Locating the critical end point using the linear sigma model coupled to quarks.

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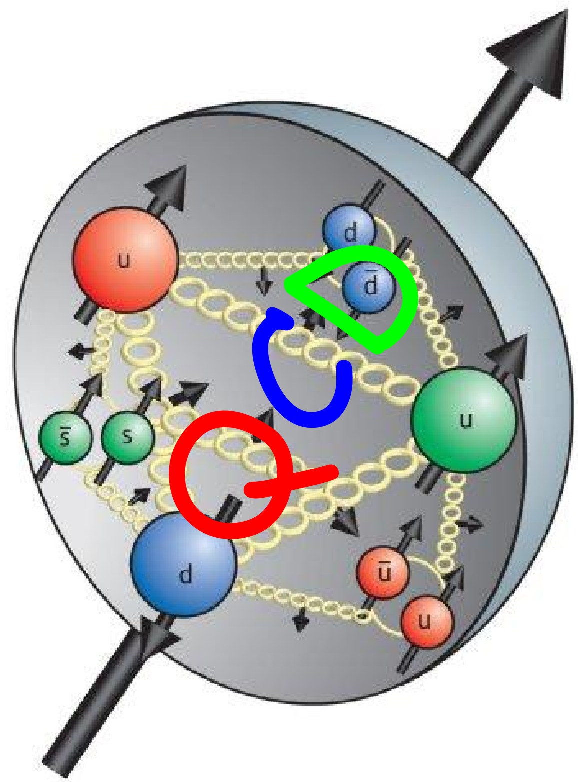
Outline

- Motivation
- The Linear Sigma model
  - High Temperature Approximation
  - Low Temperature Approximation
- Preliminary Results
- Final Comments

Motivation

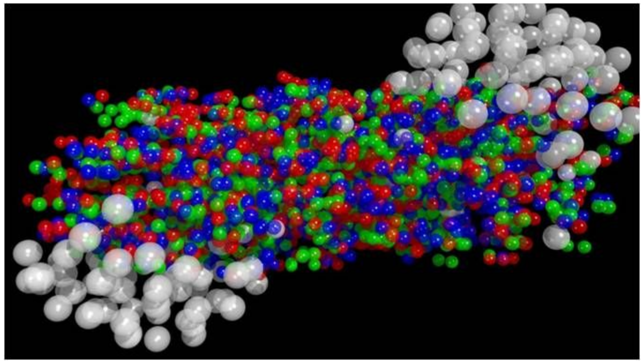
There are several phases of QCD. We are interested in studying the transition between these phases.

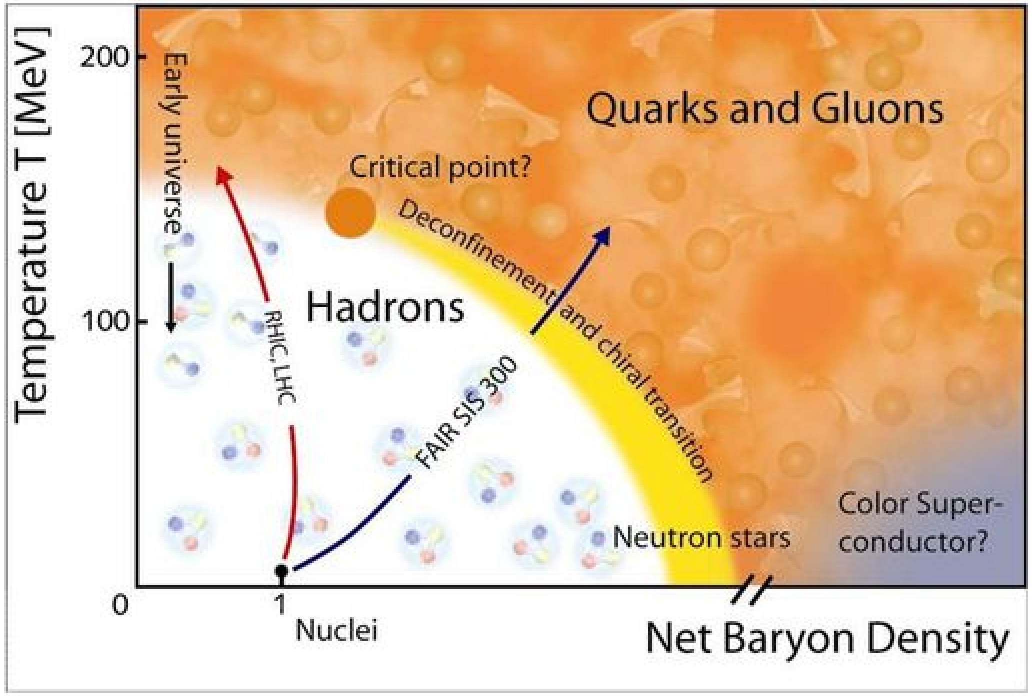
- **High Energies**
  - Asymptotic freedom
- **Low Energies**
  - Confinement



Motivation

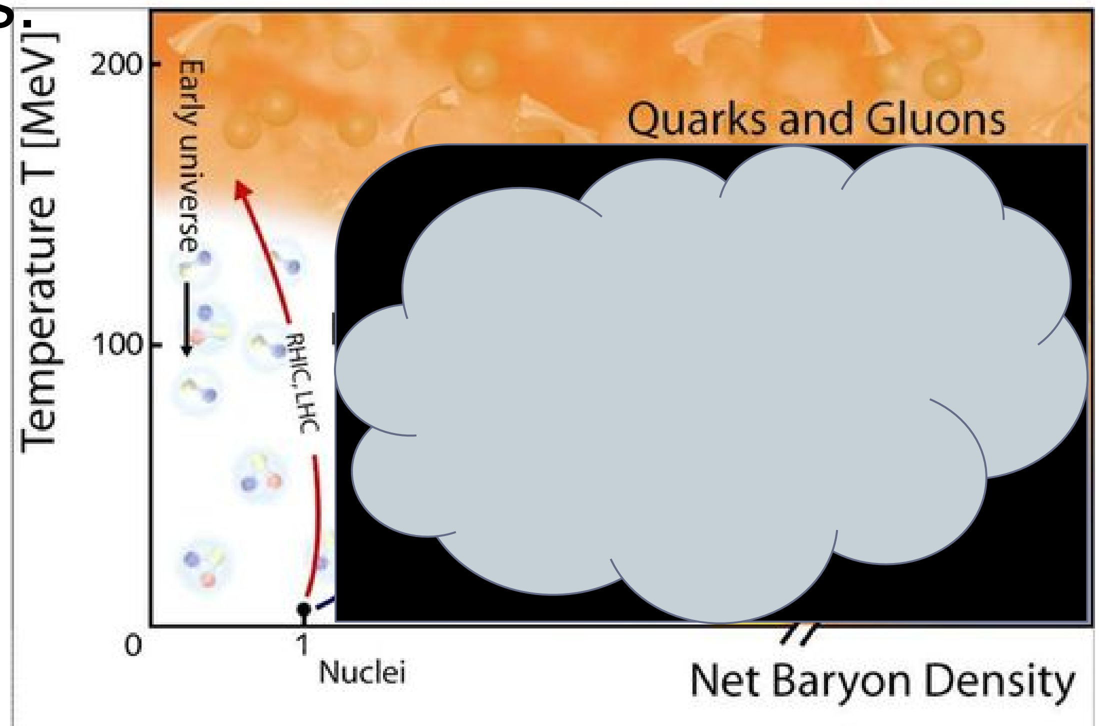
- QCD under extreme conditions (temperature and finite quark density) play an important role in understanding the transitions that took place in the early universe.





Motivation

- There is only reliable information at low densities.
- There are experimental efforts to dissipate doubts at higher densities.
  - NICA
  - RHIC(BES)
  - JPARC
  - FAIR



Linear Sigma Model

- Effective model for low-energy QCD.

- Effects of quarks and mesons on the chiral phase transition.

- Implement ideas of chiral symmetry and spontaneous symmetry breaking

Linear Sigma Model

- Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\vec{\pi})^2 + \frac{a^2}{2}(\sigma^2 + \vec{\pi}^2) - \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2)^2 + i\bar{\psi}\gamma_\mu\partial^\mu\psi - g\bar{\psi}(\sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi})\psi,$$

- To allow for spontaneous symmetry breaking

$$\sigma \to \sigma + v$$

$$\langle\sigma\rangle = v; \qquad \langle\pi\rangle = 0.$$

- where v is identified as the order parameter

Linear Sigma Model

- After the shift

$$\mathcal{L} = -\frac{1}{2}[\sigma(\partial_\mu + iqA_\mu)^2\sigma]\underline{-\frac{1}{2}\left(3\lambda v^2 - a^2\right)\sigma^2}$$
$$-\frac{1}{2}[\vec{\pi}(\partial_\mu + iqA_\mu)^2\vec{\pi}]\underline{-\frac{1}{2}\left(\lambda v^2 - a^2\right)\vec{\pi}^2}$$
$$+ i\bar{\psi}\gamma^\mu D_\mu\psi\underline{-gv\bar{\psi}\psi}+\frac{a^2}{2}v^2 - \frac{\lambda}{4}v^4$$
$$-\frac{\lambda}{4}[(\sigma^2 + \pi_0^2)^2 + 4\pi^+\pi^-(\sigma^2 + \pi_0^2 + \pi^+\pi^-)]$$
$$-g\hat{\psi}(\sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi})\psi$$

with masses

$$m_\sigma^2 = 3\lambda v^2 - a^2$$
$$m_\pi^2 = \lambda v^2 - a^2$$
$$m_f = gv$$

$$a = \sqrt{\frac{m_\sigma^2 - 3m_\pi^2}{2}}$$

Linear Sigma Model

- We calculate the effective potential for fermions and bosons at finite temperature and quark chemical potential beyond the mean field approximation.

$$V_b = s_b T \sum_n \int \frac{d^3k}{(2\pi)^3} \ln\left(D^{-1}\right)^{1/2}, \qquad V_f = s_f T \sum_n \int \frac{d^3k}{(2\pi)^3} \ln\left(S^{-1}\right).$$

$$V^{ring} = \frac{1}{2} s_b T \sum_n \int \frac{d^3k}{(2\pi)^3} \ln\left[1 + \Pi D\right]$$

where boson and fermion propagators are given by

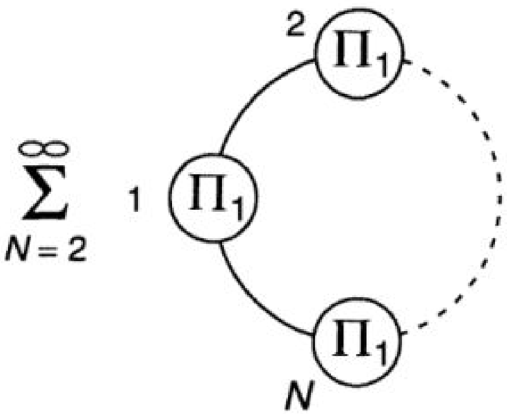
$$D = \frac{1}{k^2 + m_b^2 + \omega_n^2}, \qquad S = \frac{\not{k} + m_f}{k^2 + m_f^2 + (\omega_n - i\mu)^2}.$$

High temperatures

with Π the self energy

$$\Pi = \lambda\frac{T^2}{2} - N_f N_c g^2 \frac{T^2}{\pi^2}[Li_2(-e^{\mu/T}) + Li_2(-e^{-\mu/T})]$$

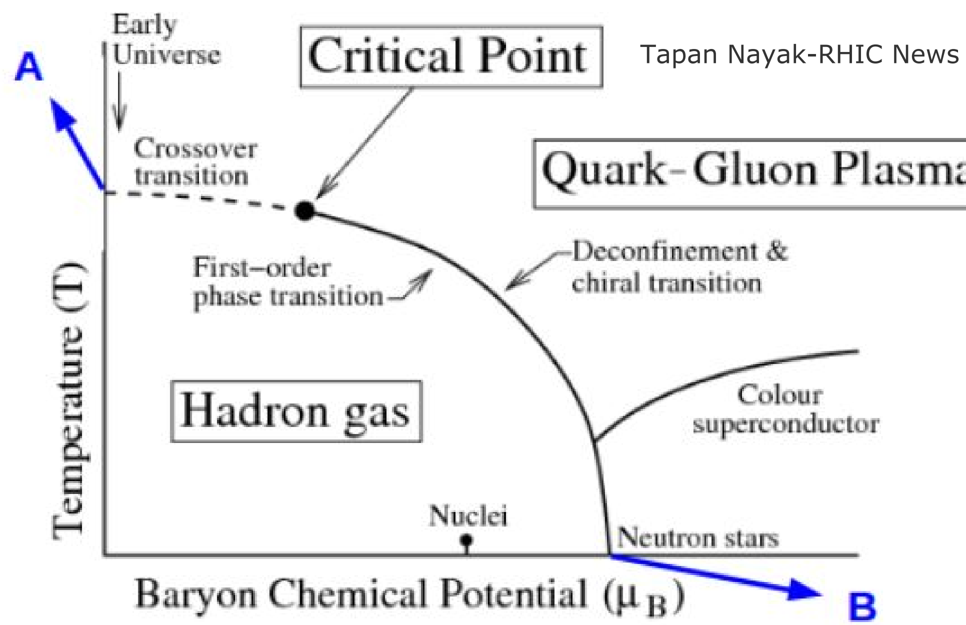
- For high temperatures we include the next term in the perturbative series, the ring diagrams (Dolan & Jackiw, Phys. Rev. D12 3320 (1974)) that considers screening properties of the plasma



M. Le Bellac, Thermal Field Theory (Cambridge Univ. Press, Cambridge 2000).

$$
\begin{aligned}
V_{HT}^{(eff)} = & \boxed{-\frac{a^2}{2}v^2 + \frac{\lambda}{4}v^4} + \sum_{i=\sigma,\pi^0} \left\{ \frac{m_i^4}{64\pi^2} \left[\ln\left(\frac{(4\pi T)^2}{2a^2} \right) - 2\gamma_E + 1 \right] \right. \\
& \left. - \frac{\pi^2 T^4}{90} + \frac{m_i^2 T^2}{24} - \frac{T}{12\pi} \boxed{(m_i^2 + \Pi)^{3/2}} \right\} \\
& - N_c \sum_{f=u,d} \left[\frac{m_f^4}{16\pi^2} \left[\ln\left(\frac{(4\pi T)^2}{2a^2} \right) + \psi^0\left(\frac{1}{2} + \frac{i\mu}{2\pi T} \right) \right. \right. \\
& \left. + \psi^0\left(\frac{1}{2} - \frac{i\mu}{2\pi T} \right) \right] + 8m_f^2 T^2 [Li_2(-e^{\mu/T}) + Li_2(-e^{-\mu/T})] \\
& \left. - 32T^4 [Li_4(-e^{\mu/T}) + Li_4(-e^{-\mu/T})] \right]
\end{aligned}
$$

The parameter space consists of the λ and g coupling constants which are determined uniquely by the Goldstone boson mass at two fixed points



(A) $$m_\pi^2(v=0, T=T_c, \mu=0) = -a^2 + \Pi_{HT} = 0$$

(B) $$m_\pi^2(v=v_1, T=0, \mu=\mu_c) = \lambda v_1^2 - a^2 + \Pi_{LT} = 0$$

Low Temperature

- For high quark chemical potential, first we compute the effective potential at T=0 and finite μ, i.e.

$$V_f^0 = N_c \sum_{f=u,d} \left\{ \frac{m_f^4}{16\pi^2} \left[\ln\left(\frac{4\pi a^2}{\mu + \sqrt{\mu^2 - m_f{}^2}} \right) + \frac{1}{2} - \gamma_E \right] - \frac{\mu\sqrt{\mu^2 - m_f{}^2}}{24\pi^2} \left(2\mu^2 - 5m_f{}^2\right) \right\}$$

$$V_b^0 = -\sum_{i=\sigma,\vec{\pi}} \left\{ \frac{m_i{}^4}{64\pi^2} \left[\ln\left(\frac{4\pi a^2}{\mu_b + \sqrt{\mu_b^2 - m_f{}^2}} \right) + \frac{1}{2} - \gamma_E \right] - \frac{\mu_b\sqrt{\mu_b^2 - m_f{}^2}}{96\pi^2} \left(2\mu_b^2 - 5m_f{}^2\right) \right\}$$

μb is a bosonic density that it is related with the quark chemical potential.

Low Temperature

- The low-T approximation can be obtained from its expression at T = 0 as

$$V_f(T,\mu) = \sum_{f=u,d} \int_{\frac{\mu - m_f}{T}}^{\infty} V_f^0(\mu + xT) h_F(x) dx,$$

$$V_b(T,\mu_b) = \sum_{i=\sigma,\vec{\pi}} \int_{\frac{\mu_b - m_i}{T}}^{\infty} V_b^0(\mu + xT) h_B(x) dx$$

where $h_F(x)$ and $h_B(x)$ are the derivate of Fermi-Dirac and Bose-Einstein distributions. (C. O. Dib & R. Espinosa, Nucl. Phys. B **612**, 492)

Low Temperature

- Now, for $T \ll \mu, \mu_b$ both potentials can be expanded in a Taylor series in $T \to 0$ because they varies slowly under the hump and then obtain the low temperature expansion.

$$V_f(T,\mu) = V_f^0(\mu) + \frac{\pi^2}{6} T^2 \frac{\partial^2}{\partial T^2} V_f^0(\mu) + \frac{7\pi^4}{360} T^4 \frac{\partial^4}{\partial T^4} V_f^0(\mu)$$

$$V_b(T,\mu) = V_b^0(\mu) + \frac{\pi^2}{12} T^2 \frac{\partial^2}{\partial T^2} V_b^0(\mu) + \frac{\pi^4}{1260} T^4 \frac{\partial^4}{\partial T^4} V_f^0(\mu)$$

Low Temperature

$$V_{LT}^{eff}(T,\mu) = \boxed{-\frac{a^2}{2}v^2 + \frac{\lambda}{4}v^4} + N_c \sum_{f=u,d} \left\{ \frac{{m_f}^4}{16\pi^2} \left[\ln\left(\frac{4\pi a^2}{\mu + \sqrt{\mu^2 - {m_f}^2}} \right) + \frac{1}{2} - \gamma_E \right] \right.$$

$$- \frac{\mu\sqrt{\mu^2 - {m_f}^2}}{24\pi^2}\left(2\mu^2 - 5{m_f}^2\right) - \frac{T^2}{6}\mu\sqrt{\mu^2 - {m_f}^2} - \frac{7\pi^2 T^4}{360} \left. \frac{\mu\left(2\mu^2 - 3{m_f}^2\right)}{\left(\mu^2 - {m_f}^2\right)^{\frac{3}{2}}} \right\}$$

$$- \sum_{i=\sigma,\vec{\pi}} \left\{ \frac{{m_i}^4}{64\pi^2} \left[\ln\left(\frac{4\pi a^2}{\mu_b + \sqrt{\mu_b^2 - {m_i}^2}} \right) + \frac{1}{2} - \gamma_E \right] - \frac{\mu_b\sqrt{\mu_b^2 - {m_i}^2}}{24\pi^2}\left(2\mu_b^2 - 5{m_i}^2\right) \right.$$

$$\left. - \frac{T^2}{12}\mu_b\sqrt{\mu_b^2 - {m_i}^2} - \frac{\pi^2 T^4}{180} \frac{\mu_b\left(2\mu_b^2 - 3{m_i}^2\right)}{\left(\mu_b^2 - {m_i}^2\right)^{\frac{3}{2}}} \right\}$$

Coupling Constants

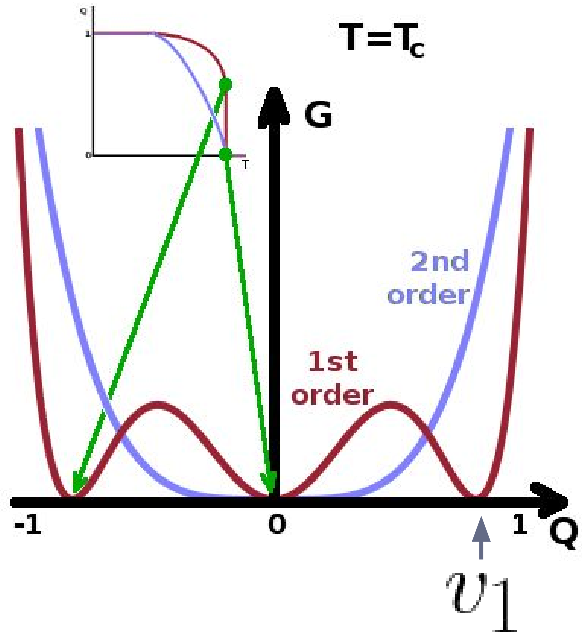
- Now, the system of equations to be solved for points A and B are

$$-a^2 + \frac{\lambda}{2}T_c^2 + N_c N_f \frac{g^2}{6} T_c^2 = 0$$

$$\lambda v_1^2 - a^2 - \frac{3\lambda}{4\pi^2}\mu_{bc}^2 + N_c N_f \frac{g^2}{2\pi^2}\mu_c^2 = 0$$

Coupling Constants

- Now the criterion to find the temperature and the chemical potential where the chiral symmetry is restored, is the following.



  - Second Order

$$\left.\frac{\partial^2}{\partial v^2} V^{eff}\right|_{v=0} = 0$$

  - First Order

$$V^{eff}(0) = V^{eff}(v_1); \quad \left.\frac{\partial}{v\partial v} V^{eff}\right|_{v=0} = \left.\frac{\partial}{v\partial v} V^{eff}\right|_{v=v_1} = 0$$

For Details...

Using the LSMq to describe the QCD phase diagram and to locate the CEP.

Flores, José Antonio.

Set of parameters:

Preliminary Results

$$T_c = 170 \text{ MeV} \quad \text{LQCD}^{1}$$

$$m_B = 100 \text{ MeV} \quad \text{Typical barion mass}$$

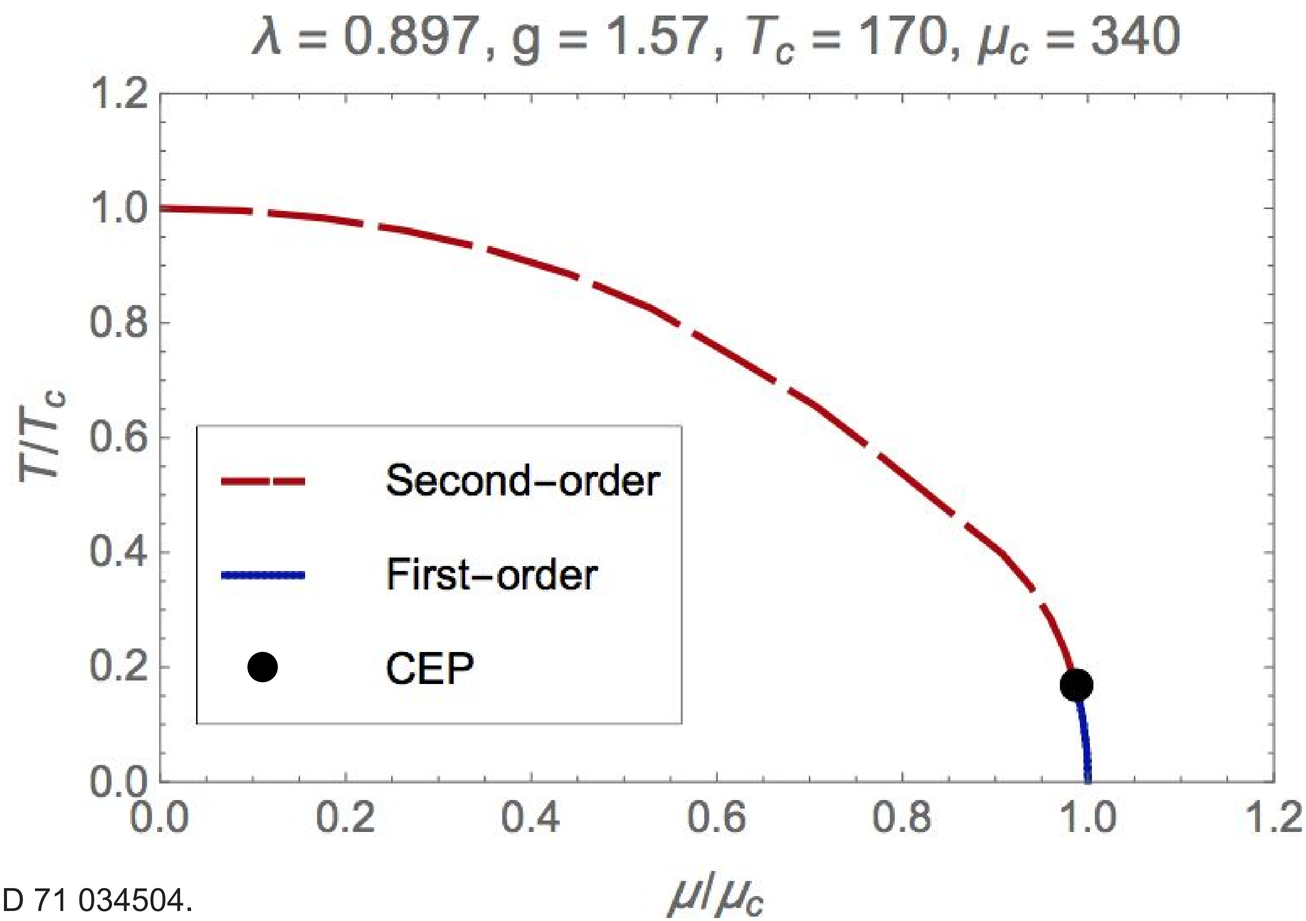
$$m_\sigma = 475 \text{ MeV}$$

$$m_\pi = 139 \text{ MeV}$$

$$N_c = 3$$

$$N_f = 2$$

$$\mu_b = \mu_c$$



[1] LC Collaboration, Phys. Rev. D 71 034504.

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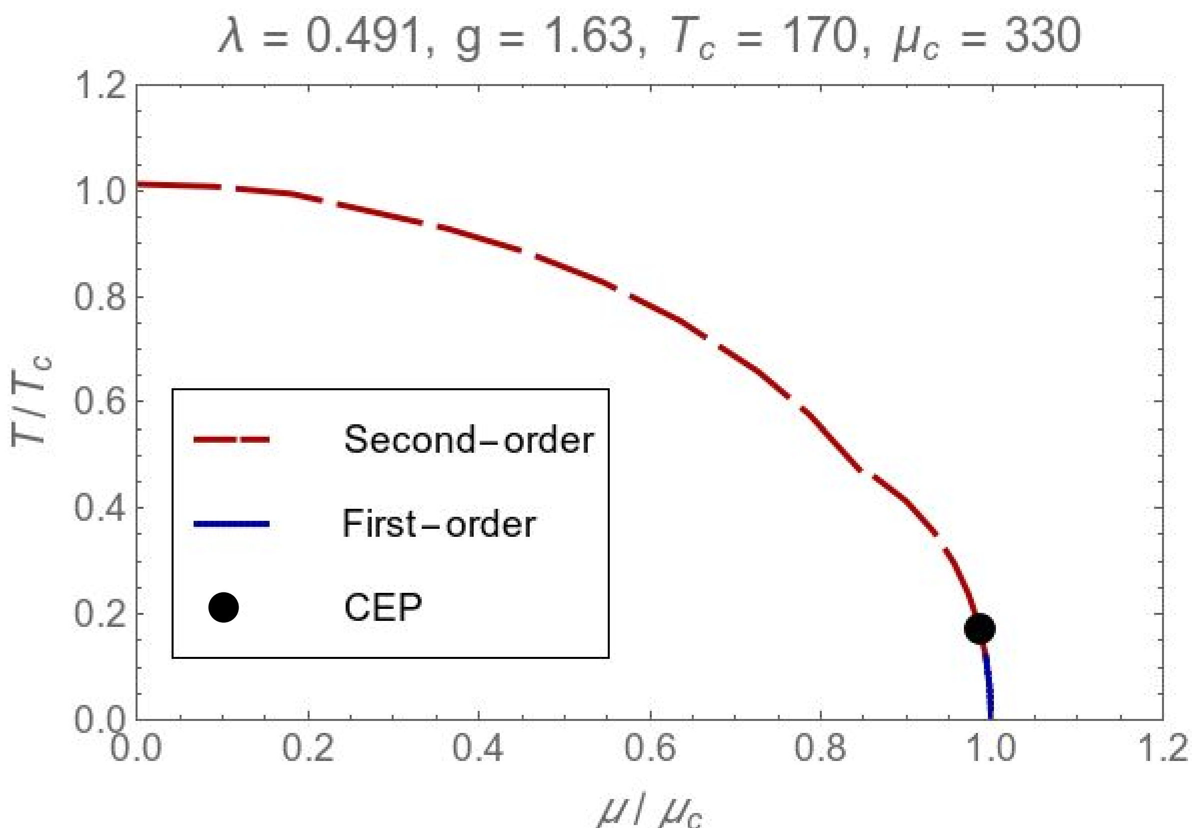
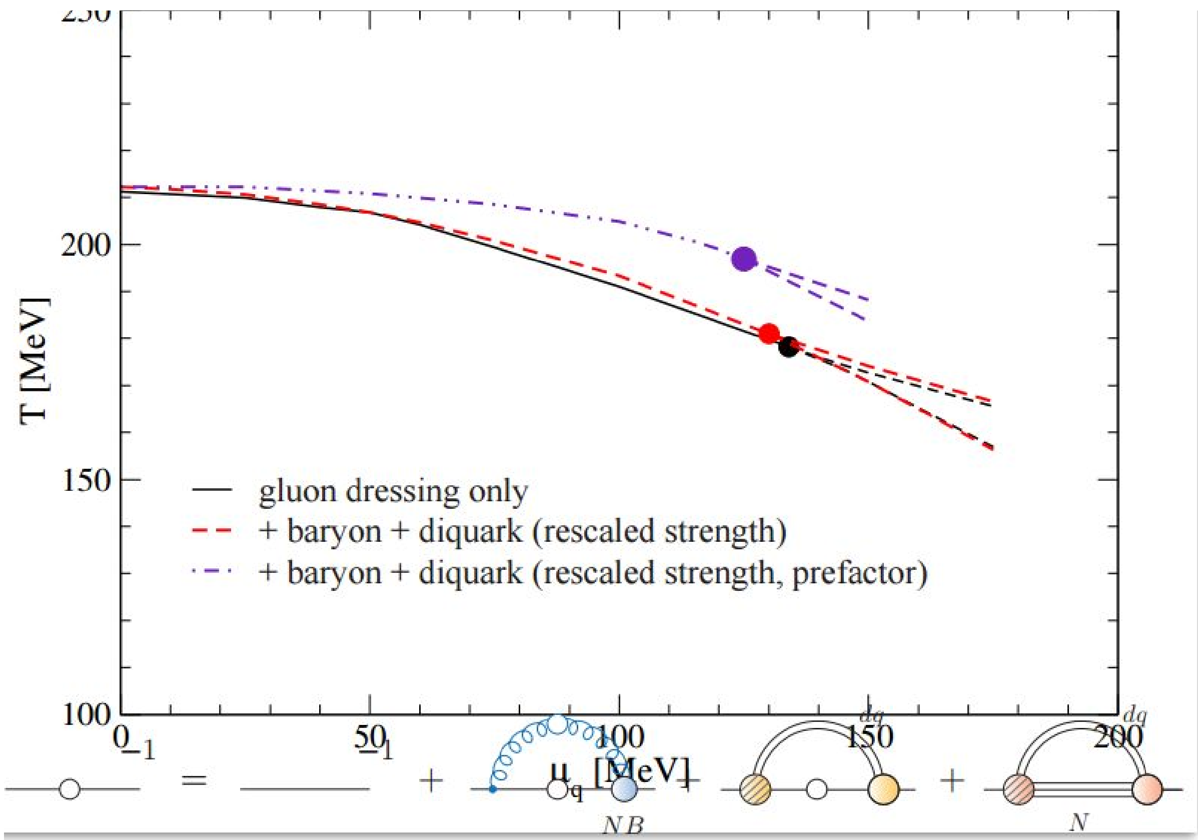


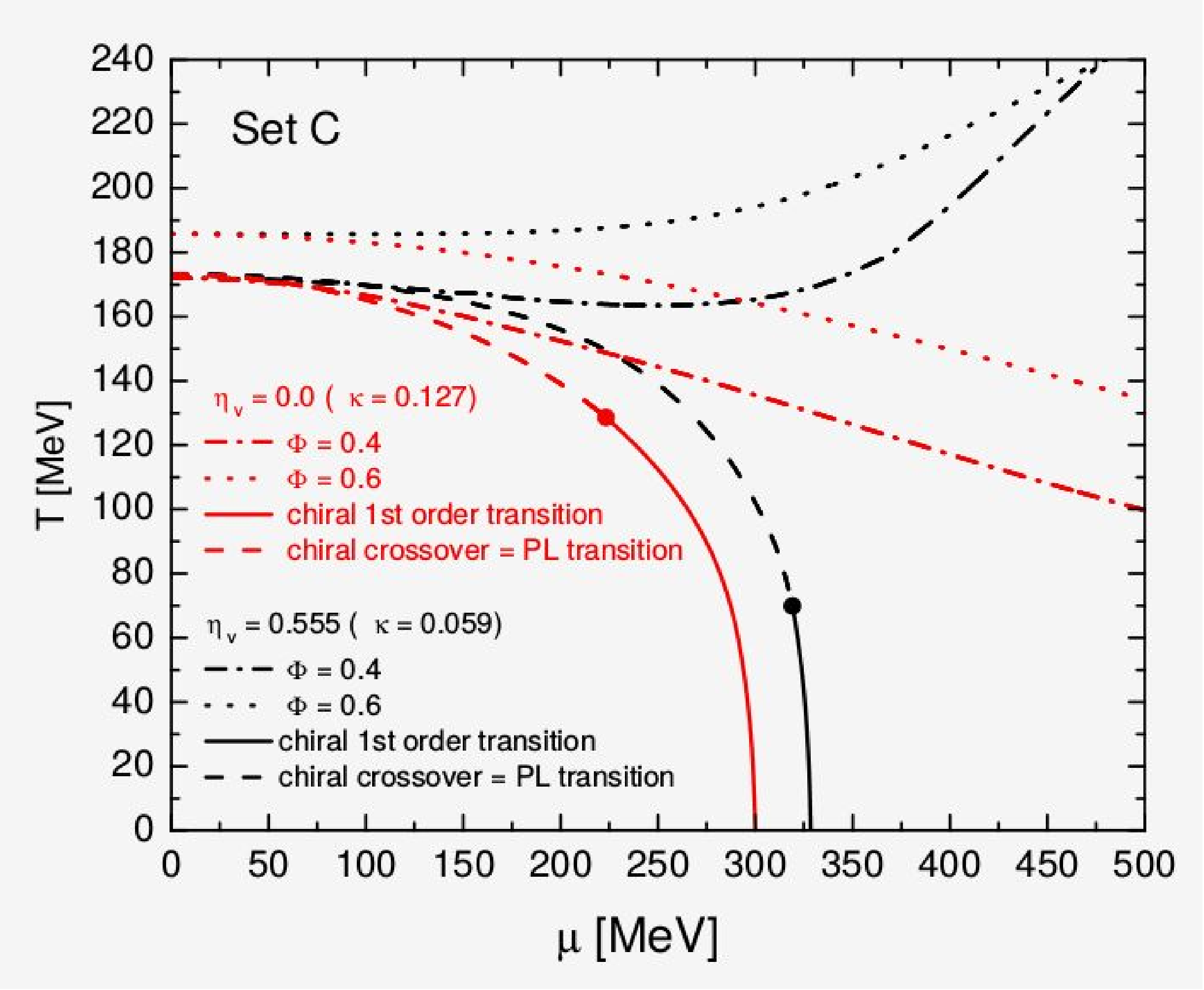
[1] LC Collaboration, Phys. Rev. D 71 034504.

Locating QCD's critical end point (with functional methods)

Christian Fisher.



Supporting the search for the CEP location with nonlocal PNJL models constrained by Lattice QCD



Final Comments

- Working in the LSMq, CEP is located in the region found by others effective models.

- We computed the effective potential and included plasma screening effects through the boson's self energy.

- We found the CEP at

$$\{\mu_{CEP} = 0.98\mu_c,\ T_{CEP} = 0.17T_c\}$$

$$\{\mu_{CEP} = 0.99\mu_c, T_{CEP} = 0.12T_c\}$$

Many Thanks!!!

Gracias!!!