THE ANALYSIS OF PREDICTABILITY OF RECENT ALPHA DECAY FORMULAE AND THE ALPHA PARTIAL HALF-LIVES OF SOME EXOTIC NUCLEI

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1. Introduction

Alpha decay is one of the two main decay modes of the heaviest nuclei, (SHE), and constitutes one of the dominant decay modes of highly neutron deficient medium mass nuclei ("exotics"). Thus identifying and characterizing the alpha decay chains form a crucial part of the identification of SHE. We report the extension of the previously developed method [1] for the detailed and systematic investigation of the reliability of the three main extant analytical formulae of alpha decay half-lives: the generalized liquid drop model based formula of Royer et al [2] (FR), the Sobiczewski modified semi-empirical Viola-Seaborg formula [3] (VSS) and the recent phenomenological formula of Sobiczewski and Parkhomenko [4] (SP).

2. Method

The ansatz based on standard bench-marking procedures in experiment. A fixed-size basis set of nuclides (the REF nuclides) with very well-characterized alpha spectroscopic and mass data are chosen to derive anew the coefficients of each of the three formulae, called the ‘calibrated formulae’. REF are the alpha energy and intensity standards used in alpha spectrometry [5]. The experimental \( Q_\alpha \), \( Q_\alpha(E) \), as well as the theoretical \( Q_\alpha \), derived from the Finite Range Droplet Model, the \( Q_\alpha(FRDM) \) [6] were used.

"Predictability" is used to connote the extent of accuracy of the three formulations itowards the prediction of the half-lives of an independent set of nuclei, i.e. nuclei whose data have not been used to arrive at the values of the coefficients. This test of accuracy necessitates the experimental nuclear data of this independent set of nuclides, called TEST, to be also well known. The TEST were taken to be nuclides that covered a (Z,A) range close to the REF nuclides to reduce the possibility of the influence of Z,A dependent differences in the underlying nuclear parameters in the data analysis, and which also possess well-known experimental data [5].

The coefficients of the calibration formulae were derived using a multi-variable regression fitting procedure using REF as the basis set. For each formula, two sets of coefficients one using the experimental half-life \( T(E) \) and \( Q_\alpha(E) \) and the other using the \( T(E) \) and the \( Q_\alpha(FRDM) \), were derived. In the case of the VSS and SP type formulae, we retain the authors’ original approach for the formulae and derive the \( a,b,c,d \) coefficients only using the e-e set, the \( h_{log} \) and \( E_i \) using the odd sets, as well as calculate the coefficients explicitly for each parity set. These formulae with re-derived coefficients are termed the FR-DS, VSS-DS and SP-DS.

3. Analysis of Predictability

For all formulae, the use of \( Q_\alpha(E) \) to derive the coefficients and for the application of the resulting formula, resulted in the lowest root mean square fractional deviation of the log half-lives (RMSFDL), despite the fact that for both the REF and TEST sets the RMSFD values for \( Q_\alpha(FRDM) \) are very low. This shows the sensitivity of the phenomena and the formulae to slight changes in decay energetics. For the VSS and SP formulae, the approach adopted in the present work (VSS-DS, SP-DS) was found to consistently produce lower average RMSFDL. The overall predictive accuracy of the 3 calibrated formulae show that the VSS-DS formula with its recalculated coefficients produced the least average RMSFDL closely followed by the FR-DS
formula and then by the SP-DS formula.

4. The calibrated formulae and the $T^{1/2}$ of SHE and Exotic alpha emitters.

The question of how well the calibration formulae obtained over an accurate but narrow basis set, can predict the half-lives of newly discovered nuclei, was also investigated. The original FR shows the lowest mean RMSFDL value, followed by the VSS and next by the SP. This is clearly the effect of the much larger basis sets for the coefficient derivation in the original formulae and the fact that these sets contained members close to the SHE and exotics.

For the calibrated formulae, the effect of the size of REF basis set is clearly discerned to affect RMSFDL: e-e with the largest number of nuclides (31) performs the best, even better than the FR original formula. To investigate the effect of the increasing the number of members for the calibration formulae without changing the values of the coefficients, the calibrated formulae were fitted by linear regression to the logT(E) of the TEST nuclides. The linearly optimized calibration formulae, were then used to calculate the logT(E) values of the SHE and exotics. The results show that the average RMSFDL is now lowered considerably; comparable to the average for the FR. This shows that the method of calibrating a formula against a REF set can also be used to reliably predict the $\alpha$ partial half-lives of a set of unknown nuclides if the REF set is sufficiently large. Thus, it is important to carry out experimental alpha spectroscopic and mass determinations to firmly establish the decay data of many more alpha emitters than the current alpha standards nuclei so that the REF basis set may be expanded.

5. Conclusion

A method to benchmark and check the predictability of the 3 extant alpha decay formulae based on the ansatz of experimental practice, is proposed. We use a calibration data set, REF, of the alpha measurement standards to derive a reliable set of formula coefficients. The calibrated formulae are used to check the predictability of the alpha partial half-lives of a set of well-studied and independent nuclei, the TEST. The Viola-Seaborg-Sobiczewski and the GLDM formula, FR, give the best matches with the VSS slightly better. The calibrated formulae with their reliable coefficients were also found to give good correspondences with the newly discovered SHE and exotics, but only for the cases where the basis set of REF was large, or enlarged by a linear optimization procedure. Amongst these the GLDM based formula of Royer [2] performed the best. In all, 408 nuclides (REF+TEST+SHE&Exotics) were analyzed.

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References:

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