Sound waves in hadronic matter

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Content:

(1) Introduction: log-periodic oscillations decorating quasi-power-like Tsallis distributions → scale-invariance → complex power index?

(2) Or, rather, → log-periodically oscillating scale parameter (temperature)?

(3) If so, one is dealing with sound waves in hadronic matter → self-similarity.

(4) Possible experimental confirmation.

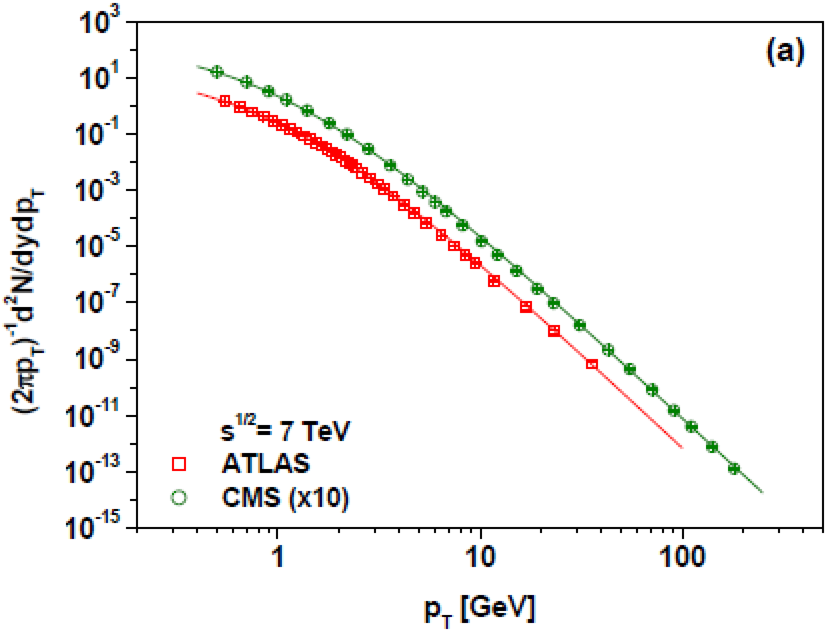
(1)

(*) The large transverse momentum distributions of particles observed in all LHC experiments exhibit a quasi-power-like behavior following the two-parameter Tsallis distribution with a scale factor T and nonextensivity q.

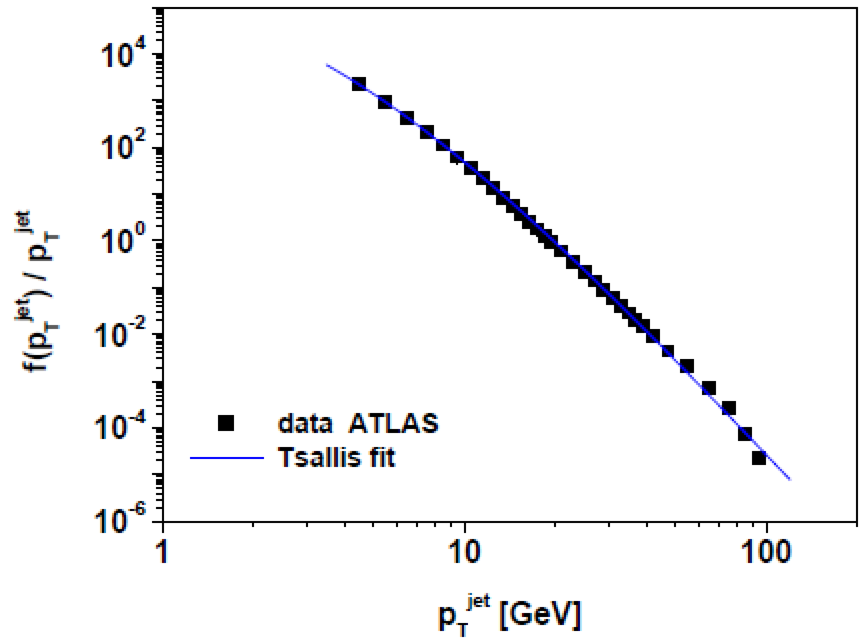
(1)

transverse momentum distributions are characterized by a quasi-power law (Tsallis distribution)

hadrons



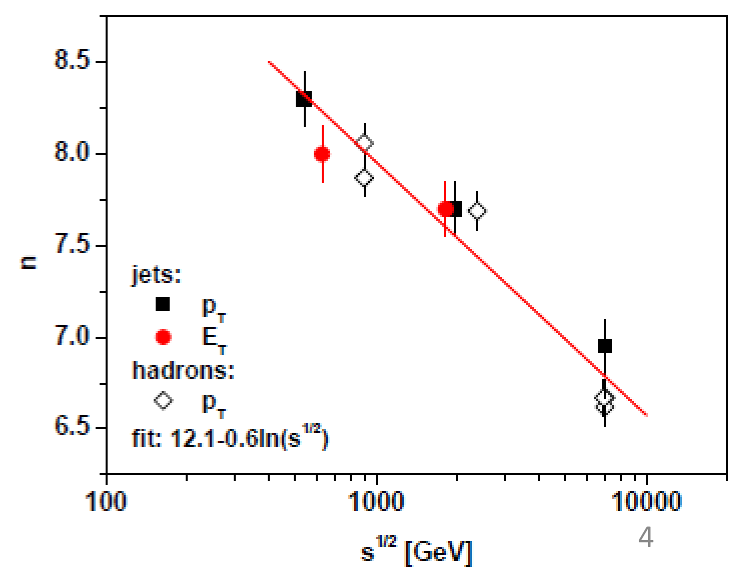
jets



$$f(E) = C\left(1+\frac{E}{mT}\right)^{-m}$$

$$T = 0.145 \;\; GeV$$

$$m = 6.7$$



Tsallis distribution successfully describes spectra, the flux of which changes by over 14 orders of magnitude.

The values of the corresponding power indices are similar, strongly indicating the existence of a common mechanism behind all these processes.

(1)

(*) The large transverse momentum distributions of particles observed in all LHC experiments exhibit a quasi-power-like behavior following the two-parameter Tsallis distribution with a scale factor T and nonextensivity q.

(*) However, looking at the ratios of the measured cross-sections to their phenomenological power-like fits, $R = f_{data}(p_T)/f_{fit}(p_T)$, one discovers some log-periodic oscillations in R.

(1)

Tsallis distribution decorated with log-periodic oscillation

transverse momentum distributions
are characterized by
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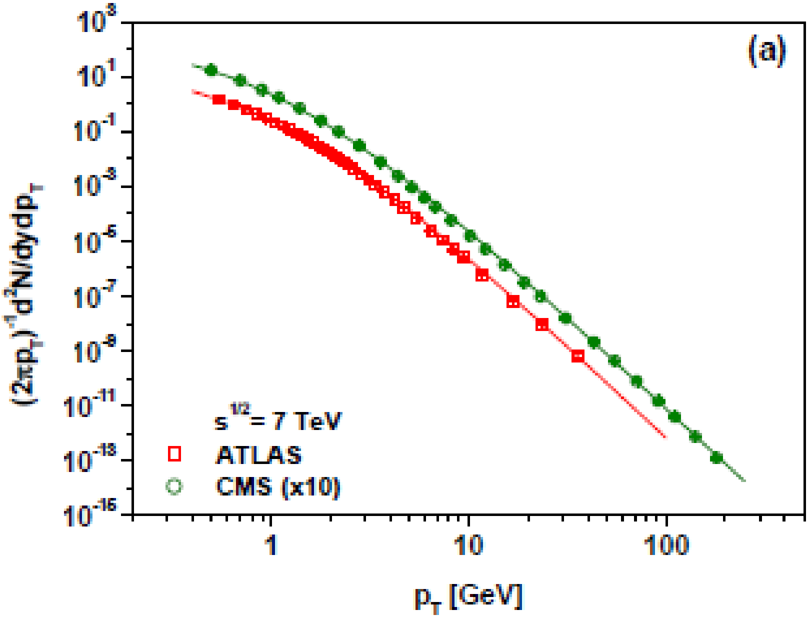
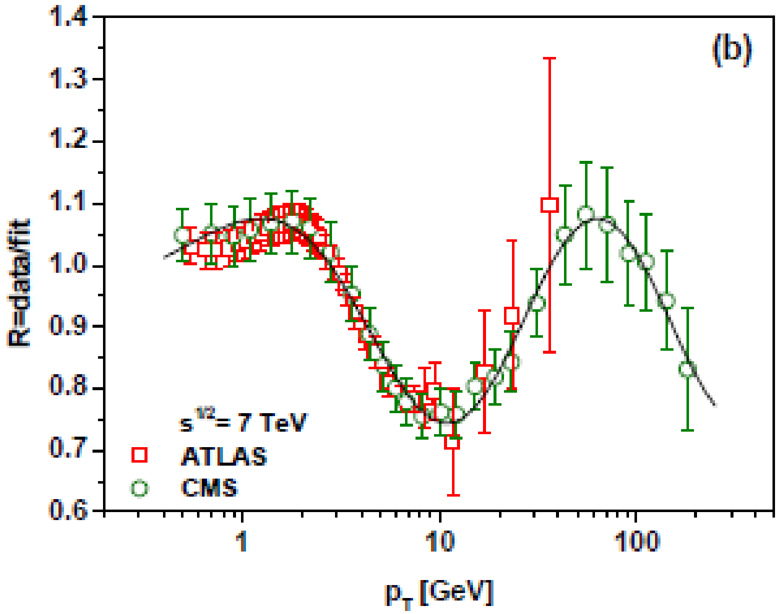
Tsallis distribution is decorated with
log-periodic oscillations





$$f(E) = C\left(1 + \frac{E}{mT}\right)^{-m}$$

$$T = 0.145\ GeV \qquad m = 6.7$$

$$R(E) = a + b\cos\left[c\ln(E + d) + f\right]$$

a = 0.909, b = 0.166, c = 1.86, d = 0.948 and f = −1.462

(1)

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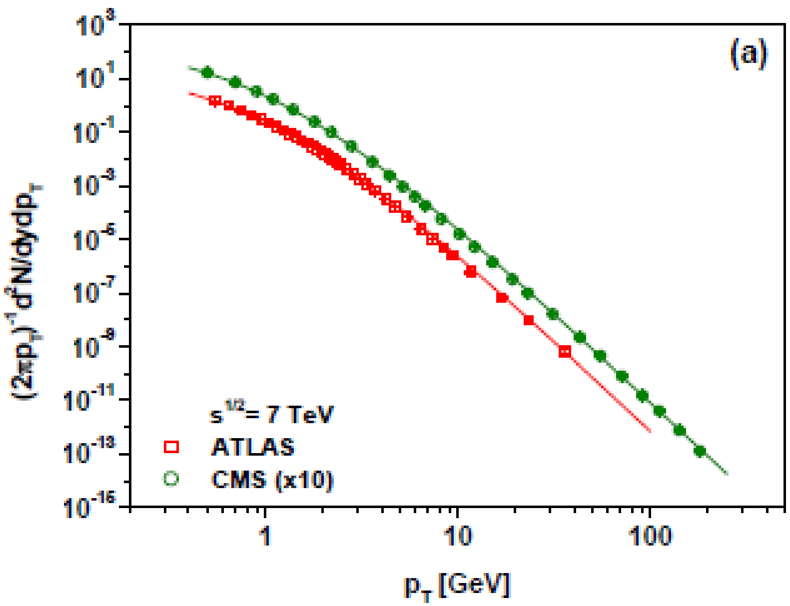
(*) However, looking at the ratios of the measured cross-sections to their phenomenological power-like fits, R = f_{data}(p_T)/f _{fit}(p_T), one discovers some log-periodic oscillations in R.

(*) This is a rather subtle effect, but it shows itself in all experiments, at all energies (provided that the range of transverse momenta observed is large enough) and also in reactions with nuclei where they grow with increasing centrality of the collision, it cannot be erased by any reasonable change of fitting parameters – in what follows we shall assume it to be real.

(1) Different experiments

transverse momentum distributions are characterized by a quasi-power law (Tsallis distribution)



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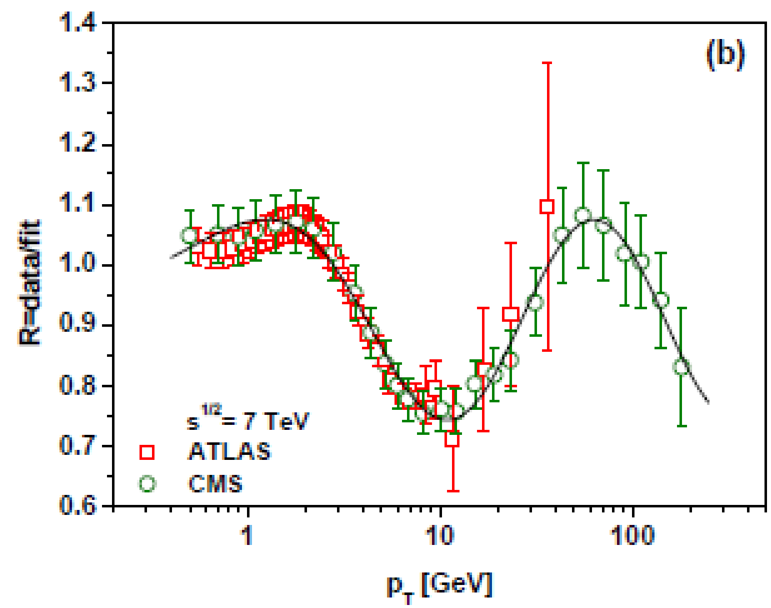
Tsallis distribution is decorated with log-periodic oscillations



R(E) = a + b cos [c ln(E + d) + f]

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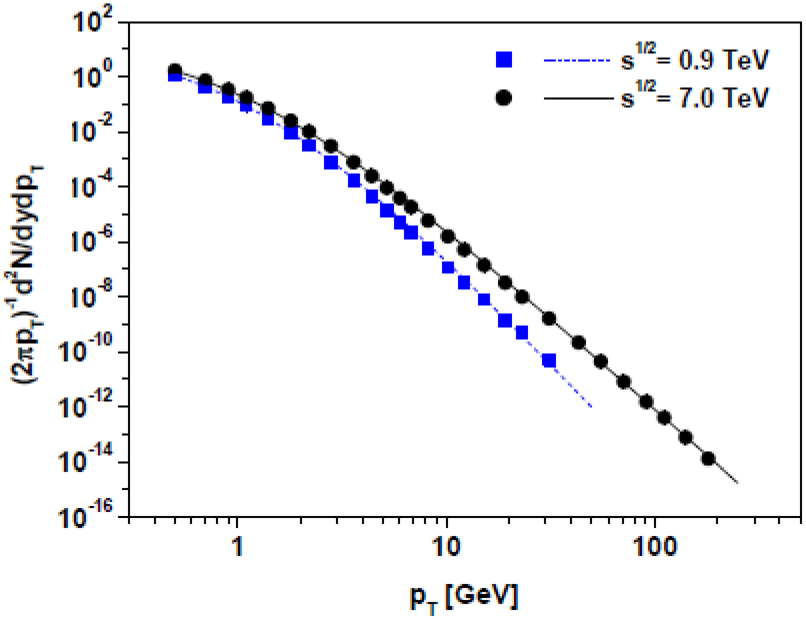
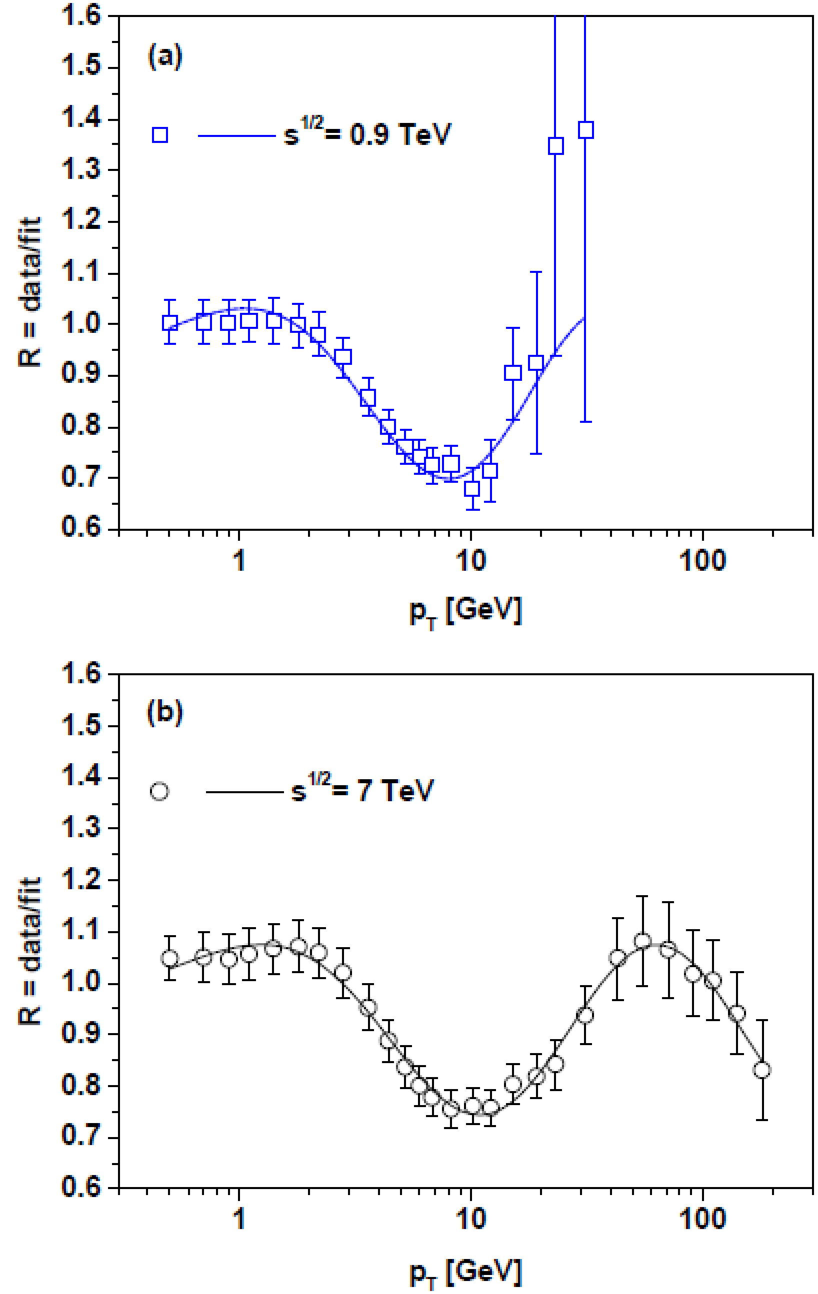
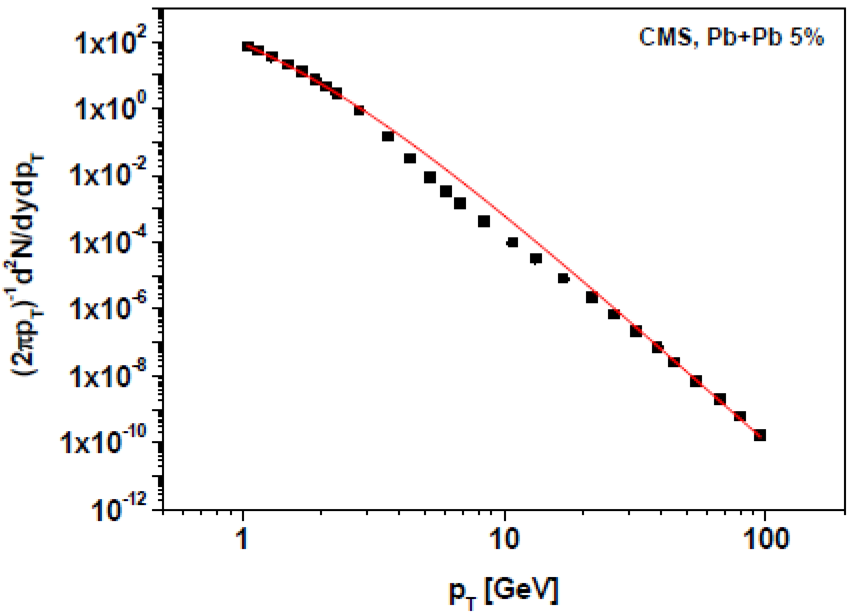
(1) Different energies

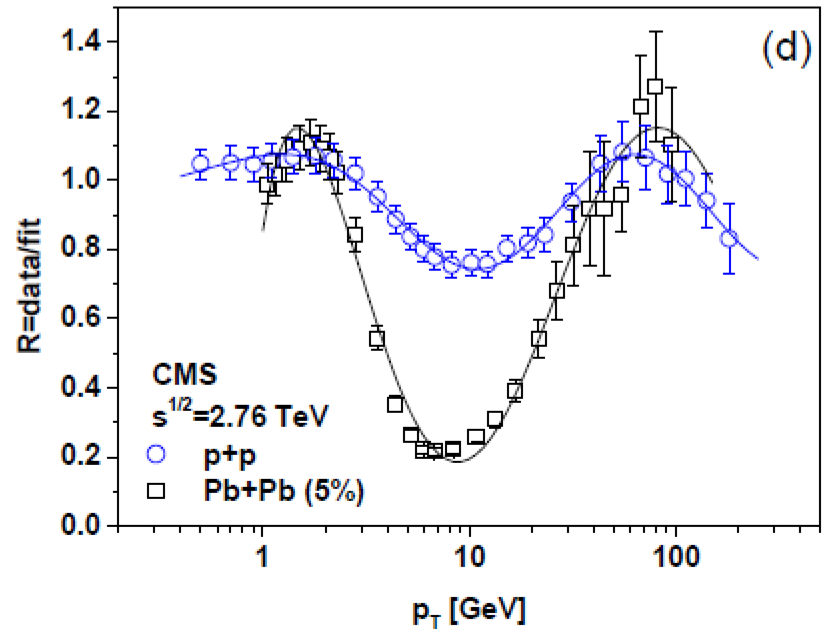


Fit to *data for pp collisions at 0.9 and 7* TeV from CMS experiment.
Parameters used are, respectively,
(T = 0.135, m = 8) and (T = 0.145, m = 6.7).

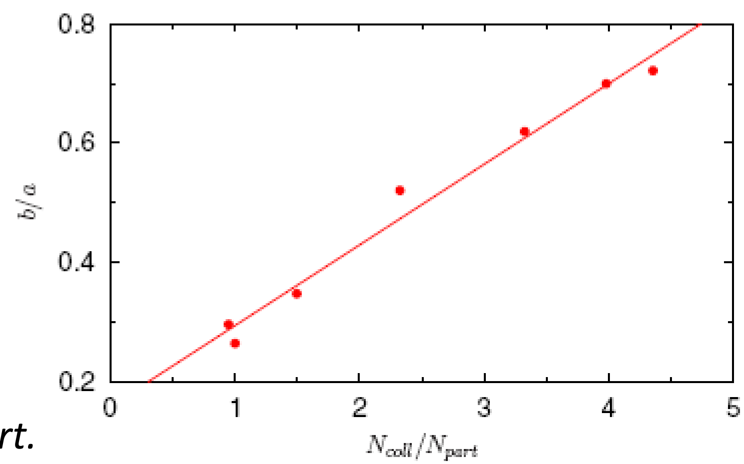


(1) Different collision systems





Fit to *data for Pb+Pb collisions (5% centrality) at 2.76* TeV from CMS experiment.
Parameters used are $T = 0.15$, $m = 7.05$



Ratio of the values of b/a parameters in R(E) as a function of number of collisions per participant nucleon, Ncoll/Npart.

(1)

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(*) However, looking at the ratios of the measured cross-sections to their phenomenological power-like fits, R = f_{data}(p_T)/f _{fit}(p_T), one discovers some log-periodic oscillations in R.

(*) This is a rather subtle effect, but it shows itself in all experiments, at all energies (provided that the range of transverse momenta observed is large enough), in reactions with nuclei where they grow with increasing centrality of the collision, it cannot be erased by any reasonable change of fitting parameters – in what follows we shall assume it to be real.

(*) In fact, such oscillations are seen in all branches of physics whenever one deals with power-like distributions. They are usually attributed to a discrete scale invariance (connected with a possible fractal structure of the process under consideration) and are described by introducing a complex power index.

(1)

Scale invariance

if for some function O(x), one finds that

$$\boxed{O(\lambda x) = \mu O(x)}$$

then it is **scale invariant** and its form follows a simple power law,

$$\boxed{O(x) = Cx^{-m}} \quad \text{with} \quad \boxed{m = -\ln \mu / \ln \lambda}$$

This relation can be written as

$$\boxed{\mu\lambda^{m} = 1 = e^{i2\pi k}}$$

where k is an arbitrary integer. It means therefore that, in general,

$$\boxed{m = -\ln \mu / \ln \lambda + i2\pi k / \ln \lambda,}$$

i.e., it is a complex number, the imaginary part of which signals a hierarchy of scales leading to

Log-periodic oscillations

(1)

$$\frac{df(E)}{dE} = -\frac{1}{T} f(E)$$

BG distribution $\Longrightarrow$

$$f(E) = \frac{1}{T}\exp\left(-\frac{E}{T}\right)$$

If the scale parameter is dependent on variable (preferential attachment)

$$T = T(E) = T_0 + (q-1)E$$

$$\frac{df(E)}{dE} = -\frac{1}{T(E)} f(E) = -\frac{1}{T_0 + (q-1)E} f(E)$$

Tsallis distribution $\Longrightarrow$

$$n = 1/(q-1)$$

$$f(E) = \frac{n-1}{nT_0}\left(1 + \frac{E}{nT_0}\right)^{-n}$$

dE → δE finite

$$f(E+\delta E) = \frac{-n\delta E + nT + E}{nT + E} f(E)$$

$\Longrightarrow$

$$\delta E = \alpha\left(nT + E\right)$$

$$\boxed{f(E + \alpha(nT + E)) = (1 - \alpha n) f(E)}$$

scale invariant relation

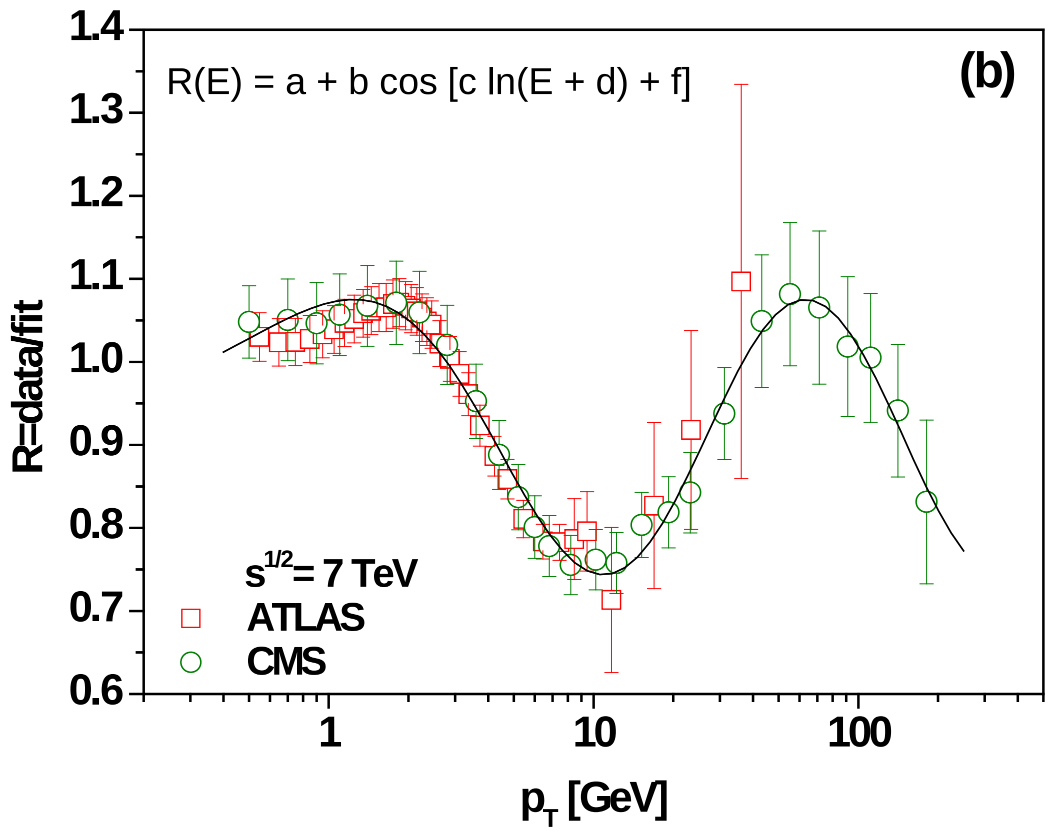
$$x = (1 + \frac{E}{nT})$$

$$\boxed{g\left((1+\alpha)x\right) = (1 - \alpha n) g\left(x\right)}$$

$$g(x) = \sum_k w_k \,\mathrm{Re}(x^{-m_k}) = x^{-\mathrm{Re}(m_k)} \sum_k w_k \cos[\mathrm{Im}(m_k)\ln(x)]$$

$$g(E) = \left(1+\frac{E}{nT}\right)^{-m_0} \boxed{\left\{ w_0 + w_1 \cos\left[\frac{2\pi}{\ln(1+\alpha)} \ln\left(1+\frac{E}{nT}\right)\right]\right\}}$$

$$g(E) = \left(1+\frac{E}{nT}\right)^{-m_0} R(E)$$



$$a/b = w_0/w_1$$

$$c = 2\pi/\ln(1+\alpha)$$

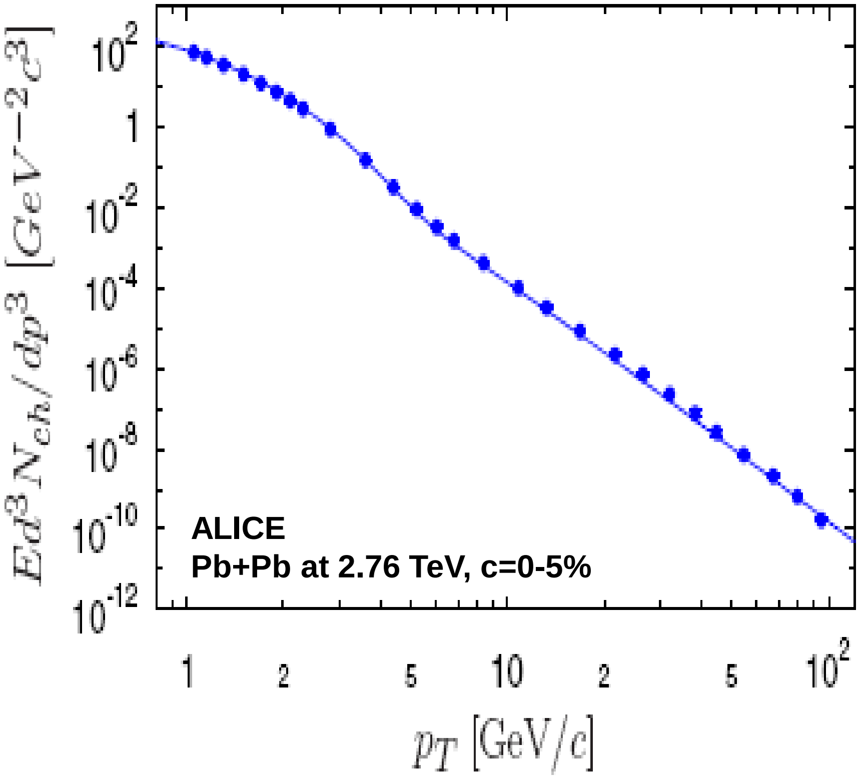
$$d = nT$$

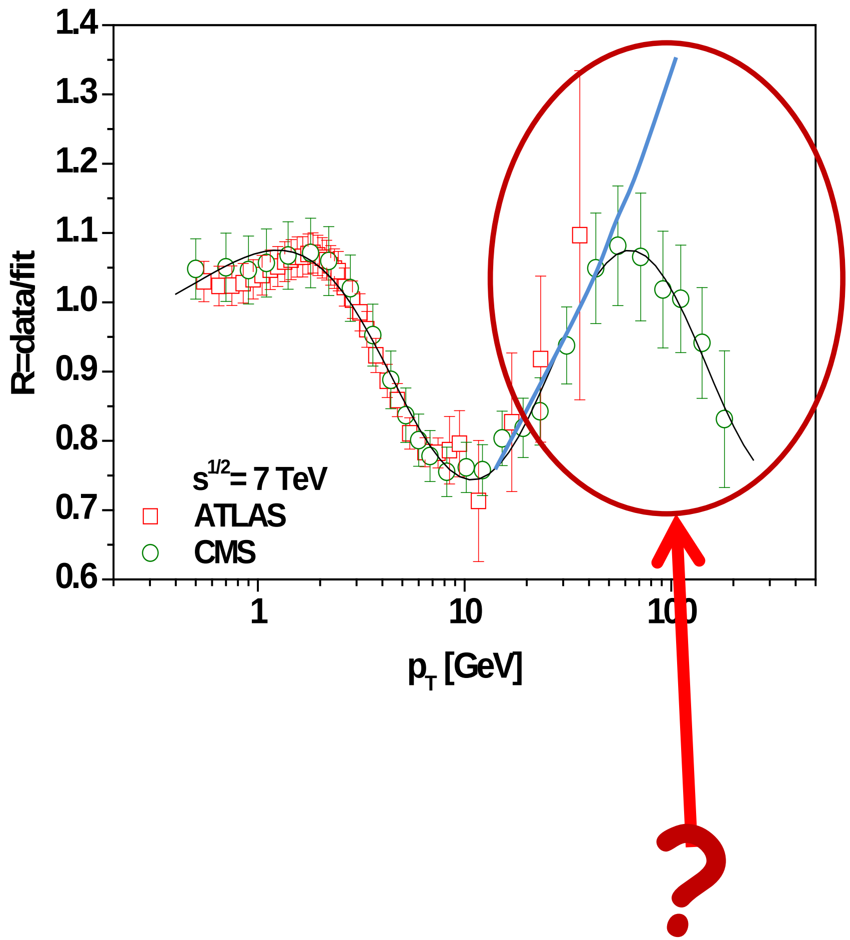
$$f = -2\pi\ln(nT)/\ln(1+\alpha)$$

(1)

Alternative - Two-component model ?

$$h(p_T) = \alpha_1 \left(1 + \frac{p_T}{m_1 T_1}\right)^{-m_1} + \alpha_2 \left(1 + \frac{p_T}{m_2 T_2}\right)^{-m_2}$$





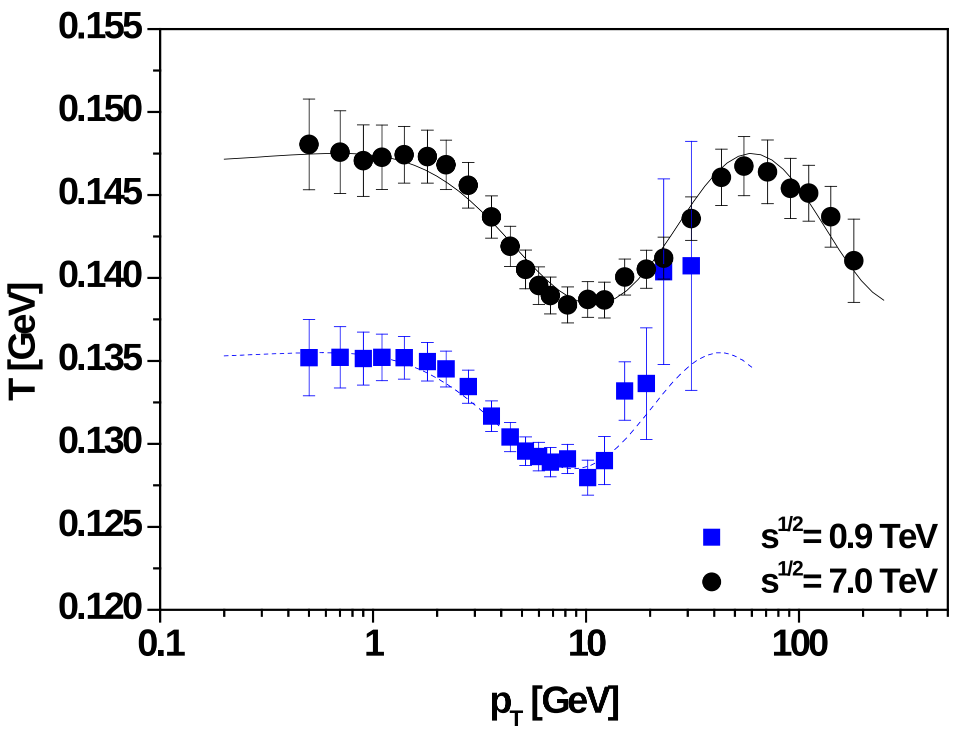
G.G.Barnafoldi et al., JPCS612(2015)012048

A „soft+hard" model…"

(2)

(*) However, one can also describe these oscillations by allowing for some specific log-periodic oscillations of the scale parameter T (both approaches are numerically equivalent).

(2) log-periodic scale parameter T



$$T = \bar{a} + \bar{b}\sin[\bar{c}(\ln(E+\bar{d}) + \bar{f}]$$

Stochastic equation for the temperature evolution in Langevin formulation with energy dependent noise $\xi(t,E)$

$$\frac{dT}{dt} + \frac{1}{\tau}T + \xi(t,E)T = \Phi$$

leads to

$$\frac{1}{n}\frac{d^2T}{d(\ln E)^2} + \left[\frac{1}{\tau} + \xi(t,E)\right]\frac{dT}{d(\ln E)} + T\frac{d\xi(t,E)}{d\ln E} = 0$$

which for the noise

$$\xi(t,E) = \xi_0(t) + \frac{\omega^2}{n}\ln E$$

(and relaxation time $\tau = const$)

is just an equation for the damping harmonic oscillator and has a solution

$$T = C\exp\left\{-n\cdot\left[\frac{1}{2\tau} + \frac{\xi(t,E)}{2}\right]\ln E\right\}\cdot\sin(\omega\ln E + \phi)$$

We could equivalently assume the energy independent noise, $\xi(t,E) = \xi_0(t)$

but allow for the energy dependent relaxation time

$$\tau = \tau(E) = \frac{n\tau_0}{n + \omega^2\ln E}$$

(3)

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In particular:

- ***The Fourier transform of the log-periodically oscillating T***

$$\boxed{T = \bar{a} + \bar{b}\sin[\bar{c}(\ln(E + \bar{d}) + \bar{f}]}$$

represents some log-periodic acoustic wave forming in the source.

- ***The corresponding wave equation has self-similar solutions of the second kind connected with the so called intermediate asymptotic (observed in phenomena which do not depend on the initial conditions because sufficient time has already passed, although the system considered is still out of equilibrium).***

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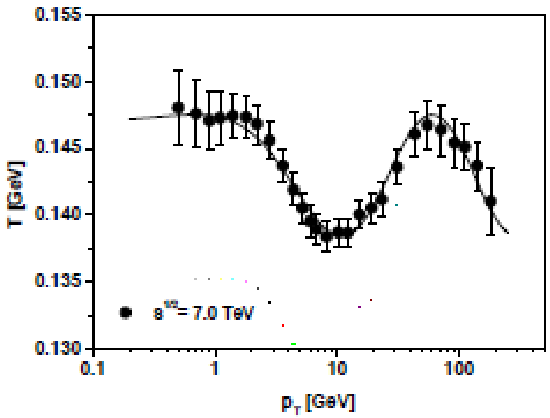
- *The corresponding wave equation has self-similar solutions of the second kind connected with the so called intermediate asymptotic (observed in phenomena which do not depend on the initial conditions because sufficient time has already passed, although the system considered is still out of equilibrium) .*

➔ ***Both in p+p and Pb+Pb one deals with an inhomogeneous medium with the density and the velocity of sound both depending on the position in the way which seems to be supported by experimental results.***

(3)

Temperature oscillations and sound waves in hadronic matter

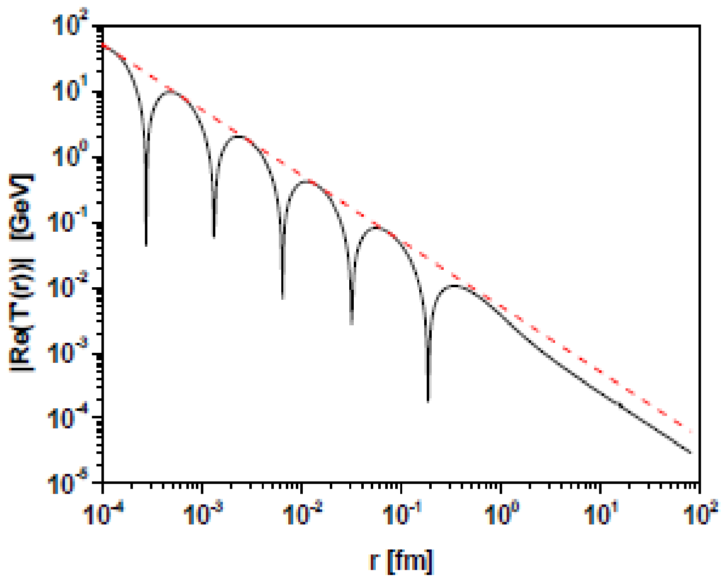


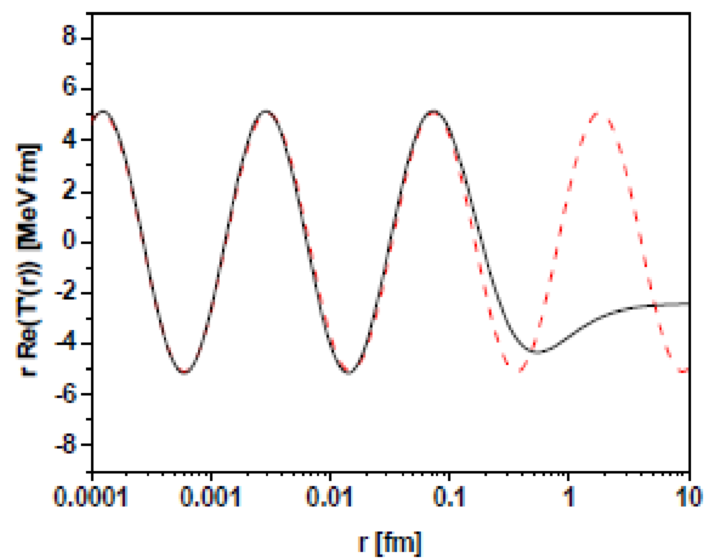
$$T\,(p_T) = a + b \sin\left[c \ln\left(p_T + d\right) + e\right]$$

$$T(r) = \sqrt{\frac{2}{\pi}} \int_0^\infty T\,(p_T)\, e^{i p_T r}\, dp_T$$

------ T(r)=0.005/r

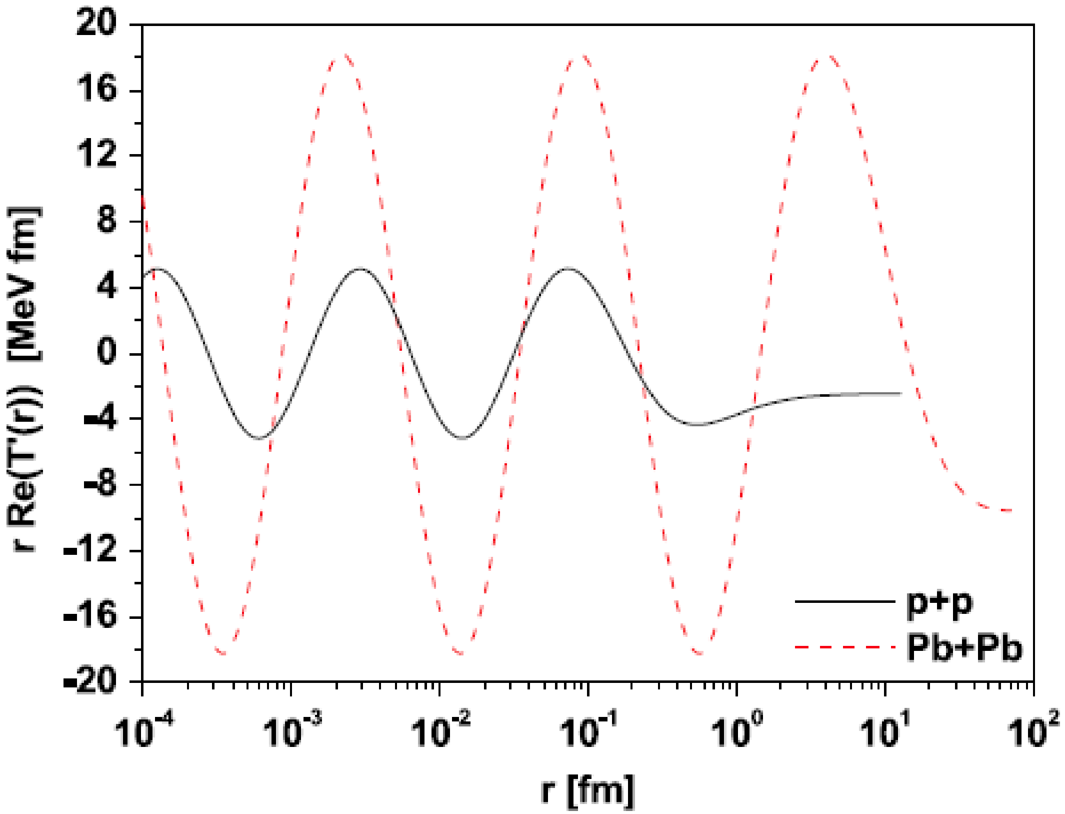
------ T(r)=5.1sin(2π)/3.2ln(1.24r)





(3)

Comparison of different collision systems:



p-p and Pb-Pb

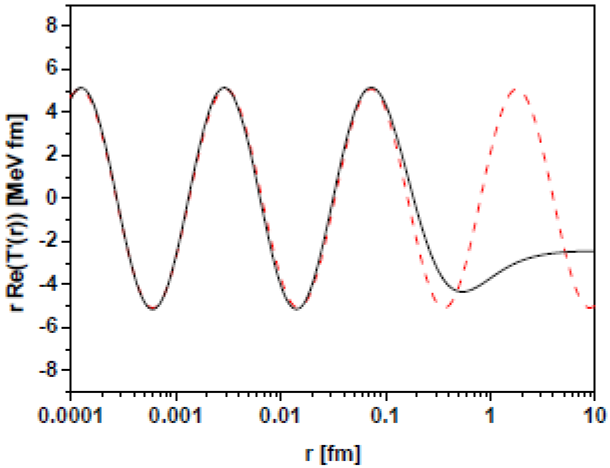
(most central)

collisions

Table 1
Values of fit parameters in $T(p_T) = a + b\sin[c\ln(p_T + d) + e]$.

Collision	a	b	c	d	e
$p + p$	0.143	0.0045	2.0	2.0	−0.4
$Pb + Pb$	0.131	0.017	1.7	0.05	0.98

(3) Temperature oscillations and sound waves in hadronic matter



Temperature oscillations $T(r) = T_0 + T'$ can be related to the velocity v in a sound wave. For small oscillations $P = P_0 + P'$ and $\rho = \rho_0 + \rho'$ we have

$$T' = (\partial T / \partial P)_S \, P'$$

and using thermodynamic formula $\left(\frac{\partial T}{\partial P}\right)_S = \frac{T}{c_P}\left(\frac{\partial V}{\partial T}\right)_P$

$$\boxed{T' = \frac{c\kappa T}{c_P} v}$$

$$\kappa = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P \quad \text{and} \quad c = \sqrt{\left(\frac{\partial P}{\partial \rho}\right)_S}$$

Introducing the velocity potential $\vec{v} = \overline{grad}(f)$

for cylindrical wave, the wave equation is given as

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f}{\partial r}\right) - \frac{1}{c^2}\frac{\partial^2 f}{\partial t^2} = 0$$

For monochromatic wave $f(r,t) = f(r)\exp(-i\omega t)$ for $f(r)$ we have

$$\frac{\partial^2 f(r)}{\partial r^2} + \frac{1}{r}\frac{\partial f(r)}{\partial r} + K^2 f(r) = 0$$

where $K=K(r)=\omega/c(r)$

In inhomogenous media, the wave number K=K(r) is dependent on r. For the wave number $K(r) = \frac{\alpha}{r}$

the solution is given by $f(r) \propto \sin[\alpha \ln(r)]$

and using $f(r) \propto vr$ we can write

$$\boxed{rT'(r) \propto \frac{c\kappa T_0}{c_P} f(r) = \frac{c\kappa T_0}{c_P} \sin(\alpha \ln r)}$$

(3) Self similar solution

Equation

$$\frac{\partial^2 f(r)}{\partial r^2} + \frac{1}{r}\frac{\partial f(r)}{\partial r} + K^2 f(r) = 0$$

with the wave number

$$K(r) = \frac{\alpha}{r}$$

Has a so-called self similar solution of the second kind.

It is known from other branches of physics and is connected with the so called intermediate asymtotics encountered in phenomena which do not depend on the initial conditions because sufficient time has already passed, although the system considered has not yet reached the equilibrium state.

Therefore , introducing the variable

$$\xi = \ln r$$

we find

$$\frac{\partial^2 F(\xi)}{\partial \xi^2} + \alpha^2 F(\xi) = 0$$

with solution

$$F(\xi) \propto \sin(\alpha\xi)$$

(3) Remarks: Correspondence between scale invariance and self similarity

$$\boxed{\frac{\partial^2 f(r)}{\partial r^2}+\frac{1}{r}\frac{\partial f(r)}{\partial r}+\left(\frac{\alpha}{r}\right)^2 f(r)=0}$$

$$f(r)=r^{i\alpha}=\exp[i\alpha\ln(r)]$$

$$\mathrm{Re}[f(r)]=\cos[\alpha\ln(r)]$$

Scale invariance

$$\boxed{\eta=\lambda r}$$

$$\frac{\partial^2 f(\eta)}{\partial \eta^2}+\frac{1}{\eta}\frac{\partial f(\eta)}{\partial \eta}+\left(\frac{\alpha}{\eta}\right)^2 f(\eta)=0$$

$$\boxed{f(\lambda r)=f(r)}$$

Self similarity

$$\boxed{\xi=\ln r}$$

traveling wave $\frac{\partial^2 F(\xi)}{\partial \xi^2}+\alpha^2 F(\xi)=0$

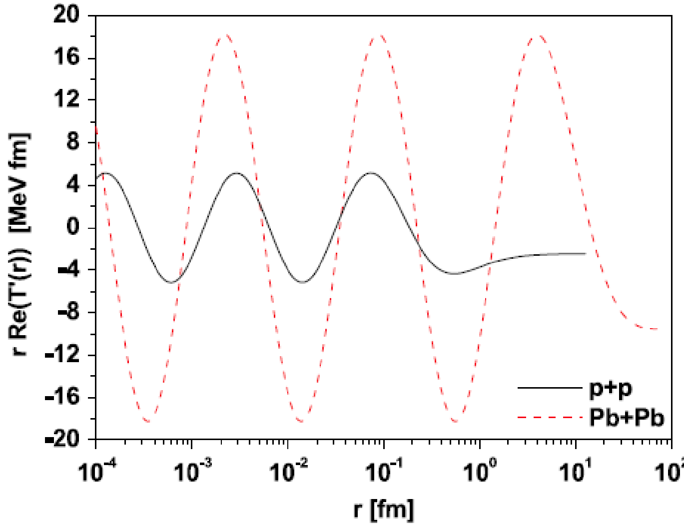
with solution $F(\xi)\propto$ sin[αξ]

$$\boxed{F(\xi+\ln\lambda)=F(\xi)}$$

$$\alpha=\frac{2\pi k}{\ln\lambda}$$

(4)

Consequences (1)



(*) The space picture of the collision (in the plain perpendicular to the collision axis and located at the collision point) presented shows the existence of some regular (on the logarithmic scale) structure for small distances.

(*) It starts to weaken quite early (at r ~ 0.1 fm) and essentially disappears when r reaches the dimension of the nucleon, i.e., for r ~ 1 fm.

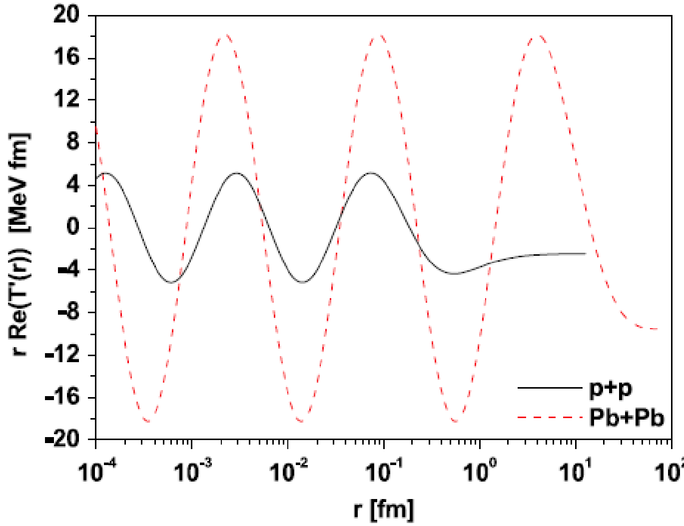
(*) For central Pb + Pb collisions we observe ***~3.6 times bigger amplitude and ~1.15 longer period of oscillations****. With decreasing centrality the amplitude decreases smoothly reaching practically the same value as for p + p collisions . With the parameters used we have, in the region of regular oscillations, that*

$$(p+p):\quad rT'(r) = 5.1\sin\left[\frac{2\pi}{3.2}\ln(r)+0.42\right];$$

$$(Pb+Pb):\quad rT'(r) = 18.53\sin\left[\frac{2\pi}{3.68}\ln(r)-0.51\right].$$

(4)

Consequences (2)



(*) *Longer period of oscillations in* ***Pb + Pb*** *collisions means a smaller value of the parameter* ***α*** *in*

$$f(r) \propto \sin[\alpha \ln(r)].$$

in ***p + p*** *collisions.*

(*) *Because* $\boldsymbol{\omega/c(r) = \alpha/r}$ ➔ *the velocity of sound* $\boldsymbol{c(r) = (\omega/\alpha)r}$

➔ *velocity of sound in the nuclear environment (**Pb + Pb**) is greater than in **p + p**.*

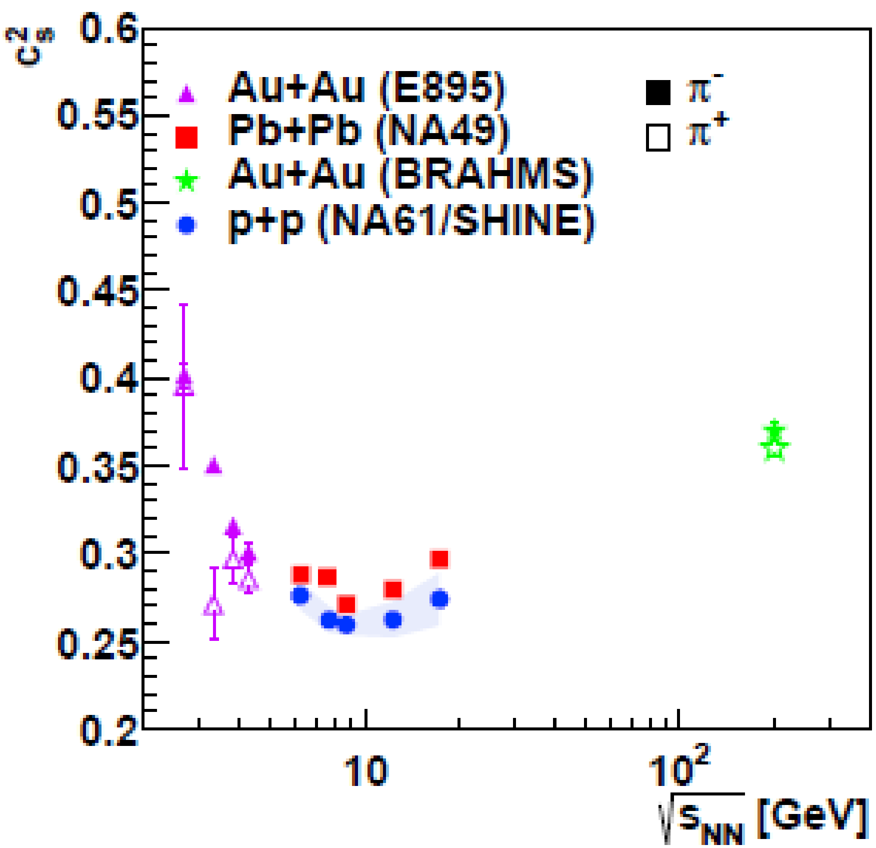
(*) *This, in turn, means that the refractive index* $\boldsymbol{n(r) = c_0 / c(r)}$ *at position* $\boldsymbol{r}$ *in nuclear collisions is smaller than in collisions of protons.*

(*) *In both cases we encounter an inhomogeneous medium with* ***r****-dependent c(r) and* ***n(r)****.*

(4)

Confrontation with experimental observations (1)

*(*) **In nuclear collisions one observes a higher speed of sound** c_s^2 (used as a parameter in the equation of state of hadronic matter described by a hydrodynamical model)*



Compilation of the energy dependence of sound velocities c_s^2 obtained from the widths of $\pi-$ rapidity spectra [PoS DIS2014(2014) 018]

(4)

Confrontation with experimental observations (2)

*(*) **This should be expected**.*

Consider the connection of the isothermal compressibility of the nuclear matter

$$\kappa_T = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T;$$

and fluctuations of the multiplicity of produced secondaries represented by the relative variance, ϖ , of the multiplicity fluctuations ($\rho_0 = \langle N\rangle/V$ denotes the equilibrium density for N particles with mass m located in volume V).

$$T\kappa_T\rho_0 = \frac{\langle N^2\rangle - \langle N\rangle^2}{\langle N\rangle} = \varpi$$

This allows the velocity of sound to be expressed by fluctuations of multiplicity because one has :

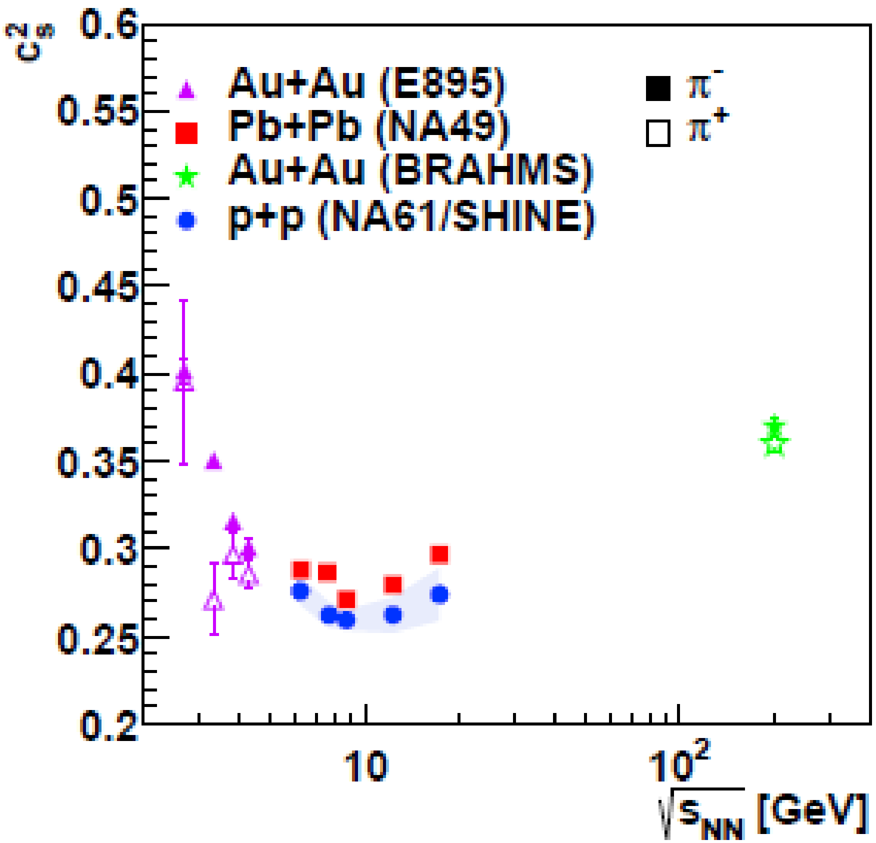
$$c = \sqrt{\frac{\gamma}{\kappa_T\rho_0 m}} = \sqrt{\frac{\gamma T}{\varpi m}} \quad \text{where} \quad \gamma = \frac{c_P}{c_V}.$$

Note: higher velocity of sound corresponds to lower fluctuations of multiplicity.

(4)

Confrontation with experimental observations (3)

Higher velocity of sound corresponds to lower fluctuations of multiplicity, this seems to be observed:



$$\frac{c_{Pb+Pb}}{c_{p+p}} \simeq 1,15$$

$$\frac{\varpi_{p+p}}{\varpi_{Pb+Pb}} \simeq 1.32$$

Fluctuations (omega) decreases with number of wounded nucleons.

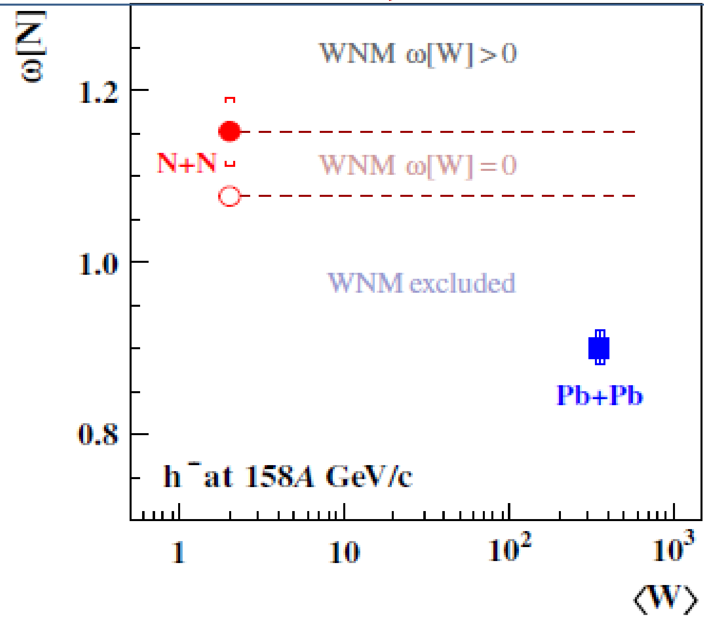
(4) Confrontation with experimental observations (4)

This agrees also with the recently obtained values $\frac{\bar{\omega}_{p+p}}{\bar{\omega}_{Pb+Pb}} \cong 1.29 \pm 0.04$

[Eur. Phys. J. C 76 (2016) 635]

*Scaled variance of the multiplicity distribution of negatively charged hadrons, **ω**, as a function of the mean number of wounded nucleons **<W>**. Results for inelastic p+p (NA61/SHINE) interactions (filled circles and the 1% most central Pb+Pb (NA49 [6]) collisions (squares).*

Fluctuations (represented by ω) decreases with number of wounded nucleons.



These results can be connected with the pair correlation function, g(2), because the scaled variance can be written as

$$\rho_0 \kappa_T T = 1 + \rho_0 \int d\vec{r} \left[g^{(2)}(r) - 1 \right]$$

J. Phys. Conf. Ser. 5 (2005) 23 ➔ for central nuclear collisions the number of binary collisions exceeds that of wounded nucleons and correlation function becoming negative which means smaller fluctuations of multiplicity.

(4)

Confrontation with experimental observations (5)

(*) In nuclear collisions participate N_{part} nucleons (wounded nucleons) experiencing N_{coll} binary NN collisions. In Glauber picture : $N_{coll} \sim (N_{part})^{4/3}$.

(*) Incompresability is proportional to the energy density:

$$(1/\kappa) \sim \varepsilon = N_{part}\,\varepsilon_0 + (N_{coll} - N_{part})\varepsilon_1$$

[ε_0 - energy produced by participating nucleons , ε_1 -energy produced in the remaining binary collisions; note that, approximately, nucleon interacts many times and in first interaction it releases energy ε_0, in each additional collision energy ε_1 (on the average)]

(*) Incompresability of nuclear matter exceed incompresability of hadronic matter and sound speed ratio is given by

$$\boxed{\frac{c_{AA}^2}{c_{pp}^2} = 1 + \left(\frac{N_{coll}}{N_{part}} - 1\right)\frac{\varepsilon_1}{\varepsilon_0}}$$

(*) For central Pb+Pb ccollisions (with $N_{part} \cong 200$, $[c_{PbP}/c_{pp}]^2 \sim 1.3$ and $N_{coll}/N_{part} \sim [N_{part}]^{1/3}$) we estimate that

$$\boxed{\varepsilon_1 / \varepsilon_0 \cong 0.06}$$

Concluding Remarks - 1

(*) Transverse momentum distributions are characterized by a quasi-power law (Tsallis distribution) decorated with log-periodic oscillations.

(*) This means that either: the system and/or the underlying physical mechanisms have characteristic <u>scale invariance behavior</u>. The discrete scale invariance and its associated complex exponents can appear spontaneously, without a pre-existing hierarchical structure.

(*) Or that: we observe a <u>sound wave in hadronic mat</u>ter (resulting in the temperature oscillations) which has self similar solution (in log-periodic form).

(*) This, in turn, can have some interesting experimental consequences.

Concluding Remarks - 2

Further reading *(GW&ZW):* *Physica A 413 (2014) 53 ;*
Entropy 17 (2015) 384;
Chaos,Sol.& Frac., 81 (2015) 487 ;
Physica A 486 (2017) 579.

Also: *D.A. Fogaça et al. Nucl. Phys. A 819 (2009) 150 [Sound waves and solitons in hot and dense nuclear matter];*
G.I. Barenblatt and Ya.B. Zeldovich, Ann. Rev. Fluid Mech. 4 (1972) 285 [Self-Similar Solutions as Intermediate Asymptotics];
D.Sornette, Phys. Rep. 297 (1998)239 [Discrete-scale invariance and complex dimensions]


Gracias por su atención
Thank you for your attention
and I look forward to your comments and questions