) depends only on \( t \)).
- Equations for \( T(t), v_w(t), \ldots \) can be solved numerically.

In general:

Relevant features:

- All bubbles nucleate in a short interval \( \delta t_\Gamma \) around the “initial” time \( t \equiv t_\Gamma \).
- The bubble number density at \( t = t_\Gamma \) determines the bubble size \( d \sim n_b^{-1/3} \).
- Soon after \( t = t_\Gamma \) the *shock waves* collide and turbulence starts.
Results

- Taking into account the general features of the dynamics, we can derive relations between $\varepsilon$, $d$, $\nu_w$, ...

- and obtain analytical expressions

  [A.M., PRD 78, 084003 (2008)]

\[
 f_p \sim 10^{-2} \text{mHz} \left( \frac{T_c}{100 \text{GeV}} \right) \left( \frac{d}{H^{-1}} \right)^{-1},
\]

\[
 \Omega_{GW}\big|_{\text{peak}} \sim 10^{-4} (\alpha \nu_w)^{8/3} \left( \frac{d}{H^{-1}} \right)^2.
\]

- For the electroweak phase transition at $T_c \sim 100 \text{GeV}$, we would need $d/H^{-1} \sim 10^{-2}$ so that $f_p \sim \text{mHz}$.

  (In general, $10^{-5} \lesssim d \lesssim 10^{-1}$)
Results

- Then, for \( d/H^{-1} \sim 10^{-2} \), \( \nu_W \sim 0.1 \) and \( \alpha \sim 1 \), we have \( \Omega_{GW} \sim 10^{-11} \).

Detecting electroweak GWs at LISA

\[ \begin{array}{c}
\hline
T_c (\text{TeV}) & \alpha & \nu_w = 0.1 & \nu_w = 0.05 & \nu_w = 0.02 \\
\hline
0.1 & 1.0 & & & \\
0.2 & 0.8 & & & \\
0.3 & 0.6 & & & \\
0.4 & 0.4 & & & \\
0.5 & 0.2 & & & \\
0.6 & 0.0 & & & \\
\hline
\end{array} \]

**Figure:** The values of \( \alpha \) and \( T_c \) that give \( f_p = 1 \text{mHz} \) and \( \Omega_{GW}(f_p) = 10^{-11} \).
Summary

- It is important to consider deflagrations as a source of GWs.
- The resulting amplitude may be comparable to the detonation case.

Outlook

- A complete numerical calculation is necessary to evaluate the quantities \( \nu_w, d, \ldots \) in specific models.
  [A.M. and A. Sánchez, work in progress]