Predictions of Finite Unified Theories

Sven Heinemeyer
Ernest Ma
Myriam Mondragón
George Zoupanos
R. Noriega Papaqui

XIII Mexican School of Particles and Fields
What happens as we approach the Planck scale?
How do we go from a fundamental theory to field theory as we know it?
How are the gauge, Yukawa and Higgs sectors related at a more fundamental level?
How do particles get their very different masses?
What is the nature of the Higgs?
Search for understanding relations between parameters

addition of symmetries.

\[ N = 1 \text{ SUSY GUTs.} \]

Complementary approach: look for RGI relations among couplings at GUT scale \( \rightarrow \) Planck scale

\[ \Rightarrow \text{ reduction of couplings} \]

\[ \Rightarrow \text{ FINITENESS} \]

resulting theory: less free parameters \( \therefore \) more predictive

scale invariant
Dimensionless sector of all-loop finite $SU(5)$ model

**prediction for $M_{\text{top}}, \text{large } \tan \beta$**

Can be extended to Soft Supersymmetry Breaking (SSB) sector expressed only in terms of

- $g$ (gauge coupling) and
- $M$ (unified gaugino mass)

**too restrictive**

Constraint can be relaxed

- sum-rule for soft scalars
- better phenomenology

**Confronting with low energy precision data**

- Discriminate among different models
- $\Rightarrow$ **Prediction for Higgs mass and s-spectra**
Reduction of Couplings

A RGI relation among couplings $\Phi(g_1, \ldots, g_N) = 0$ satisfies

\[
\mu \frac{d\Phi}{d\mu} = \sum_{i=1}^{N} \beta_i \frac{\partial \Phi}{\partial g_i} = 0.
\]

$g_i$ = coupling, $\beta_i$ its $\beta$ function

Finding the $(N-1)$ independent $\Phi$'s is equivalent to solve the reduction equations (RE)

\[
\beta_g \left( \frac{dg_i}{dg} \right) = \beta_i ,
\]

$i = 1, \ldots, N$

- completely reduced theory contains only one independent coupling and its $\beta$ function
- complete reduction: power series solution of RE
- uniqueness of the solution can be investigated at one-loop
The complete reduction might be too restrictive, one may use fewer $\Phi$’s as RGI constraints.

Reduction of couplings is essential for finiteness.

**finiteness:** absence of $\infty$ renormalizations

$$\Rightarrow \beta^N = 0$$

In SUSY no-renormalization theorems

$$\Rightarrow$$ only study one and two-loops

guarantee that is gauge and reparameterization invariant at all loops.
Finiteness

A chiral, anomaly free, $N = 1$ globally supersymmetric gauge theory based on a group $G$ with gauge coupling constant $g$ has a superpotential

$$ W = \frac{1}{2} m^{ij} \Phi_i \Phi_j + \frac{1}{6} C^{ijk} \Phi_i \Phi_j \Phi_k , $$

Requiring one-loop finiteness $\beta_g^{(1)} = 0 = \gamma_i^{(1)}$ gives the following conditions:

$$ \sum_i T(R_i) = 3 C_2(G) , \quad \frac{1}{2} C_{ipq} C^{ipq} = 2 \delta^j_i g^2 C_2(R_i) . $$

$C_2(G) =$ quadratic Casimir invariant, $C_{ijk} =$ Yukawa coup., $T(R_i)$ Dynkin index of $R_i$.

- restricts the particle content of the models
- relates the gauge and Yukawa sectors
One-loop finiteness $\Rightarrow$ two-loop finiteness

One-loop finiteness restricts the choice of irreps $R_i$, as well as the Yukawa couplings

Cannot be applied to the susy Standard Model (SSM):

$C_2[U(1)] = 0$

The finiteness conditions allow only SSB terms

It is possible to achieve all-loop finiteness $\beta^n = 0$:

1. One-loop finiteness conditions must be satisfied
2. The Yukawa couplings must be a formal power series in $g$, which is solution (isolated and non-degenerate) to the reduction equations
Supersymmetry is essential. It has to be broken, though…

\[-\mathcal{L}_{\text{SB}} = \frac{1}{6} h^{ijk} \phi_i \phi_j \phi_k + \frac{1}{2} b^{ij} \phi_i \phi_j + \frac{1}{2} (m^2)_i^j \phi^* i \phi_j + \frac{1}{2} M \lambda \lambda + \text{H.c.}\]

The RGI method has been extended to the SSB of these theories.

- One- and two-loop finiteness conditions for SSB have been known for some time

  Jack, Jones, et al.

- It is also possible to have all-loop RGI relations in the finite and non-finite cases

  Kazakov; Jack, Jones, Pickering
SSB terms depend only on $g$ and the unified gaugino mass $M$

universality conditions

$$h = -MC,$$  
$$m^2 \propto M^2,$$  
$$b \propto M_{\mu}$$

Very appealing! But too restrictive; it leads to phenomenological problems:

- The lightest susy particle (LSP) is charged.  
  Yoshioka; Kobayashi et al
- It is incompatible with radiative electroweak breaking.  
  Brignole, Ibáñez, Muñoz

Possible to relax the universality condition to a sum-rule for the soft scalar masses

$$\Rightarrow$$ better phenomenology.

Kobayashi, Kubo, Mondragón, Zoupanos
Soft scalar sum-rule for the finite case

Finiteness implies

\[ C^{ijk} = g \sum_{n=0}^{\infty} \rho_{(n)}^{ijk} g^{2n}, \]

The one- and two-loop finiteness for \( h \) gives

\[ h^{ijk} = -MC^{ijk} + \cdots = -M\rho_{(0)}^{ijk} g + O(g^5). \]

Assume that lowest order coefficients \( \rho_{(0)}^{ijk} \) and \((m^2)_j^{i}\) satisfy diagonality relations

\[ \rho_{ipq(0)}^{jpq} \propto \delta_i^j, \quad (m^2)_j^{i} = m_j^2 \delta_i^j \quad \text{for all } p \text{ and } q. \]

We find the following soft scalar-mass sum rule

\[ \frac{(m_i^2 + m_j^2 + m_k^2)}{MM^\dagger} = 1 + \frac{g^2}{16\pi^2} \Delta^{(1)} + O(g^4) \]

for \( i, j, k \) with \( \rho_{(0)}^{ijk} \neq 0 \), where \( \Delta^{(1)} \) is the two-loop correction,

\[ \Delta^{(1)} = -2 \sum_l [\left( m_l^2 / MM^\dagger \right) - (1/3)] T(R_l), \]

which vanishes for the universal choice.
All-loop sum rule

One can generalize the sum rule for finite and non-finite cases to all-loops!!

Possible thanks to renormalization properties of $N=1$ susy gauge theories.

Kazakov et al; Jack, Jones et al; Yamada; Hisano, Shifman

The sum-rule in the NSVZ scheme is

$$m_i^2 + m_j^2 + m_k^2 = |M|^2 \left\{ \frac{1}{1 - g^2 C(G)/(8\pi^2)} \frac{d \ln C^{ijk}}{d \ln g} + \frac{1}{2} \frac{d^2 \ln C^{ijk}}{d(\ln g)^2} \right\} + \sum_l \frac{m_l^2 T(R_l)}{C(G) - 8\pi^2/g^2} \frac{d \ln C^{ijk}}{d \ln g}.$$

Kobayashi, Kubo, Zoupanos

Interesting: Finite sum rule satisfied also in certain certain class of orbifold models in which the massive states are organized into $N = 4$ supermultiples, if $d \ln C^{ijk}/d \ln g = 1$. 
Several aspects of Finite Models have been studied

- **$SU(5)$ Finite Models studied extensively**
  
  Rabi et al; Kazakov et al; López-Mercader, Quirós et al; M.M, Kapetanakis, Zoupanos; etc

- One of the above coincides with a non-standard Calabi-Yau $SU(5) \times E_8$

  Greene et al; Kapetanakis, M.M., Zoupanos

- Finite theory from compactified string model also exists (albeit not good phenomenology)

  Ibáñez

- Criteria for getting finite theories from branes exist

  Hanany, Strassler, Uranga

- Realistic models involving all generations exist

  Babu, Eckbahrt, Gogoladze

- Some models with $SU(N)^k$ finite $\iff$ 3 generations, good phenomenology with $SU(3)^3$

  Ma, M.M, Zoupanos

- Relation between commutative field theories and finiteness studied

  Jack and Jones

- Proof of conformal invariance in finite theories

  Kazakov
SU(5) Finite Models

We study two models with $SU(5)$ gauge group. The matter content is

$$3\, \overline{5} + 3\, 10 + 4\, \{5 + \overline{5}\} + 24$$

The models are finite to all-loops in the dimensionful and dimensionless sector. In addition:

- The soft scalar masses obey a sum rule
- At the $M_{GUT}$ scale the gauge symmetry is broken and we are left with the MSSM
- At the same time finiteness is broken
- The two Higgs doublets of the MSSM should mostly be made out of a pair of Higgs $\{5 + \overline{5}\}$ which couple to the third generation

The difference between the two models is the way the Higgses couple to the 24

Kapetanakis, Mondragón, Zoupanos; Kazakov et al.
The superpotential which describes the two models takes the form

$$W = \sum_{i=1}^{3} \left[ \frac{1}{2} g_{i}^{u} \bar{10}_{i} 10_{i} H_{i} + g_{i}^{d} \bar{10}_{i} \bar{5}_{i} \bar{H}_{i} \right] + g_{23}^{u} \bar{10}_{2} 10_{3} H_{4}$$

$$+ g_{23}^{d} \bar{10}_{2} \bar{5}_{3} \bar{H}_{4} + g_{32}^{d} \bar{10}_{3} \bar{5}_{2} \bar{H}_{4} + \sum_{a=1}^{4} g_{a}^{f} H_{a} 24 \bar{H}_{a} + \frac{g_{\lambda}}{3} (24)^{3}$$

find isolated and non-degenerate solution to the finiteness conditions
The finiteness relations give at the $M_{GUT}$ scale

**Model A**

- $g_t^2 = \frac{8}{5} g^2$
- $g_{b,\tau}^2 = \frac{6}{5} g^2$
- $m_{H_u}^2 + 2m_{10}^2 = M^2$
- $m_{H_d}^2 + m_5^2 + m_{10}^2 = M^2$

- **3 free parameters:**
  - $M$, $m_5^2$ and $m_{10}^2$

**Model B**

- $g_t^2 = \frac{4}{5} g^2$
- $g_{b,\tau}^2 = \frac{3}{5} g^2$
- $m_{H_u}^2 + 2m_{10}^2 = M^2$
- $m_{H_d}^2 - 2m_{10}^2 = -\frac{M^2}{3}$
- $m_5^2 + 3m_{10}^2 = \frac{4M^2}{3}$

- **2 free parameters:**
  - $M$, $m_5^2$
Phenomenology

The gauge symmetry is broken below $M_{GUT}$, and what remains are boundary conditions of the form $C_i = \kappa_i g$, $h = -MC$ and the sum rule at $M_{GUT}$, below that is the MSSM.

- We assume a unique susy breaking scale
- The LSP is neutral
- The solutions should be compatible with radiative electroweak breaking
- No fast proton decay

We also

- Allow 5% variation of the Yukawa couplings at GUT scale due to threshold corrections
- Include radiative corrections to bottom and tau, plus resummation (very important!)
- Estimate theoretical uncertainties
We look for the solutions that satisfy the following constraints:

- Right masses for top and bottom
- The decay $b \rightarrow s\gamma$
- The branching ratio $B_s \rightarrow \mu^+\mu^-$
- Cold dark matter density $\Omega_{CDM}h^2$

The lightest MSSM Higgs boson mass
The SUSY spectrum

FeynHiggs, Suspect, FUT
FUTA: $M_{top} \sim 183$ GeV
FUTB: $M_{top} \sim 172$ GeV

Theoretical uncertainties $\sim 4\%$
Δb and Δtau included, resummation done

FUTB μ < 0 favoured

uncertainties ~ 8 %
FUTB: \( M_{Higgs} = 122 \sim 126 \text{ GeV} \)
Uncertainties \( \pm 3 \text{ GeV} \) (FeynHiggs)

\[ \Omega_{CDM} h^2 < 0.3 \]
LOSP and coloured particles that satisfy B physics and loose CDM constraint

Challenging for LHC
Results

When confronted with low-energy precision data

only FUTB $\mu < 0$ survives

No solution for g-2, very constrained from dark matter

$\sim M_{top} \sim 172 \text{ GeV}$ 4 %
$\sim m_{bot}(M_Z) \sim 2,8 \text{ GeV}$ 8 %
$\sim M_{Higgs} \sim 122 - 126 \text{ GeV}$ 3 GeV
$\sim \tan \beta \sim 44 - 46$

Extension to 3 fams on its way with flavour symmetry; with $R \Rightarrow$ neutrino masses

in this case dark matter candidate is not LSP, results may change
Finite $SU(N)^k$ Unification

Consider $N = 1$ supersymmetric gauge theories based on the group

$$SU(N)_1 \times SU(N)_2 \times \ldots \times SU(N)_k$$

with matter content

$$(N, N^*, 1, \ldots, 1) + (1, N, N^*, \ldots, 1) + \ldots + (N^*, 1, 1, \ldots, N)$$

with $\beta$-function coefficient in the renormalization-group equation of each $SU(N)$ gauge given by

$$b = \left( -\frac{11}{3} + \frac{2}{3} \right) N + n_f \left( \frac{2}{3} + \frac{1}{3} \right) \left( \frac{1}{2} \right) 2N = -3N + n_f N.$$ 

$$n_f = 3 \Leftrightarrow b = 0,$$ 

**FINITE independently of the values of $N$ and $k$**

Ma, M.M., Zoupanos
Possible Models

Minimum requirements:

- leads to the SM or the MSSM at low energies
- it predicts correctly $\sin^2 \theta_W$.

MODELS:

- $SU(3)_C \times SU(3)_L \times SU(3)_R$ ✓
- $SU(3)^4 \rightarrow SU(3)_C$ predicted value of $\alpha_s$ be too small. ✗
- $SU(4)^4$ non-susy unification at scale of $4 \times 10^{11}$ GeV. ✗
- $SU(4)^3$ either $\sin^2 \theta_W$ wrong or an unbroken $U(1)$ coupled to everything. ✗

Lots of interest lately in these finite or reduced theories, since they could provide a bridge between strings or branes and ordinary GUTs

Ibáñez; Kachru and Silverstein
Finite $SU(3)^3$

Invariant is $(N, N^*, 1)(1, N, N^*)(N^*, 1, N)$
Could come from the compactification of $E_8 \rightarrow E_6$ over a
Calabi-Yau manifold, or via coset space dimensional reduction,
with a Wilson line

$$E_8 \rightarrow E_6 \rightarrow SU(3)^3 \rightarrow MSSM \rightarrow SM$$

We consider the $SU(3)^3$ between $M_{GUT}$ and $M_{Planck}$,
below MSSM
For the unification of couplings to hold the cyclic symmetry $Z_3$
must be imposed

$$q \rightarrow \lambda \rightarrow q^c \rightarrow q$$

Now we have $\beta_g = 0$, search for unique solutions.
$SU(3)_C \times SU(3)_L \times SU(3)_R$ with quarks transforming as

De Rújula, Georgi, and Glashow; Lazarides, Panagiotakopoulos, and Shafi

$$q = \begin{pmatrix} d & u & h \\ d & u & h \\ d & u & h \end{pmatrix} \sim (3, 3^*, 1), \quad q^c = \begin{pmatrix} d^c & d^c & d^c \\ u^c & u^c & u^c \\ h^c & h^c & h^c \end{pmatrix} \sim (3^*, 1, 3)$$

and leptons transforming as

$$\lambda = \begin{pmatrix} N & E^c & \nu \\ E & N^c & e \\ \nu^c & e^c & S \end{pmatrix} \sim (1, 3, 3^*)$$

The breaking down of

$$SU(3)^3 \to SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{Y_L+Y_R}$$

is achieved with the $(3,3)$ entry of $\lambda$, and the further breaking of $SU(2)_R \times U(1)_{Y_L+Y_R}$ to $U(1)_Y$ with the $(3,1)$ entry.
The superpotential is

\[
f \, \text{Tr}(\lambda q^c q) + \frac{1}{6}f' \epsilon_{ijk} \epsilon_{abc}(\lambda_i \lambda_j \lambda_k + q^c_{ia} q^c_{jb} q^c_{kc} + q_{ia} q_{jb} q_{kc})
\]

With 3 families: most general superpotential contains 11\(f\) couplings, and 10\(f'\) couplings, subject to 9 conditions, due to the vanishing of the anomalous dimensions of each superfield:

\[
\sum_{j,k} f_{ijk}(f_{ijk})^* + \frac{2}{3} \sum_{j,k} f'_{ijk}(f'_{ijk})^* = \frac{16}{9} g^2 \delta_{il},
\]

where \(f_{ijk} = f_{jki} = f_{tki}; \quad f'_{ijk} = f'_{kji} = f'_{kij} = f'_{jki} = f'_{jik}\)

Quarks and leptons receive masses when the scalar part of the superfields \(\tilde{N}_{1,2,3}\) and \(\tilde{N}^c_{1,2,3}\) obtain vevs

\[
(M_d)_{ij} = \sum_k f_{kij} \langle \tilde{N}_k \rangle, \quad (M_u)_{ij} = \sum_k f_{kij} \langle \tilde{N}^c_k \rangle,
\]

\[
(M_e)_{ij} = \sum_k f'_{kij} \langle \tilde{N}_k \rangle, \quad (M_\nu)_{ij} = \sum_k f'_{kij} \langle \tilde{N}^c_k \rangle.
\]
Since we have MSSM ⇒ two Higgs doublets we choose the linear combinations coupled to the third generation

\[ \tilde{N}_c = \sum_i a_i \tilde{N}_i^c \]

and

\[ \tilde{N} = \sum_i b_i \tilde{N}_i \]

this can be done by choosing appropriately the masses in the superpotential, León et al

- Then these two Higgs doublets couple to the three families differently providing the freedom to understand their different masses and mixings
- We need to fulfill the second (and most difficult) finiteness requirement for all-loop finite theories
- Solutions give all-loop or two-loop finite models, with Universal soft terms or with the sum rule
Phenomenology

Phenomenology of the models was analyzed for an all-loop finite and a two-loop finite case. Best results (so far) for the two-loop finite model:

\[ m_{\text{top}} \sim 170 - 173 \text{ GeV} \quad \tan \beta \sim 58 \quad M_{\text{Higgs}} \sim 120 - 125 \text{ GeV}, \]

with a charged LSP \( \tilde{\tau} \)

\[ LSP = \chi^0 \sim 300 - 600 \text{ GeV} \]

Notice: it involves three generations, it requires a discrete symmetry.
A more thorough analysis is under way.

Heinemeyer, Ma, M.M., Zoupanos
Conclusions

- Finiteness: powerful, interesting and intriguing principle $\Rightarrow$ reduces greatly the number of free parameters
- completely finite theories
  i.e. including the SSB terms, that satisfy the sum rule.
- Confronting the $SU(5)$ models with low-energy precision data does distinguish among models:
  - FUTB $\mu < 0$ survives (remarkably)
  - large tan $\beta$
  - s-spectrum starts above $\sim 400$ GeV
  - a prediction for the Higgs $M_h \sim 122 - 126$ GeV
  - no solution for $g - 2$, constrained from dark matter
- Extension to three fams with $\mathcal{R}$ on its way
- Detailed study of finite $SU(3)^3$ $\iff$ 3 generations in progress