HYPERON SEMILEPTONIC DECAYS:
SOME THEORETICAL ISSUES

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**Background**

▶ Cabibbo\(^1\) proposed a model for weak hadronic currents based on SU(3) symmetry: it led to detailed predictions for hyperon semileptonic decays (HSD).

The Lorentz structure of the current is \( V - A \), and \( \theta_c \) — the Cabibbo angle — is a parameter to be determined from experimental data.

▶ Kobayashi and Maskawa\(^2\) generalized Cabibbo universality to three generations of quarks, which could accommodate CP violation.

The matrix \( V \) is known as **Cabibbo-Kobayashi-Maskawa (CKM)** matrix and

\[
V_{ud} \approx \cos \theta_c, \quad V_{us} \approx \sin \theta_c.
\]

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\(^1\)N. Cabibbo, Phys. Rev. Lett. 10, 531 (1963)
\(^2\)M. Kobayashi, T. Maskawa, Prog. Theor. Phys. 49, 652 (1973)
Analysis of $K_{e3}$ decays yields

$$|V_{us}| = 0.2196 \pm 0.0023$$

... The analysis of hyperon decay data has larger theoretical uncertainties because of first order SU(3) symmetry breaking effects in the axial-vector couplings... but due account of symmetry breaking... gives the corrected value of $0.222 \pm 0.003$. We average these results to obtain

$$|V_{us}| = 0.2205 \pm 0.0018$$

$|V_{us}|$ may be determined from kaon decays, hyperon decays, and tau decays. Previous determinations have most often used $K\ell 3$ decays...

...with the Leutwyler-Roos calculation of $f_+ (0)$ gives

$$|V_{us}| = \lambda = 0.2255 \pm 0.0019.$$  \hfill (11)

It should be mentioned that hyperon semileptonic decay fits suggest [5]

$$|V_{us}| = 0.2250(27) \quad \text{Hyperon decays} \quad (16)$$

modulo SU(3) breaking effects that could shift that value up or down... Similarly, strangeness changing tau decays gives [45]

$$|V_{us}| = 0.2208(34) \quad \text{Tau decays} \quad (17)$$

where the central value depends on the strange quarks mass.

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Hyperon Semileptonic Decays

\[ B_1 \rightarrow B_2 + \ell + \bar{\nu}_\ell \]

4-momentum \hspace{2cm} mass

\[ \begin{align*}
B_1 & \quad p_1 = (E_1, \mathbf{p}_1) \quad M_1 \\
B_2 & \quad p_2 = (E_2, \mathbf{p}_2) \quad M_2 \\
\ell & \quad l = (E, \mathbf{l}) \quad m \\
\bar{\nu}_\ell & \quad p_\nu = (E_\nu^0, \mathbf{p}_\nu) \quad m_\nu
\end{align*} \]
The low-energy weak interaction Hamiltonian for HSD is given by

\[ H_W = \frac{G_V}{\sqrt{2}} J_\alpha L^\alpha + \text{H.c.} \]

Here the leptonic current is

\[ L^\alpha = \bar{\psi}_e \gamma^\alpha (1 - \gamma_5) \psi_{\nu_e} + \bar{\psi}_\mu \gamma^\alpha (1 - \gamma_5) \psi_{\nu_\mu} \]

and the hadronic current expressed in terms of the vector \((V_\alpha)\) and axial-vector \((A_\alpha)\) currents is

\[ J_\alpha = V_\alpha - A_\alpha , \]

\[ V_\alpha = V_{ud} \bar{u} \gamma_\alpha d + V_{us} \bar{u} \gamma_\alpha s , \]

\[ A_\alpha = V_{ud} \bar{u} \gamma_\alpha \gamma_5 d + V_{us} \bar{u} \gamma_\alpha \gamma_5 s . \]

\(G_V\) is the weak coupling constant.
Matrix Elements of the Hadronic Current

The matrix elements of $J_\mu$ between spin-1/2 states can be written as

$$\langle B_2|J_\alpha|B_1\rangle = V_{\text{CKM}} \mathbf{u}_{B_2}(p_2) \left[ f_1(q^2)\gamma_\alpha + \frac{f_2(q^2)}{M_1}\sigma_{\alpha\beta}q^{\beta} + \frac{f_3(q^2)}{M_1}q_\alpha \right. $$

$$\left. + \left( g_1(q^2)\gamma_\alpha + \frac{g_2(q^2)}{M_1}\sigma_{\alpha\beta}q^{\beta} + \frac{g_3(q^2)}{M_1}q_\alpha \right) \gamma_5 \right] u_{B_1}(p_1),$$

$q = p_1 - p_2$ is the momentum transfer, $V_{\text{CKM}}$ is either $V_{ud}$ or $V_{us}$, as the case may be, and

- $f_1$ vector f.f.
- $f_2$ weak magnetism f.f.
- $f_3$ induced scalar f.f.
- $g_1$ axial-vector f.f.
- $g_2$ weak electricity f.f.
- $g_3$ induced pseudoscalar f.f.
In the limit of exact flavor SU(3) symmetry

- $V_\mu$ and $A_\mu$ belong to SU(3) octets and

$$f_k(q^2) = C^{B_2B_1}_F F_k(q^2) + C^{B_2B_1}_D D_k(q^2),$$
$$g_k(q^2) = C^{B_2B_1}_F F_{k+3}(q^2) + C^{B_2B_1}_D D_{k+3}(q^2),$$

$F_i(q^2)$ and $D_i(q^2)$ are reduced form factors.

- The weak vector currents and the em current are members of the same SU(3) octet, which fixes $f_1$ and $f_2$: 

$$F_1(0) = 1, \quad D_1(0) = 0, \quad F_2(0) = \kappa_p + \frac{1}{2}\kappa_n, \quad D_2(0) = -\frac{3}{2}\kappa_n.$$

- $F_3(q^2) = D_3(q^2) = 0$ by conservation of the em current so $f_3(q^2) = 0$.

- $g_1$ is given in terms of $F$ and $D$ (undetermined parameters).

- $g_2 = 0$ by hermiticity and time reversal invariance for diagonal matrix elements of hermitian currents.
**Transition Amplitude for HSD**

The transition amplitude $M_0$ for the process

$$B_1 \rightarrow B_2 + \ell + \nu_\ell$$

is given by

$$M_0 = \frac{G_V}{\sqrt{2}}[\bar{u}_{B_2}(p_2)W^\mu(p_1, p_2)u_{B_1}(p_1)][\bar{u}_\ell(l)O_\mu\nu_\nu(p_\nu)],$$

where

$$W_\mu = f_1(q^2)\gamma_\mu + \frac{f_2(q^2)}{M_1}\sigma_{\mu\nu}q^\nu + \frac{f_3(q^2)}{M_1}q_\mu$$

$$+ \left[ g_1(q^2)\gamma_\mu + \frac{g_2(q^2)}{M_1}\sigma_{\mu\nu}q^\nu + \frac{g_3(q^2)}{M_1}q_\mu \right] \gamma_5,$$

and $O_\mu = \gamma_\mu(1 - \gamma_5)$. Hereafter $f_i \equiv f_i(0)$, $g_i \equiv g_i(0)$. 
**Differential Decay Rate $d\Gamma$**

$d\Gamma$ is obtained from $M_0$. Different choices of the variables in the final states lead to appropriate expressions:

- In the rest frame of $B_1 [B_2]$ when it is polarized along the direction $s_1 [s_2]$, and with $\ell$ and $\nu_\ell$ going into the solid angles $d\Omega_\ell$ and $d\Omega_\nu$.

- In the rest frame of $B_1 [B_2]$ when it is polarized along the direction $s_1 [s_2]$, by leaving $E$ and $E_2$ as the relevant variables.

This choice allows detailed studies of the Dalitz plot (DP):

$$d\Gamma(E, E_2) = d\Phi_3 \left[ A'_0(E, E_2) - A''_0(E, E_2) \hat{s}_i \cdot \hat{p} \right],$$

with $\hat{p} = \hat{1}, \hat{p}_2$. 
**Integrated Observables in HSD**

The (uncorrected) decay rate $R$ for HSD (rough approximation) is

$$R^0 = G_V^2 \frac{(\Delta M)^5}{60\pi^3} \left[ \left( 1 - \frac{3}{2}\beta + \frac{6}{7}\beta^2 \right) f_1^2 + \frac{4}{7}\beta^2 f_2^2 \right. $$

$$+ \left. \left( 3 - \frac{9}{2}\beta + \frac{12}{7}\beta^2 \right) g_1^2 + \frac{12}{7}\beta^2 g_2^2 + \frac{6}{7}\beta^2 f_1 f_2 - (4\beta - 6\beta^2) g_1 g_2 \right]$$

where $\Delta M = M_{B_1} - M_{B_2}$, and $\beta = \Delta M/M_1$.

The $q^2$-dependence of the form factors can be incorporated as

$$f_1(q^2) = f_1(0) + \frac{q^2}{M_1^2} \lambda^f_1,$$

$$g_1(q^2) = g_1(0) + \frac{q^2}{M_1^2} \lambda^g_1,$$

where $\lambda^f_1$ and $\lambda^g_1$ are slope parameters of order unity.
$R^0$ then gets the contribution

$$G^2 V \frac{(\Delta M)^5}{60\pi^3} \left( \frac{4}{7} \beta^2 \right) \left( f_1 \lambda_1^f + 5 g_1 \lambda_1^g \right).$$

The angular spin-asymmetry coefficients are defined as

$$\alpha^0_A = \frac{2}{N} \frac{N (\theta_A < \frac{1}{2} \pi) - N (\theta_A > \frac{1}{2} \pi)}{N (\theta_A < \frac{1}{2} \pi) + N (\theta_A > \frac{1}{2} \pi)}$$

$A = B_2, \ell, \nu_\ell$. $\theta_A$ is the angle between the $A$-direction and polarization direction of $B_1$. Likewise, $\alpha_{e\nu}$ can be defined.

For more precise formulas and when the charged lepton mass is retained, $d\Gamma(E, E2)$ and $\alpha_A(E, E2)$ should be integrated numerically:

$$R^0 = \sum_{i \leq j = 1}^{6} a_{i j}^R f_i f_j + \sum_{i \leq j = 1}^{6} b_{i j}^R (f_i \lambda f_j + f_j \lambda f_i).$$
Radiative Corrections

Current high-statistics experiments in HSD make Dalitz-plot measurements feasible. The analysis of such measurements requires the application of radiative corrections (RC) so it is necessary that theoretical expressions as general and accurate as possible be available.

In computing RC one faces some difficulties. They depend on:

- An ultraviolet cutoff.
- The strong interaction, and on details of the weak interaction other than the effective $V - A$ theory. In a few words, RC have a model-dependent part.
- The charge assignments of the decaying and emitted baryons.
- The observed kinematical and angular variables, and on certain experimental conditions.
### Advances in RC

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<th>Polarized</th>
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<td>$B_1 \begin{cases} &amp; \text{Order } (\alpha/\pi)(q/M_1)^0 \ &amp; \text{Order } (\alpha/\pi)(q/M_1) \end{cases}$</td>
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<tr>
<td>$B_2 \begin{cases} &amp; \text{Order } (\alpha/\pi)(q/M_1)^0 \ &amp; \text{Order } (\alpha/\pi)(q/M_1) \end{cases}$</td>
<td>$B_2 \begin{cases} &amp; \text{Order } (\alpha/\pi)(q/M_1)^0 \ &amp; \text{Order } (\alpha/\pi)(q/M_1) \end{cases}$</td>
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</tbody>
</table>

- $R \ [1]$; $R \ [2]$
- $R \ [3]$; $R \ [4,5]$
- $R \ [1]$
- Work in progress
- $\alpha_{e\nu}, \alpha_e, \alpha_{\nu} \ [1]; \alpha_B \ [6]; \alpha_e \ [8]; \alpha_B \ [9]; \alpha_{e\nu}, \alpha_e, \alpha_B, \alpha, \beta \ [7]; \alpha_B \ [10]; \alpha_e \ [11]; \alpha_B \ [12]$
- $\hat{\alpha}_e, \hat{\alpha}_{e\nu}, A, B \ [1]; \hat{\alpha}_e \ [13]$
- Work in progress

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Virtual Radiative Corrections

Sirlin\textsuperscript{5} implemented an approach to deal with the RC to the $\beta$ decay of a physical nucleon. He showed that the virtual RC can be separated out in two parts:

- One is model-independent, finite in the ultraviolet region and contains the infrared divergence.
- The other one contains all the complications due to strong interactions and the presence of the intermediate vector boson and the ultraviolet divergence (model-dependent part).

This separation procedure is gauge invariant.

Later, it was shown\textsuperscript{6} that the method could be generalized to other observables and that it remained valid even when $q$ is not negligible.

\textsuperscript{5}A. Sirlin, Phys. Rev. 164, 1767 (1967).
\textsuperscript{6}A. Garcia and R. Juarez W., Phys. Rev. D 22, 1132 (1980)
**Order $\mathcal{O}(\alpha)$ Virtual Radiative Corrections**
**Decay Amplitude with Virtual RC**

The transition amplitude with virtual RC for the process

\[ B_1 \rightarrow B_2 + \ell + \nu_\ell \]

is\(^7\)

\[ M_V = M_0 + M_v + S, \]

All the model-dependence is contained in \( S \),

\[ S = \frac{\alpha}{\pi} \bar{u}_{B_2} (c \gamma_\mu + d \gamma_\mu \gamma_5) u_{B_1} \bar{u}_\ell O^\mu \nu \nu, \]

where \( c \) and \( d \) constants so that

\[ f'_1(0) \equiv f_1(0) + \frac{\alpha}{\pi} c, \quad g'_1(0) \equiv g_1(0) + \frac{\alpha}{\pi} d, \]

and the transition amplitude reduces to

\[ M_V = M_0' + M_v. \]

**Decay Rate with Virtual RC**

The polarization of $B_1$ can be studied with the projection operator

$$\Sigma(s_1) = \frac{1 - \gamma_5 \not{s}_1}{2}, \quad u_{B_1}(p_1) \rightarrow \Sigma(s_1)u_{B_1}(p_1)$$

with the conditions $s_1 \cdot s_1 = -1$ and $s_1 \cdot p_1 = 0$.

In the rest system of $B_1$, with $p_2$ along the $z$ axis, the DP can be written as

$$d\Gamma_V = d\Phi_3 \left\{ A'_0 + \frac{\alpha}{\pi} (A'_1 \phi + A''_1 \phi') - \hat{s}_1 \cdot \hat{p}_2 \left[ A''_0 + \frac{\alpha}{\pi} (A'_2 \phi + A''_2 \phi') \right] \right\}$$

The infrared divergence is contained in $\phi(E)$.

$A'_0, A''_0, A'_i, \text{ and } A''_i$ depend on the kinematical variables and are quadratic functions of the form factors.
Bremsstrahlung Radiative Corrections

The inner-bremsstrahlung contributions must be added to the virtual RC so we need to consider the four-body process

\[ B_1(p_1) \rightarrow B_2(p_2) + \ell(l) + \nu_\ell(p_\nu) + \gamma(k), \]

where \( \gamma \) represents a photon with momentum \( k = (\omega, k) \).

- The bremsstrahlung RC is a four-body decay whose DP covers entirely the DP of the three-body decay.

We define:

- Three-body region (TBR): The DP of the three-body decay.
- Four-body region (FBR): The non-overlap of the DP of the four-body decay and the DP of the three-body decay.

- The FBR region is present when real photons cannot be discriminated in an experimental analysis of HSD.
**Kinematical Allowed Region for the Four-Body Decay**

An event in $\mathbf{B}$ demands the existence of a fourth particle which carries away finite energy and momentum. In $\mathbf{A}$ this may or may not be the case. Thus $\mathbf{B}$ corresponds to the FBR whereas $\mathbf{A}$ is the TBR.

The analysis of bremsstrahlung RC considers the process $B_1 \rightarrow B_2 + \ell + \nu_\ell + \gamma$ with both TBR and FBR contributions.
Order $\mathcal{O}(\alpha)$ Bremsstrahlung Radiative Corrections
**Decay Amplitude with Bremsstrahlung RC**

- To order $\mathcal{O}(\alpha q/\pi M_1)$, the amplitude for the four-body decay can be obtained in a model-independent fashion by virtue of the Low theorem.\(^8\)

- The model-dependence will show up when including terms of order $\mathcal{O}(\alpha q^2/\pi M_1^2)$ or higher.

The amplitude with bremsstrahlung RC can be summarized as\(^9\)

$$M_B = M_1 + M_2 + M_3,$$

where $M_3$ is one order in $q$ higher than $M_1$ and $M_2$, and is written in terms of the electromagnetic static parameters of the baryons.

The calculation of $d\Gamma_B$ is performed with standard techniques.

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**DP with Bremsstrahlung RC**

The DP with bremsstrahlung RC, \( d\Gamma_B \), can be organized as

\[
d\Gamma_B = d\Gamma_{iR}^B + d\Gamma_{TBR}^B + d\Gamma_{FBR}^B,
\]

where

\[
d\Gamma_{iR}^B = \frac{\alpha}{\pi} I_0(\lambda) \, d\Omega + d\Gamma_0^B,
\]

contains the infrared divergence in the first summand whereas the remaining terms are finite.

\( d\Gamma_{TBR}^B \) and \( d\Gamma_{FBR}^B \) still must be integrated over the photon variables with the appropriate limits. Two ways to do it:

- Numerical integration
- Analytical integration
DP with RC: Putting Everything Together

In the rest system of $B_1$, with $p_2$ along the $z$ axis, the complete radiatively corrected DP to order $\mathcal{O}(\alpha q/\pi M_1)$ can be cast into\textsuperscript{10}

$$
\begin{align*}
    d\Gamma(B_1 \rightarrow B_2 \ell \overline{\nu}_\ell) &= d\Gamma_V + d\Gamma_B \\
    &= d\Omega \left\{ A'_0 + \frac{\alpha}{\pi} (\Phi_1 + \Phi_{1F}) \\
    &\quad - \hat{s}_1 \cdot \hat{p}_2 \left[ B''_0 + \frac{\alpha}{\pi} (\Phi_2 + \Phi_{2F}) \right] \right\}
\end{align*}
$$

where the $\Phi_i(E, E_2)$ depend quadratically on the form factors.

In the rest system of $B_1$, with $\ell$ along the $z$ axis, a similar expression can be obtained. The decay of charged and neutral baryons need to be analyzed separately.

\textsuperscript{10} J.J. Torres et. al., PRD 74, 077501 (2006).
Features

- To order $\mathcal{O}(\alpha q/\pi M_1)$, the expression for $d\Gamma$ has no infrared divergences, it does not contain an ultraviolet cutoff, and is model-independent.

- It can be useful in the analysis of the Dalitz plot of precision experiments involving light and heavy quarks.

- It is not compromised to fixing the form factors at prescribed values.
**Spin-asymmetry coefficient \( \alpha_B \)**

The DP so organized allows the calculation of \( \alpha_B \):

\[
\alpha_B = 2 \frac{N^+ - N^-}{N^+ + N^-}
\]

\( N^+ [N^-] \) denotes the number of emitted hyperons with momenta in the forward [backward] hemisphere with respect to \( s_1 \). One gets

\[
\alpha_B = - \frac{\Delta_2 + (\alpha/\pi)(\Psi_2 + \Psi_{2F})}{\Delta_1 + (\alpha/\pi)(\Psi_1 + \Psi_{1F})},
\]

where

\[
\Delta_2 = \int_{m}^{E_m} \int_{E_2^\text{min}}^{E_2^\text{max}} B_0'' dE_2 dE,
\]

\[
\Delta_1 = \int_{m}^{E_m} \int_{E_2^\text{min}}^{E_2^\text{max}} A_0' dE_2 dE,
\]

\[
\Psi_i = \int_{m}^{E_m} \int_{E_2^\text{min}}^{E_2^\text{max}} \Phi_i dE_2 dE,
\]

\[
\Psi_{iF} = \int_{m}^{E_b} \int_{M_2^\text{min}}^{E_2^\text{min}} \Phi_{iF} dE_2 dE,
\]
\begin{table}[h]
\centering
\begin{tabular}{cccccccccccc}
\hline
$\sigma$ & (a) & (b) \\
\hline
0.8067 & 0.5 & 0.1 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.1 \\
0.8043 & 50.7 & 0.3 & 0.1 & 0.0 & 0.0 & 0.0 & 0.0 & 0.1 & 0.1 & 0.3 \\
0.8020 & 1.2 & 0.3 & 0.2 & 0.1 & 0.1 & 0.1 & 0.0 & 0.1 \\
0.7997 & 5.4 & 0.7 & 0.3 & 0.2 & 0.1 & 0.1 & 0.1 & 0.1 \\
0.7974 & 1.6 & 0.6 & 0.3 & 0.2 & 0.1 & 0.1 & 0.1 & 0.1 \\
0.7951 & 4.4 & 1.1 & 0.5 & 0.3 & 0.2 & 0.1 & 0.1 & 0.1 \\
0.7928 & 19.8 & 2.1 & 0.9 & 0.4 & 0.2 & 0.1 & 0.1 & 0.1 \\
0.7904 & 5.2 & 1.5 & 0.7 & 0.3 & 0.2 & 0.1 & 0.1 & 0.1 \\
0.7881 & 3.2 & 1.1 & 0.3 & 0.2 & 0.1 & 0.1 & 0.1 & 0.1 \\
0.7858 & 9.8 & 2.2 & 0.2 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\
\hline
\hline
$\delta$ & 0.05 & 0.15 & 0.25 & 0.35 & 0.45 & 0.55 & 0.65 & 0.75 & 0.85 & 0.95 \\
\hline
\end{tabular}
\caption{Percentage $\delta\alpha_B(E, E_2)$ with RC over the TBR in $\Sigma^- \rightarrow n e\bar{\nu}$ decay (a) to order $\mathcal{O}(\alpha/\pi)$ and (b) to order $\mathcal{O}(\alpha q/\pi M_1)$.}
\end{table}
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Table 2: Percentage \( \delta \alpha_B(E, E_2) \) with RC over the TBR in \( \Sigma^- \rightarrow ne\bar{\nu} \) decay (a) to order \( O(\alpha q/\pi M_1) \) and (b) computed by Gluck and Toth [Phys. Rev. D 46, 2090 (1992)].
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<th>$\delta$</th>
<th>0.05</th>
<th>0.15</th>
<th>0.25</th>
<th>0.35</th>
<th>0.45</th>
<th>0.55</th>
<th>0.65</th>
<th>0.75</th>
<th>0.85</th>
<th>0.95</th>
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Table 3: Percentage $\delta \alpha_B(E, E_2)$ with RC over the TBR and FBR in $\Sigma^- \to n e \nu$ decay (a) to order $O(\alpha/\pi)$ and (b) computed by Gluck and Toth [Phys. Rev. D 46, 2090 (1992)].
**Integrated $\alpha_B$ with RC: TBR Contribution**

<table>
<thead>
<tr>
<th>Decay</th>
<th>$\alpha_B^0$</th>
<th>$\delta\alpha_B$</th>
<th>$\mathcal{O}(\alpha q/\pi M_1)$</th>
<th>$\mathcal{O}(\alpha/\pi)$</th>
<th>Toth &amp; Gluck ’92</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Lambda \to p e \bar{\nu}$</td>
<td>-58.6</td>
<td>-0.09</td>
<td>-0.2</td>
<td>-0.1</td>
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<tr>
<td>$\Sigma^+ \to n e \bar{\nu}$</td>
<td>66.7</td>
<td>0.05</td>
<td>0.1</td>
<td>0.0</td>
<td></td>
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<tr>
<td>$\Sigma^- \to \Lambda e \bar{\nu}$</td>
<td>7.2</td>
<td>0.08</td>
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</table>

Table 4: Values of $\alpha_B$ and comparison with other works. We set $f_1 = 1.27$, $g_1 = 0.89$, and $f_2 = 1.20$ for $\Lambda \to p e \bar{\nu}$; $f_1 = 1$, $g_1 = -0.34$, and $f_2 = -0.97$ for $\Sigma^- \to n e \bar{\nu}$; and $f_1 = 0$, $g_1 = 0.60$, and $f_2 = 1.17$ for $\Sigma^- \to \Lambda e \bar{\nu}$. 
Weak Form Factors

Fits to the data on HSD used to be made under the assumption of exact SU(3) symmetry in order to extract $V_{us}$. Currently, the experiments are precise enough to the extent that this assumption no longer provides a reliable fit.

Thus, the determination of $V_{us}$ from HSD requires an understanding of the SU(3) symmetry breaking effects in the form factors.

For the leading form factors:

- $f_1$ is protected by the Ademollo-Gatto theorem against SU(3) breaking corrections to lowest order in $(m_s - \hat{m})$.
- $g_1$ gets first-order SU(3) breaking effects so it introduces larger theoretical uncertainties.
The remaining form factors

- The contributions of $f_2$ to the different observables of HSD in the SU(3) limit are first-order symmetry breaking contributions because of the kinematic factor of $q$.

- Reasonable shifts\textsuperscript{11} from the SU(3) predictions of $f_2$ do not have any observable effect upon $\chi^2$ or $g_1$ in a global fit to experimental data. $f_2$ can be used in its SU(3) symmetric value.

- The data are not accurate enough for an extraction of the small $g_2$-dependence of the decay amplitudes, so we use the value $g_2 = 0$.

- Contributions of $f_3$ and $g_3$ in the different HSD observables are proportional to the square mass electron and can be safely ignored.

Some Approaches to Determine HSD Form Factors

- Quark model (relativistic and non-relativistic)
  - F. Schlumpf, PRD 51, 2262 (1995): $f_1, g_1$
  - J. Donoghue, et. al., PRD 35, 934 (1987): $f_1, g_1$

- Chiral perturbation theory
  - J. Anderson et. al., PRD 47, 4975 (1993): $f_1$
  - E. Jenkins and A. Manohar, PLB 255, 558 (1991); 259, 353 (1991): $g_1$
  - A. Faessler et al, Phys. Rev. D77, 114007 (2008): $f_1, g_1$

- The $1/N_C$ expansion
  - J. Dai, E. Jenkins, A. Manohar, PRD 53, 273 (1996): $g_1$
  - RFM, E. Jenkins, A. Manohar, PRD 58, 094028 (1998): $f_1, g_1$
  - RFM, PRD 70, 114036 (2004): $f_1, g_1$

- The combination of HBCHPT and the $1/N_C$ expansion
  - RFM, C.P. Hofmann, PRD 74, 094001 (2006): $g_1$

- Lattice Gauge Theory
Fits to the Experimental Data

The experimentally measured quantities in HSD are

- the total decay rate $R$,
- the angular correlation coefficients $\alpha_{e\nu}$, and
- the angular spin-asymmetry coefficients $\alpha_{e}$, $\alpha_{\nu}$, $\alpha_{B}$, $A$, and $B$

An alternative choice are $R$ and the ratio $g_1/f_1$.

Fits in HSD should include

- the radiative corrections to the different observables
- the momentum-transfer contribution of the form factors

$$f_1(q^2) = f_1(0) \left( 1 + 2 \frac{q^2}{M_V} \right), \quad g_1(q^2) = g_1(0) \left( 1 + 2 \frac{q^2}{M_A} \right),$$

where $M_V, M_A \sim 1 \text{ GeV}$.
Fitting the data [RFM, PRD 70, 114036 (2004)]

<table>
<thead>
<tr>
<th>Process</th>
<th>RFM</th>
<th>Anderson &amp; Luty</th>
<th>Donoghue et al.</th>
<th>Krause</th>
<th>Schlumpf</th>
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</thead>
<tbody>
<tr>
<td>$\Lambda \to p$</td>
<td>$1.02 \pm 0.02$</td>
<td>1.024</td>
<td>0.987</td>
<td>0.943</td>
<td>0.976</td>
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<tr>
<td>$\Sigma^- \to n$</td>
<td>$1.04 \pm 0.02$</td>
<td>1.100</td>
<td>0.987</td>
<td>0.987</td>
<td>0.975</td>
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<tr>
<td>$\Xi^- \to \Lambda$</td>
<td>$1.04 \pm 0.04$</td>
<td>1.059</td>
<td>0.987</td>
<td>0.957</td>
<td>0.976</td>
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<tr>
<td>$\Xi^- \to \Sigma^0$</td>
<td>$1.07 \pm 0.05$</td>
<td>1.011</td>
<td>0.987</td>
<td>0.943</td>
<td>0.976</td>
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<tr>
<td>$\Xi^0 \to \Sigma^+$</td>
<td>$1.07 \pm 0.05$</td>
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</table>

$V_{us} = 0.2199 \pm 0.0026$  
$V_{us} = 0.2177 \pm 0.0019$  
$V_{us} = 0.2244 \pm 0.0019$  
$V_{us} = 0.2274 \pm 0.0019$  
$V_{us} = 0.2256 \pm 0.0019$

Table 5: SB pattern for $f_1$. The entries correspond to $f_1/f_1^{SU(3)}$. The fit to data was performed by using the decay rate and asymmetry coefficients.
From $K_{l3}$ decays (Particle Data Group, 2008)

$$V_{us} = 0.2255 \pm 0.0019$$


$$V_{us} = 0.2250 \pm 0.0027$$

with no indication of flavor SU(3)-breaking effects.

In the context of the $1/N_c$ expansion

$$V_{us} = 0.2238 \pm 0.0019 \quad \text{(No SB effects)}$$

$$V_{us} = 0.2230 \pm 0.0019 \quad \text{(First order SB in $g_1$)}$$

$$V_{us} = 0.2199 \pm 0.0026 \quad \text{(Second order SB in $f_1$)}$$
Conclusions

• Analyses in HSD still face some interesting theoretical problems.
• On the one hand, the computation of RC to order $\mathcal{O}(\alpha q/\pi M_1)$ is under reasonable control. Model-independent theoretical expression for the Dalitz plot which cover both the three-body and the four-body regions are now available.
• The results are suitable for high-statistics experiments.
• On the other hand, a deep understanding of the SU(3) symmetry breaking effects in the weak form factors is not settled down yet.
• The $1/N_c$ expansion has been useful in the analysis of SB effects. Fits to data prefer the values $f_1/f_1^{SU(3)}>1$ (up to 7%), opposite to the value $<1$ predicted by the quark model (relativistic and non-relativistic).
• The value of $V_{us}$ from HSD is similar in precision to the one derived from $K_{l3}$.
• More work, theoretical and experimental, will be welcome in the near future.