HBT overview

with emphasis on multiparticle correlations

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Overview on fundamentals:
Hanbury Brown and Twiss

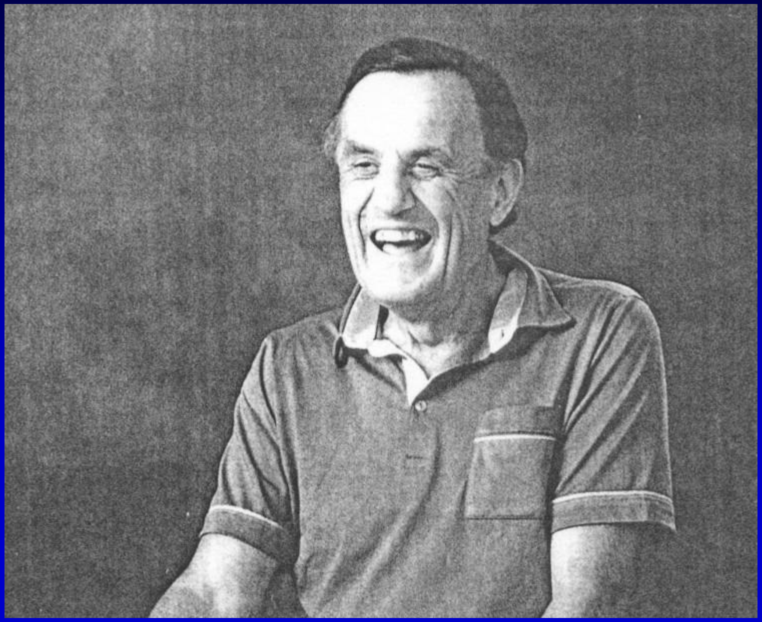
Positive definitene form – or not?
Two-particle symmetrization effect – or not?
Bose-Einstein condensation of charged particles – or not?
Gaussian shape – or not?

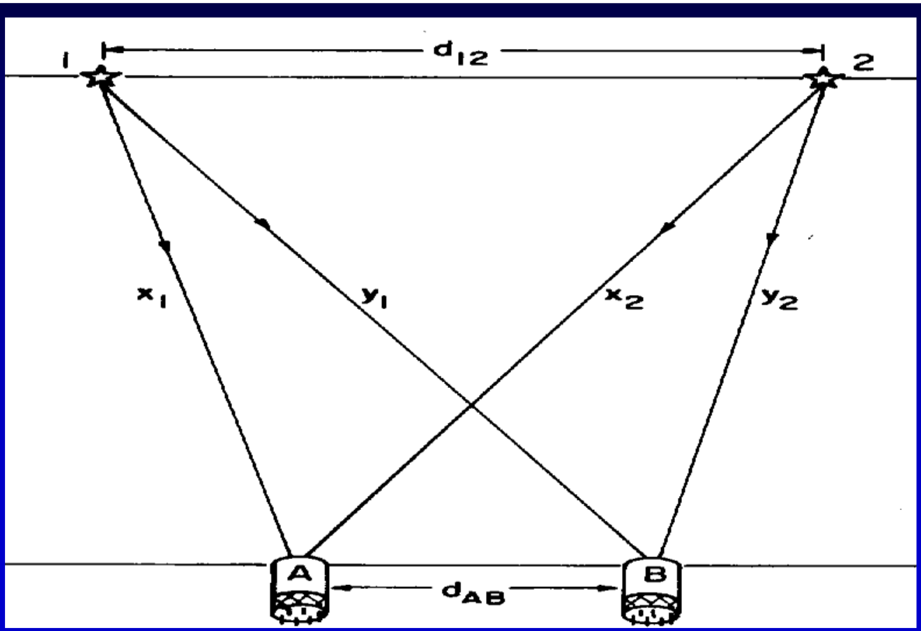
Sensitive to $U_A(1)$ symmetry restoration

Summary, conclusions

Based on HBT Overview talks at HDNM 2017, Rehovot and ISMD 2017, Amsterdam
Supported by NKTIH and EFOP-3.6.1-16-2016-00001 (Hungary)

HBT: Robert Hanbury Brown – Richard Quincy Twiss





Two people: Robert Hanbury Brown and Richard Quincy Twiss

– Robert, Hanbury as well as Richard and Quincy: can be given names, but…

– Sir Robert Hanbury Brown had a *compound family* name,

– just like Sir Christopher Llywellyn Smith, whose *compound family* name is sometimes hyphenated

R. Hanbury Brown and R. Q. Twiss: Engineers, who worked in radio and optical astronomy

„Interference between two different photons can never occur."

P. A. M. Dirac, The Principles of Quantum Mechanics, Oxford, 1930

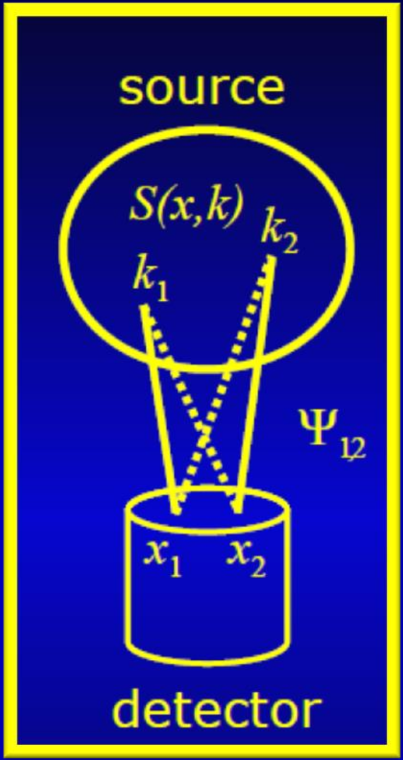
„As an engineer my education in physics had stopped far short of the quantum theory.
Perhaps just as well … ignorance is sometimes a bliss in science."

R. H. Brown: Boffin: A Personal Story … ISBN 0-7503-0130-9

HBT: 1 + positive definite term

Two plane waves
Symmetrized, + for bosons, - for fermions
Expansion dynamics, final state interactions,
multiparticle symmetrization effects: **negligible**

$$\Psi_1 = \mathrm{e}^{-ik_1x_1}$$
$$\Psi_2 = \mathrm{e}^{-ik_2x_2}$$



$$A_{12} \propto \frac{1}{\sqrt{2}} \left[\mathrm{e}^{ik_1x_1+ik_2x_2} \pm \mathrm{e}^{ik_1x_2+ik_2x_1} \right],$$

$$N_2(k_1,k_2) \propto \int dx_1 \rho(x_1) \int dx_2 \rho(x_2)\ |A_{12}|^2$$

$$C_2(k_1,k_2) = \frac{N_2(k_1,k_2)}{N_1(k_1)N_2(k_2)} = 1 \pm |\tilde{\rho}(k_1-k_2)|^2$$

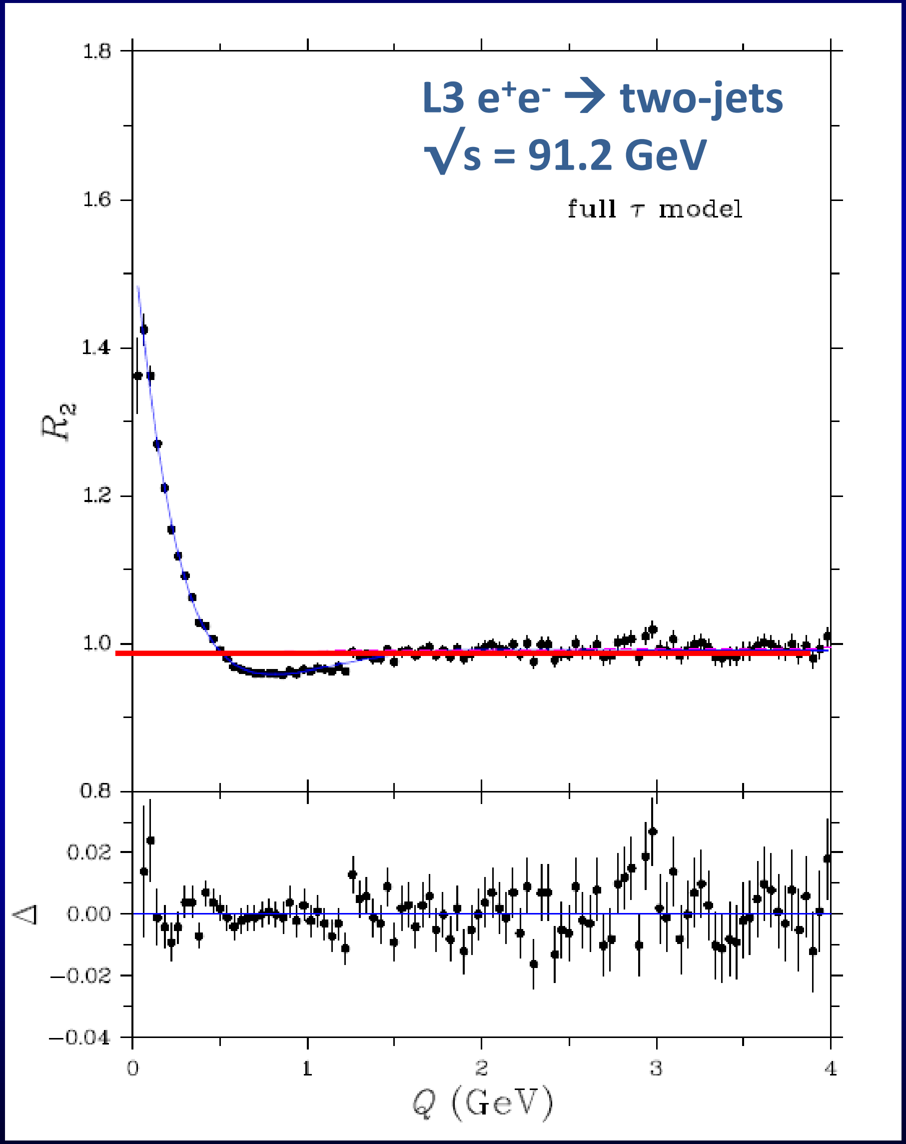
Two particle HBT correlations:
1 + positive definite term
1+ |Fourier–transform of the source|^2,

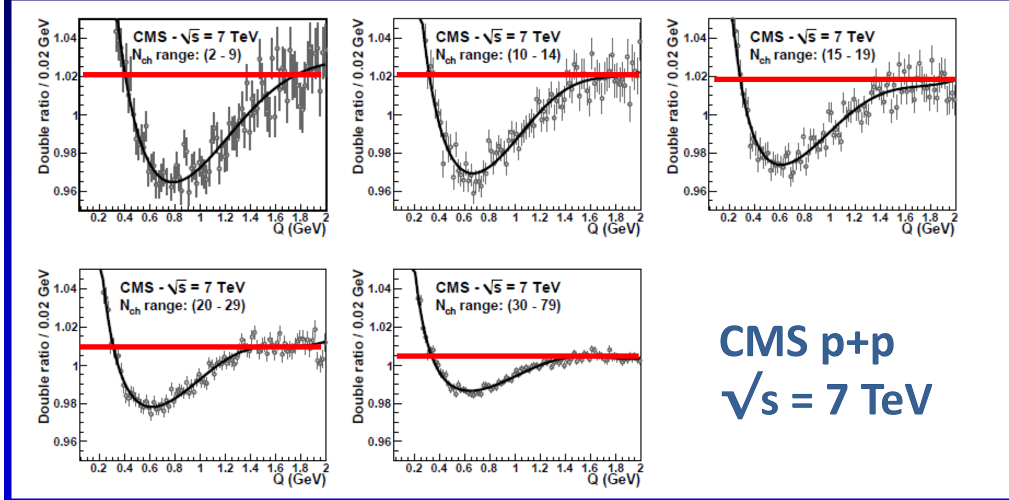
$$\tilde{\rho}(q) = \int dx\ \mathrm{e}^{iqx}\ \rho(x)$$

Usually evaluated in **Gaussian approximation**
Dependence on mean momentum:
expansion dynamics ρ(x) → S(x,k)

Dubna school: use it as a tool
Kopylov and Podgoretskii: x <-> k
1+ |Fourier–transform of the source|^2

HBT: 1 + positive definite term - or not ?





Two experimental results:

L3 for Bose-Einstein in e^+e^- at LEP
arXiv:1002.1303 [hep-ex]

CMS for Bose-Einstein in pp at LHC
arXiv:1101.3518 [hep-ex]

Expansion dynamics: role of jets?

→ strongly correlated phase-space,
τ-model: $x \sim k \rightarrow \Delta x\, \Delta k \sim Q_{inv}^2$,
$C(Q_{inv}) \neq 1$ + positive definite form
arXiv:0803.3528 [hep-ph]

See the WPCF2017 talk of Wes Metzger for details

HBT: 1 + positive definite term?

Example: Levy expansions

1st-order Lévy polynomial	$\gamma\left[1+\lambda e^{-R^\alpha Q^\alpha}\left[1+c_1 L_1(Q\|\alpha,R)\right]\right]$
3rd-order Lévy polynomial	$\gamma\left[1+\lambda e^{-R^\alpha Q^\alpha}\left[1+c_1 L_1(Q\|\alpha,R)+c_3 L_3(Q\|\alpha,R)\right]\right]$

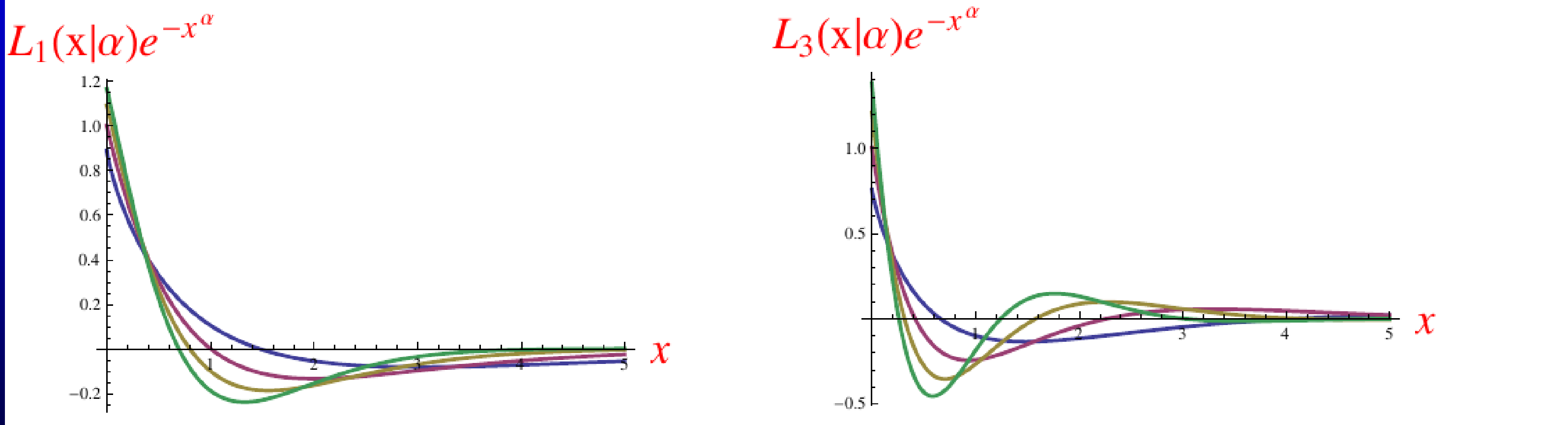
$$L_1(x\,|\,\alpha)=\det\begin{pmatrix}\mu_{0,\alpha} & \mu_{1,\alpha}\\ 1 & x\end{pmatrix}$$

$$L_2(x\,|\,\alpha)=\det\begin{pmatrix}\mu_{0,\alpha} & \mu_{1,\alpha} & \mu_{2,\alpha}\\ \mu_{1,\alpha} & \mu_{2,\alpha} & \mu_{3,\alpha}\\ 1 & x & x^2\end{pmatrix}$$

$$\mu_{r,\alpha} = \int_0^\infty dx\ x^r f(x\,|\,\alpha) = \frac{1}{\alpha}\Gamma\left(\frac{r+1}{\alpha}\right)$$

Model-independent but:

- Generalizes exponential (α =1) and Gaussian(α = 2)
- ubiquoutous in nature
- How far from a Levy?
- Not necessarily positive definite!
- Notation: x = Q R



Lévy polynomials of first and third order times the weight function e^{-x^α} for $\alpha = 0.8, 1.0, 1.2, 1.4$.

HBT: 1 + positive definite term?

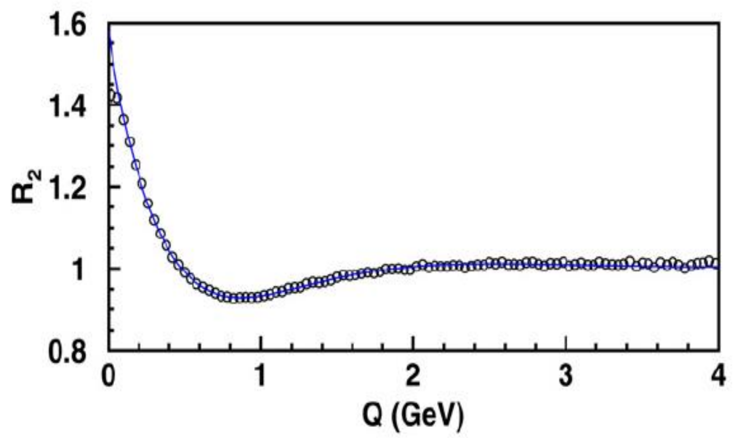
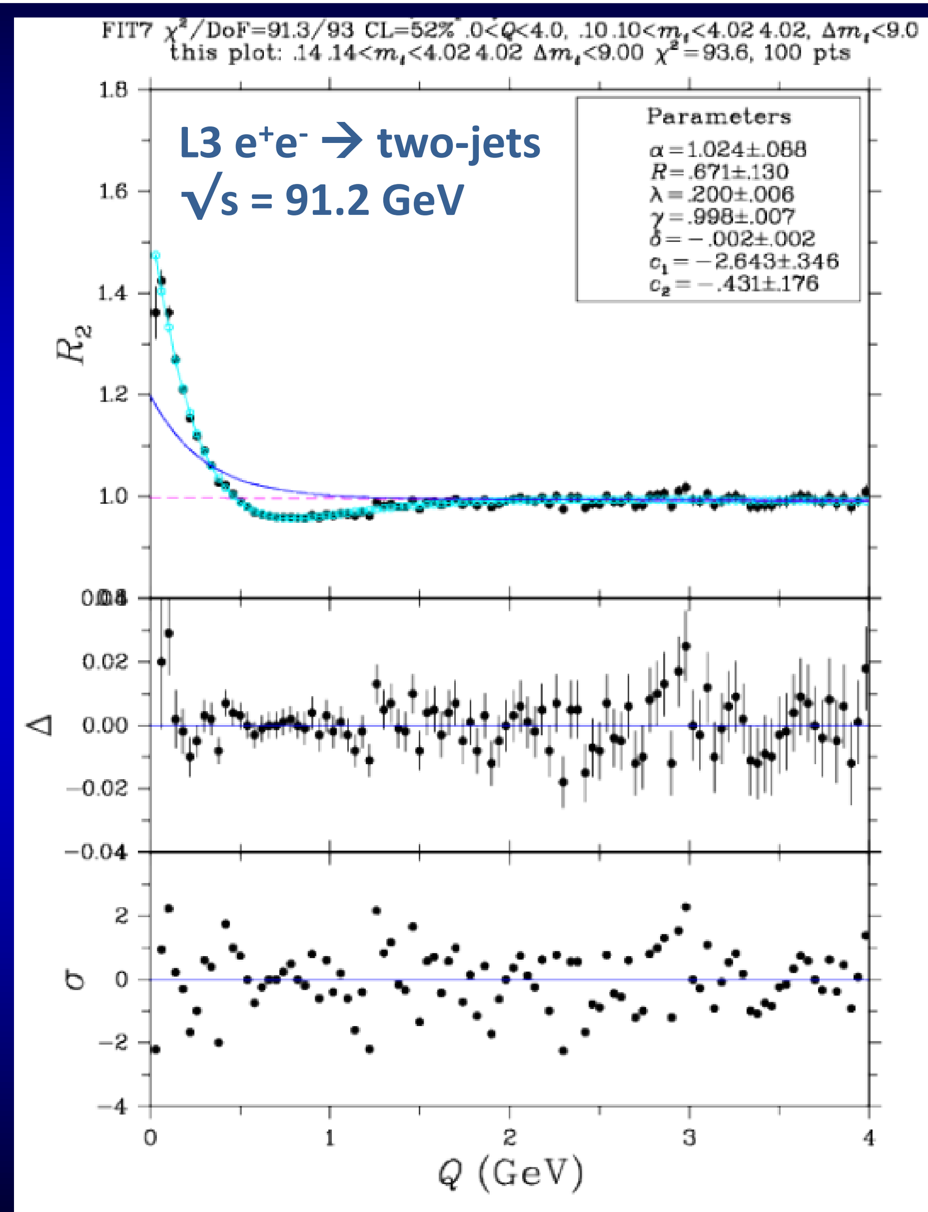


Fig. 1. The Bose–Einstein correlation function R_2 for events generated by PYTHIA. The curve corresponds to a fit of the one-sided Lévy parametrization, Eq. (13).

T. Csörgő et al. / Physics Letters B 663 (2008) 214–216



Check dip and background with Levy/Laguerre/Edgeworth/Gauss model independent expansions

$$t = QR$$

$$C_2(t) = N\left\{1 + \lambda \exp(-t^{\alpha})\left[1 + \sum_{n=1}^{\infty} c_n L_n(t|\alpha)\right]\right\}$$

See the talk of T. Novák at Low-x 2016

HBT: 1 + positive definite term?

Levy expansions for 1+ positive definite forms

$$C_2(k_1, k_2) = 1 + |\tilde{f}(q_{12})|^2,$$

$$t = \left(\sum_{i,j=\mathrm{side,out,long}} R^2_{i,j} q_i q_j \right)^{1/2},$$

$$C_2(t) = N \left\{ 1 + \lambda \exp(-t^\alpha) \left| 1 + \sum_{n=1}^{\infty} (a_n + i b_n) L_n(t|\alpha) \right|^2 \right\}$$

where $\{c_n = a_n + i b_n\}_{n=1}^{\infty}$ are now complex valued expansion coefficients,

Model-independent but:

- **Generalizes exponential (α =1) and Gaussian(α = 2)**
- **In this case, for 1+ positive definite forms**
- **ubiquoutous in nature**
- **How far from a Levy?**
- **Works also for cross-sections in elastic scattering**

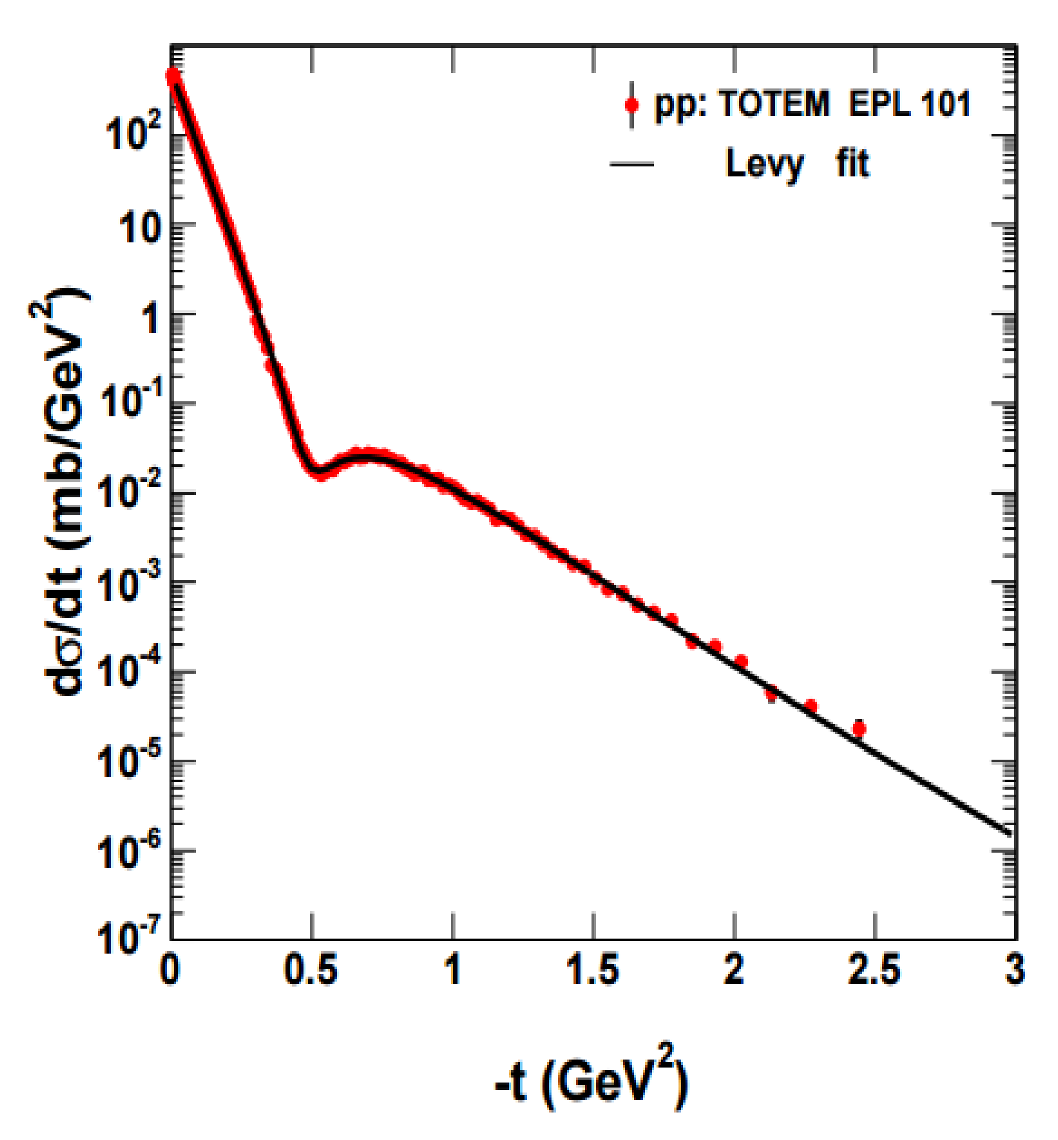
$$L_1(x\,|\,\alpha) = \det \begin{pmatrix} \mu_{0,\alpha} & \mu_{1,\alpha} \\ 1 & x \end{pmatrix}$$

$$L_2(x\,|\,\alpha) = \det \begin{pmatrix} \mu_{0,\alpha} & \mu_{1,\alpha} & \mu_{2,\alpha} \\ \mu_{1,\alpha} & \mu_{2,\alpha} & \mu_{3,\alpha} \\ 1 & x & x^2 \end{pmatrix}$$

$$\mu_{r,\alpha} = \int_0^\infty dx\ x^r f(x\,|\,\alpha) = \frac{1}{\alpha}\Gamma\left(\frac{r+1}{\alpha}\right)$$

T. Novák, T. Cs., H. C. Eggers, M. de Kock: arXiv:1604.05513 [physics.data-an]

Example: Levy expansion for $|f|^2$



T. Cs, W. Metzger, T. Novák, A. Ster, talk at Low-x 2016

HBT: Has to be a Gaussian, IF …

Model-independent but Gaussian IF we assume:

- 1 + positive definite forms
- Plane wave approximation
- Two-particle symmetrization (only)
- IF f(q) is analytic at q = 0 and
- IF means and variances are finite
- Follows an approximate Gaussian(α = 2)

$$C_2(k_1,k_2) = 1 + |\tilde{f}(q_{12})|^2,$$

$$\tilde{f}(q_{12}) = \int \mathrm{d}x \, \exp(\mathrm{i}q_{12}x)\, f(x),$$

$$q_{12} = k_1 - k_2.$$

$$\tilde{f}(q) \approx 1 + \mathrm{i}q\langle x\rangle - q^2\langle x^2\rangle/2 + \ldots,$$

$$C(q) = 1 + |\tilde{f}(q)|^2 \approx 2 - q^2(\langle x^2\rangle - \langle x\rangle^2) \approx 1 + \exp(-q^2R^2),$$

$$R = \sqrt{\langle x^2\rangle - \langle x\rangle^2}.$$

Model-independent but non-Gaussian IF we assume:

- 1 + positive definite form (same as above)
- Plane wave approximation (same)
- Two-particle symmetrization only (same)
- IF f(q) is NOT analytic at q = 0 and
- IF means and variances are NOT finite
- IF Generalized Central Limit theorems are valid
- Follows a Levy shape(0 < α ≤ 2)
- Earlier Gaussian recovered for α = 2

$$f(x) = \int \Pi_{i=1}^n \mathrm{d}x_i \, \Pi_{j=1}^n f_j(x_j)\, \delta(x - \sum_{k=1}^n x_k).$$

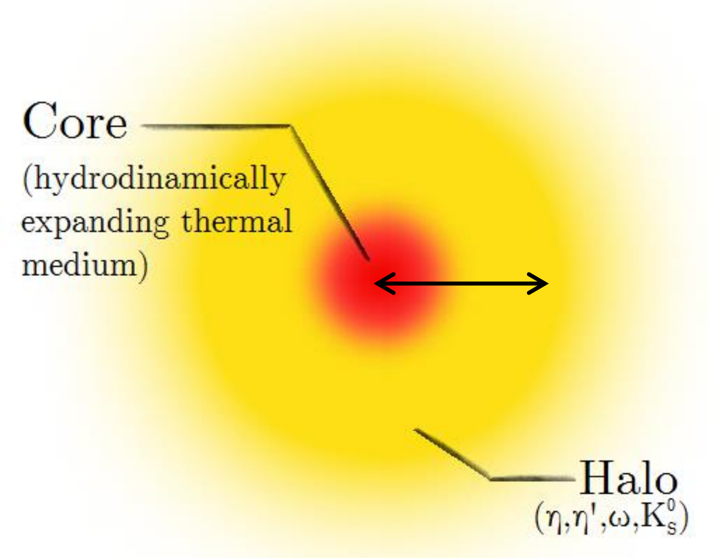
$$\tilde{f}(q) = \prod_{i=1}^n \tilde{f}_i(q)$$

$$C(q;\alpha) = 1 + \lambda \exp\left(-|qR|^\alpha\right).$$

Cs. T, S. Hegyi, W. A. Zajc, nucl-th/0310042

But: core/halo model, resonances

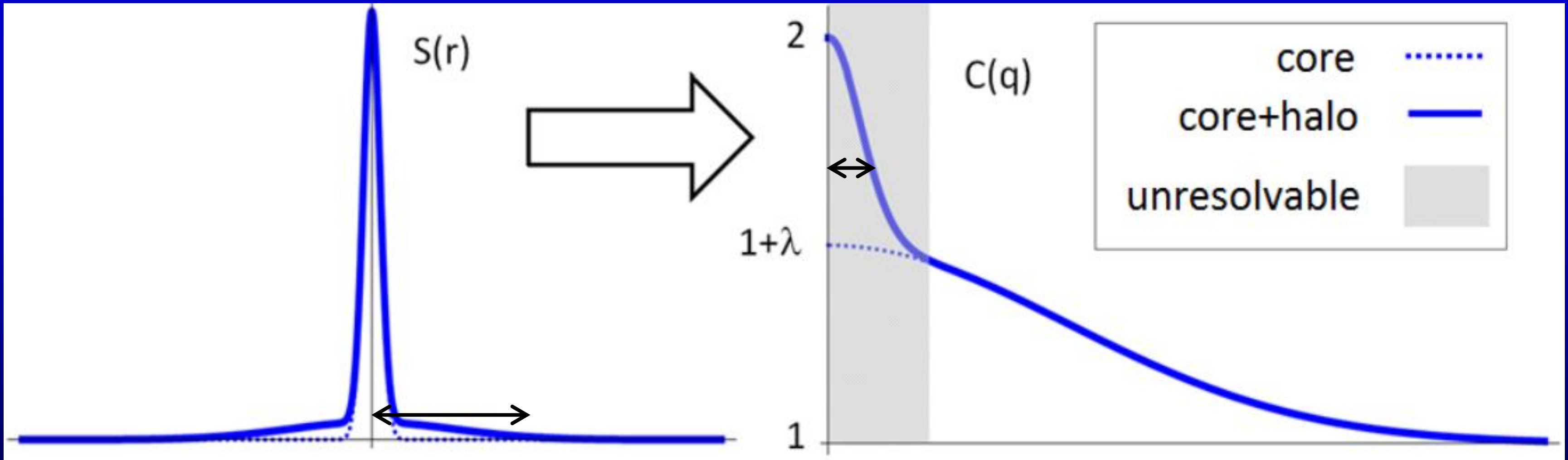
For details: D. Kincses, poster at QM17



Resonance pions reduce the corr. strength [1, 2]
Core-Halo model: $S = S_C + S_H$
Primordial pions - Core $\lesssim 10$ fm
Resonance pions - from very far regions - Halo
Corr. strength→C-H ratio: $\lambda = \left(\frac{N_C}{N_C+N_H}\right)^2$

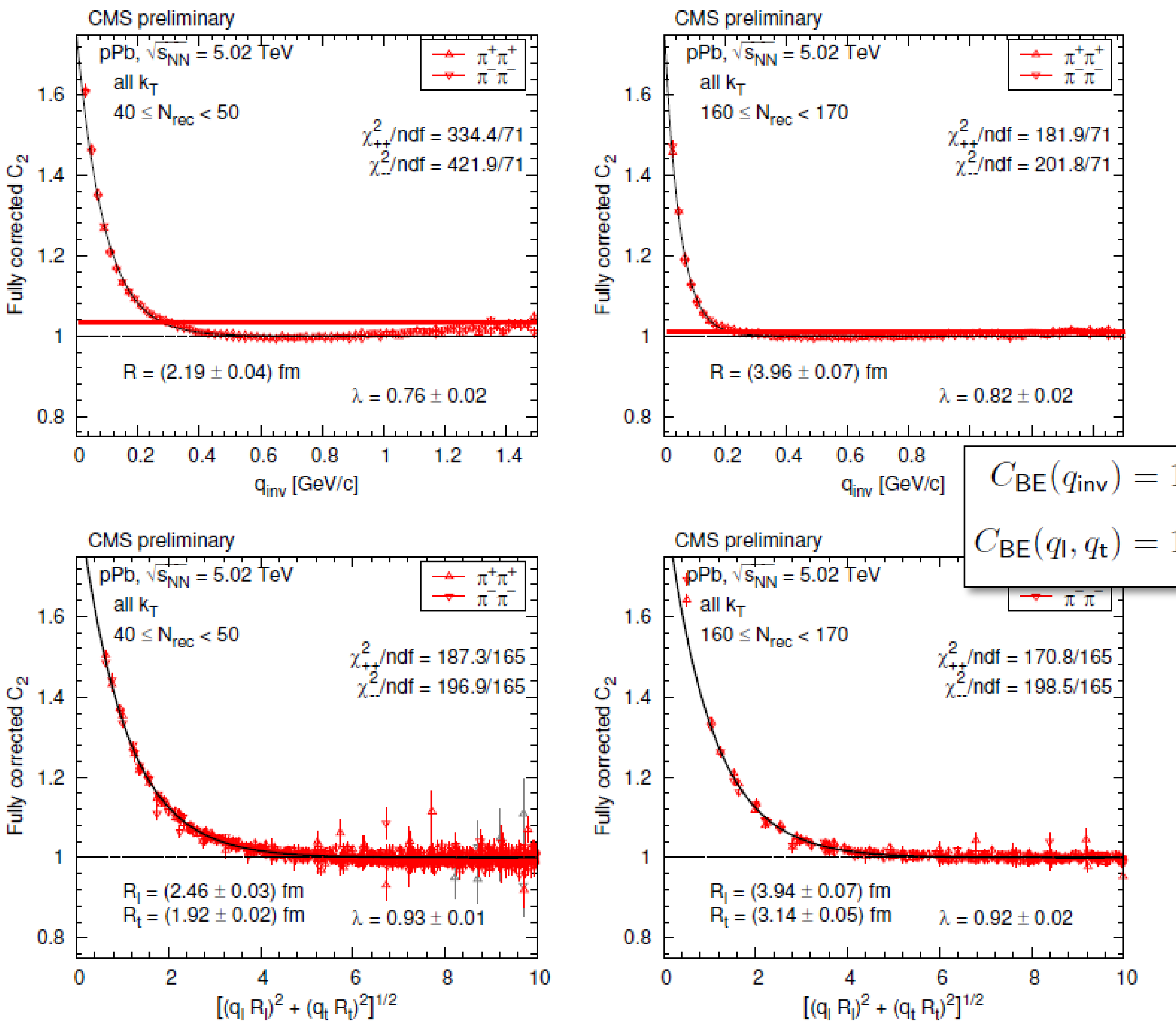
$R = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$.

Variance: halo dominated!



[1] J. Bolz et al: Phys.Rev. D47 (1993) 3860-3870
[2] T. Cs, B. Lörstad, J. Zimányi: hep-ph/9411307

HBT: Is C(Q) a Gaussian?



$$C_{\mathrm{BE}}(q_{\mathrm{inv}}) = 1 + \lambda \exp\left[-q_{\mathrm{inv}} R\right] ,$$

$$C_{\mathrm{BE}}(q_{\mathrm{l}}, q_{\mathrm{t}}) = 1 + \lambda \exp\left[-\sqrt{(q_l R_l)^2 + (q_t R_t)^2}\right] ,$$

CMS Preliminary

pPb@ √s = 5.02 TeV:

arXiv:1411.6609

- **1 + positive definite ?**
- **CL of the fits?**
- **NOT Gaussian !**
- **BUT: Exponential !**
- **IF α ≠ 2 → α = 1 ! (?)**

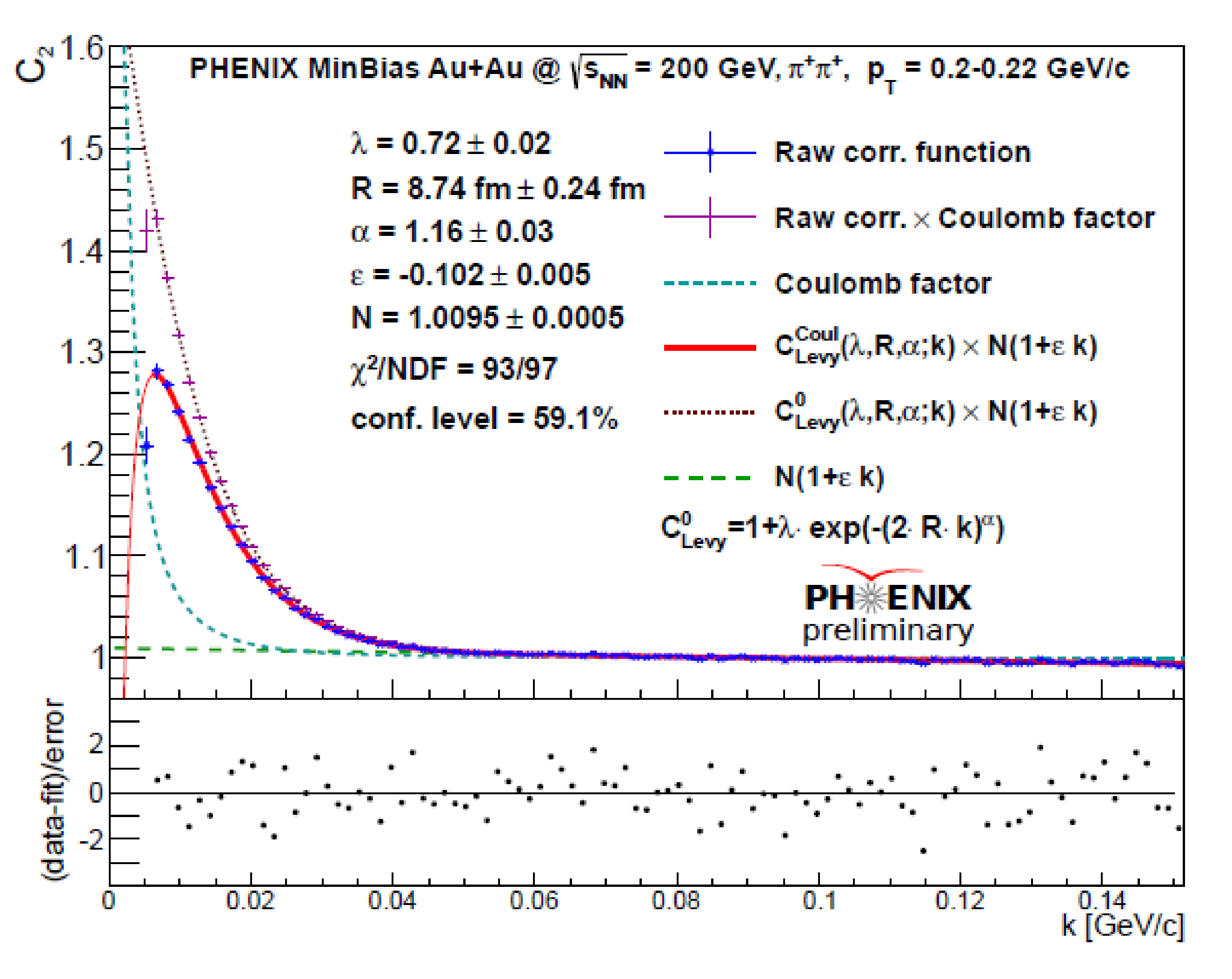
As the dimensionality increases from d= 1 to 3, shape analysis degrades

arXiv:1411.6609

See WPCF2017 talk of Sandra Padula for methods

HBT: Is C(Q) indeed exponential?

PHENIX Preliminary min. bias Au+Au@ $\sqrt{s_{NN}}$ = 200 GeV from arXiv:1610.05025



- 1 + positive definite
- Levy expansion: no 1st order correction
- CL = 59.1 %
- NOT Gaussian !
- NOT Exponential !
- $1 < \alpha < 2$
- $\alpha = 1.16 \pm 0.03$
- m_t dependent

What are the systematics of the source parameters,

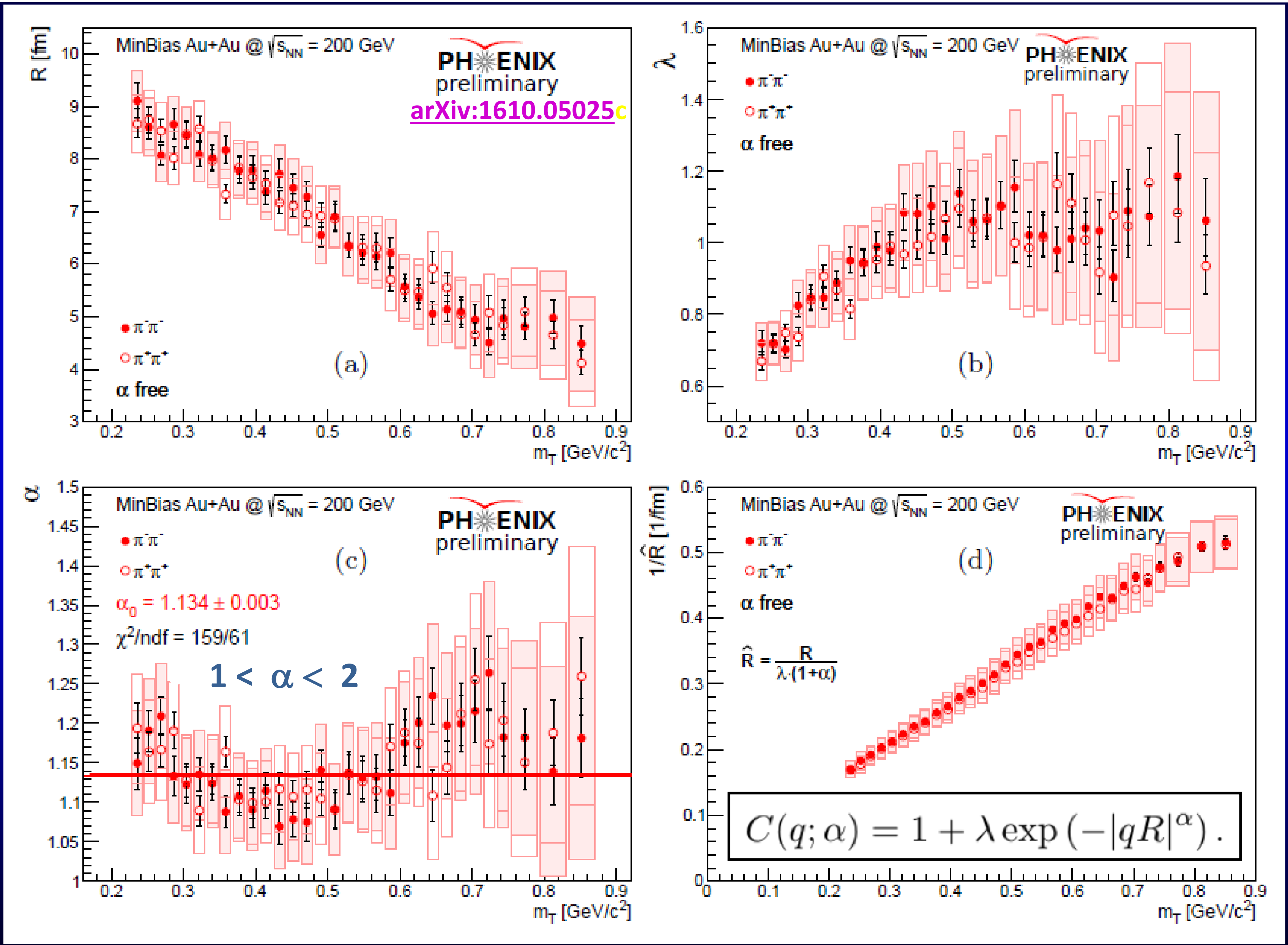
$\lambda = \lambda(m_t)$,

$R = R(m_t)$,

$\alpha = \alpha(m_t)$?

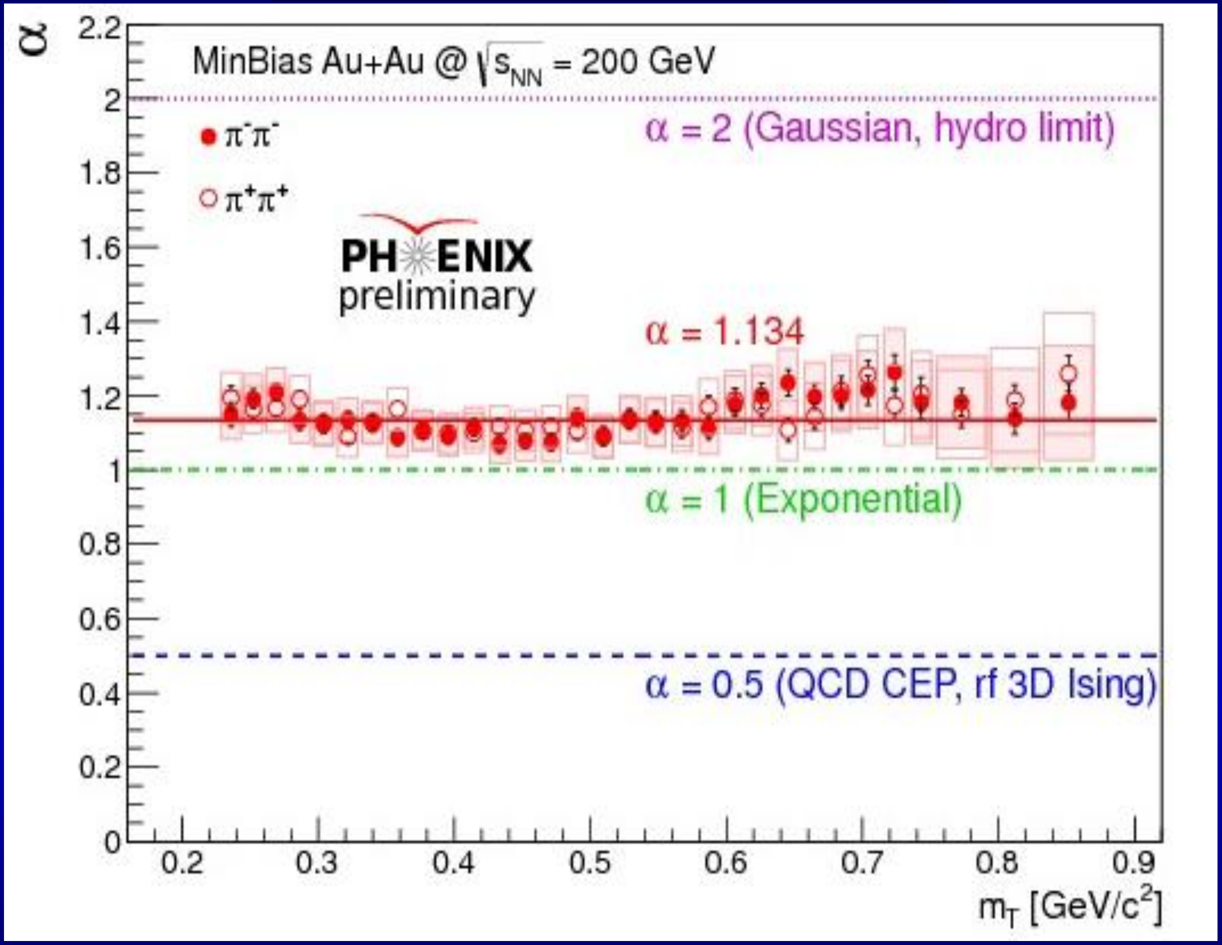
See the WPCF 2017 talk of M. Csanád for details

HBT: Is C(Q) an exponential?



Interpretation of α

$$C(q;\alpha) = 1 + \lambda \exp\left(-|qR|^{\alpha}\right).$$



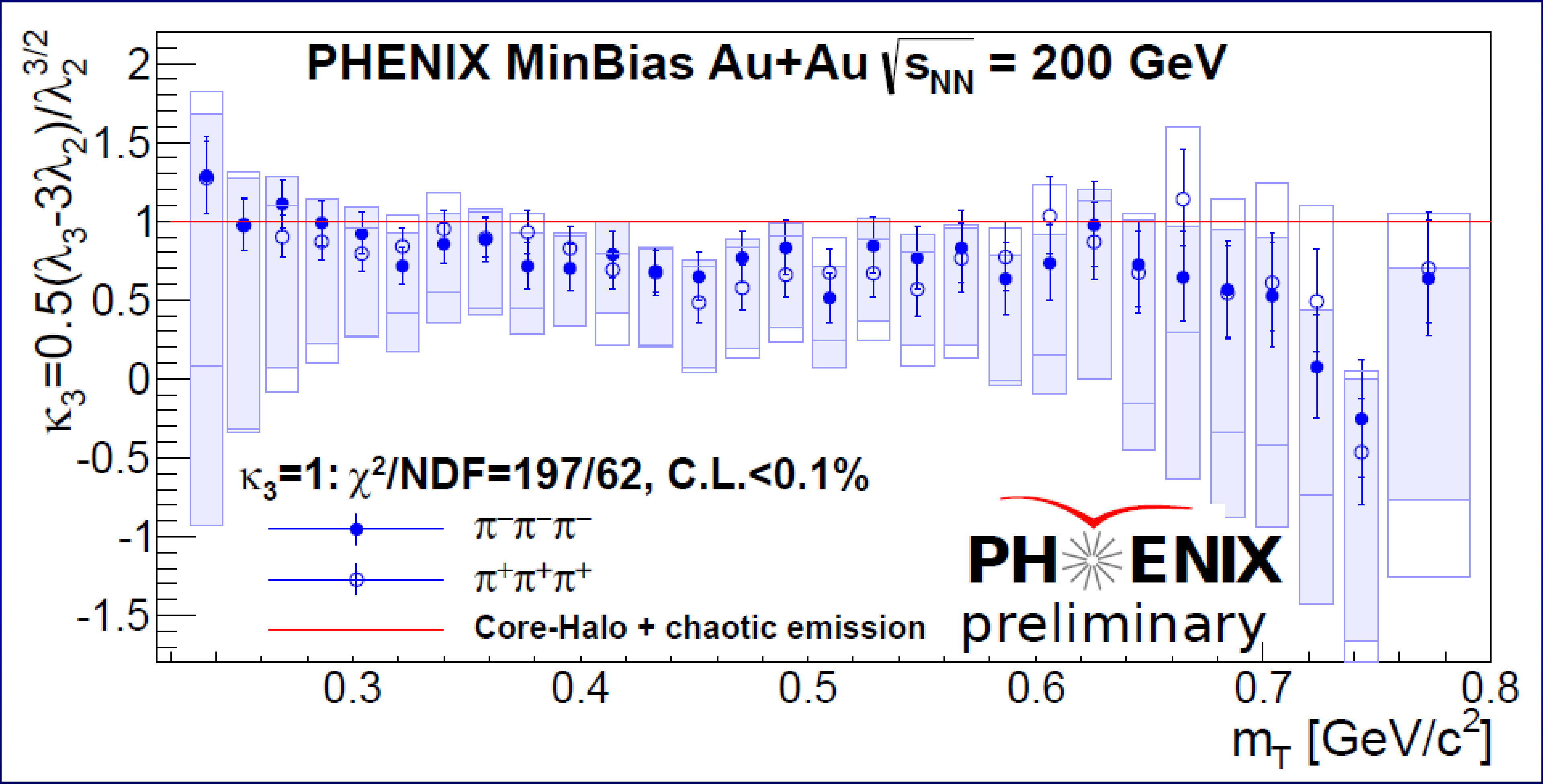
Prediction: at QCD CEP, $\alpha = \eta_c \leq 0.5$ (critical exponent of the correlation function)

T. Cs, S.Hegyi, T. Novák, W.A. Zajc, nucl-th/0512060 T. Cs, arXiv.org:0903.0669

Search for the QCD critical point with α (m_T, $\sqrt{s}$, %, …)

HBT: Two-particle symmetrization - or not ?

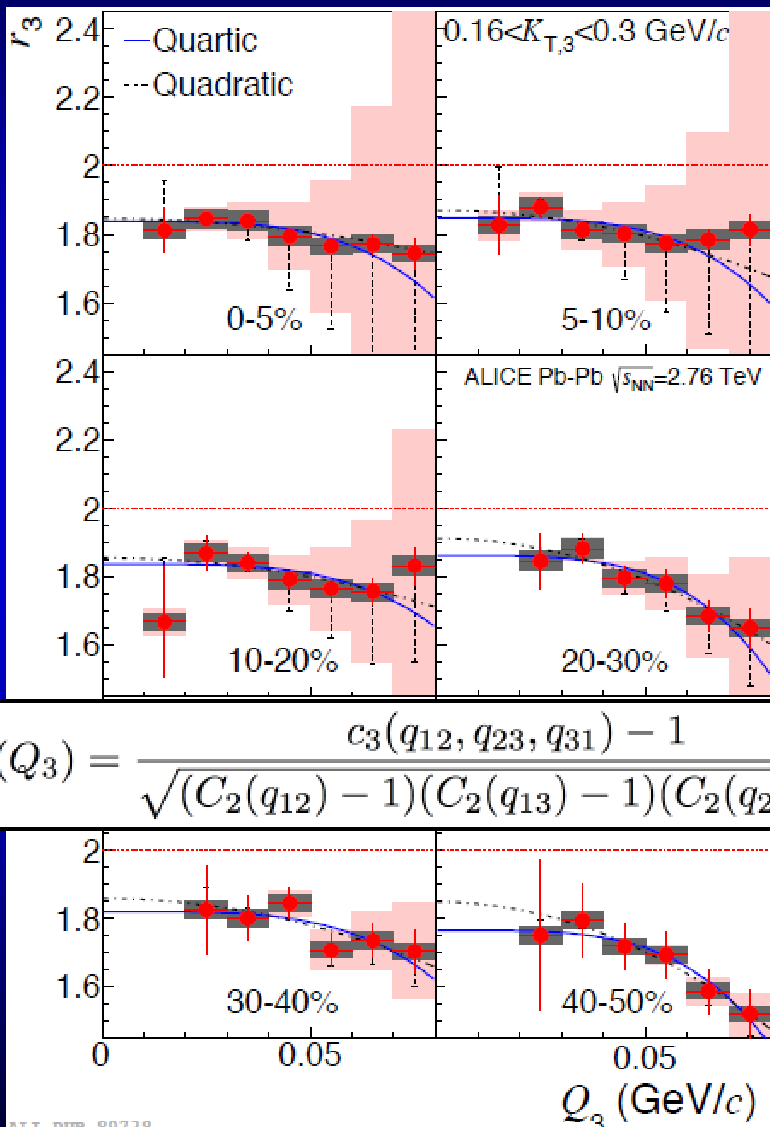
See the WPCF 2017 talk of Tamás Novák for details



PHENIX preliminary data from A.Bagoly, poster at QM17

Centrality dependence? Excitation function? Partial coherence measurement possible!

HBT: Two-particle symmetrization - or not ?



$$r_3(Q_3) = \frac{c_3(q_{12}, q_{23}, q_{31}) - 1}{\sqrt{(C_2(q_{12}) - 1)(C_2(q_{13}) - 1)(C_2(q_{23}) - 1)}}$$

ALICE Pb+Pb @ $\sqrt{s_{NN}}$= 2.76 TeV

Centrality dependence!

Partial coherence if

$r_3(Q=0) \neq 2$

Result:

$r_3(Q=0) < 2$

$p_c = 0.23 \pm 0.08$

First 3σ (+?) indication of

Bose-Einstein condensation

in a system of charged particles!

ALICE, Phys. Rev. C89 (2014) 024911

For details, see D. Gangadharan's CERN talk

Magnetic catalysis of B-E condensation of charged particles in finite V

A. Ayala, P. Mercado, C. Villavicencio, arXiv:1609.02595

Checks out the magnitude of the effect for transient magnetic fields in Pb+Pb at LHC

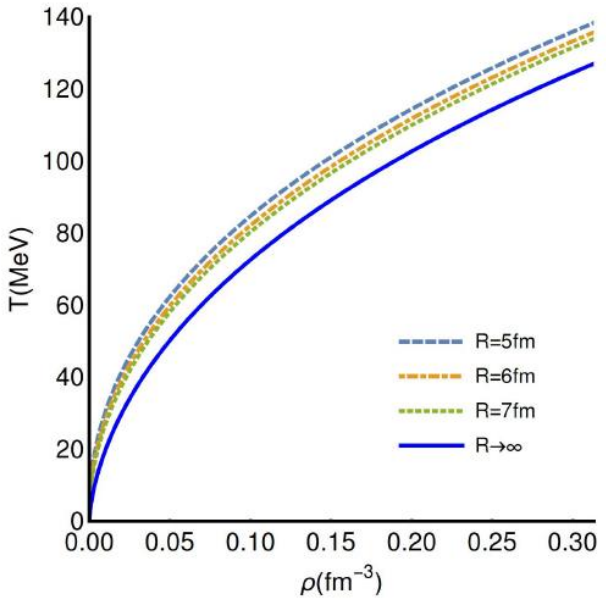


FIG. 1. Critical temperature for BEC as a function of the system's density in the absence of magnetic field effects. For a given value of the density, the critical temperature increases as the system's size decreases. For comparison we also show the case where the volume is taken to infinity.

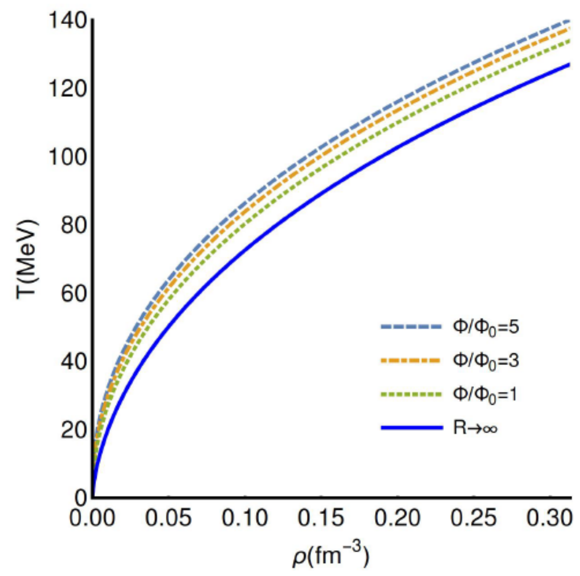


FIG. 2. Critical temperature for BEC as a function of the density for a fixed system's radius $R = 7$ fm and several values of the magnetic flux. For a given value of the density, the critical temperature increases as the magnetic flux increases. For comparison we also show the case where the volume is taken to infinity in the absence of a magnetic field.

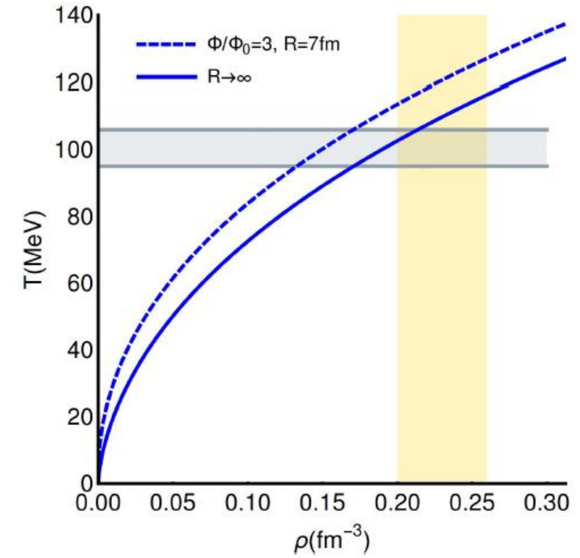


FIG. 3. Critical temperature for BEC as a function of the system's density. For comparison we show a range of freeze-out temperatures from central to semi-central collisions and a range of densities and magnetic fluxes in semi-central collisions ($R \sim 7$ fm) at LHC energies. Notice that even for moderate magnetic fluxes the critical temperatures obtained are above the freeze-out temperature range.

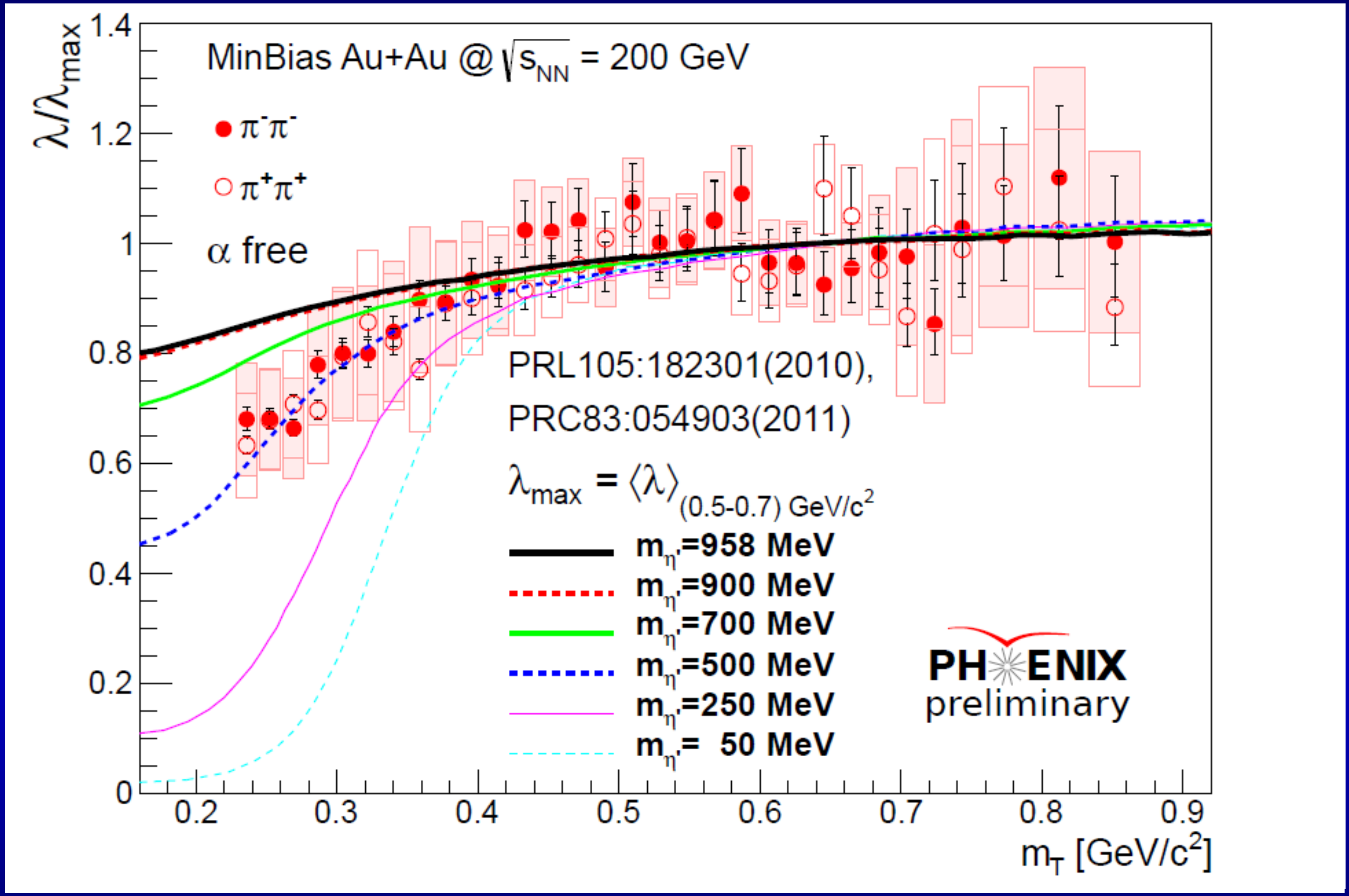
In peripheral heavy ion collisions at LHC, strong magnetic field and R ~ 7 fm

Critical temperature of Bose-Einstein condensation of charged pions above the freeze-out T.

Interpretation of λ

$$C(q;\alpha) = 1 + \lambda \exp\left(-|qR|^{\alpha}\right).$$

See the talk of Máté Csanád at WPCF 2017 for details



PHENIX preliminary data from arXiv:1610.05025

Method: S. Vance, T. Cs., D. Kharzeev: PRL 81 (1998) 2205-2208 , nucl-th/9802074

Predictions: Cs. T., R. Vértesi, J. Sziklai, arXiv:0912.5526 [nucl-ex] arXiv:0912.0258 [nucl-ex]

Cross-check, Sinyukov-Tolstykh model

$$\rho(x_i) = (\pi R^2/2)^{-3/2} \exp(-2\mathbf{x}_i^2/R^2).$$

The momentum spectrum

$$f(p) = \tilde{f}^2(p) = (2\pi p_0^2)^{-3/2} \exp(-\mathbf{p}^2/2p_0^2)$$

$$C_{\pi^-\pi^-}(p_1, p_2) = \chi\left(1 + \lambda e^{-\frac{\Delta \mathbf{p}^2 R_{\rm int}^2}{4}}\right).$$

$$\lambda = \left[1 + \frac{1-\alpha}{\alpha(1+p_0^2 R^2)^{3/2}}\right]^{-1}.$$

Intercept parameter λ

Independent of momentum p

Decrease controlled by

source radii R and $p_0{}^2 = m\, T_{eff}$

$p_0\, R$ ~ average phase space volume

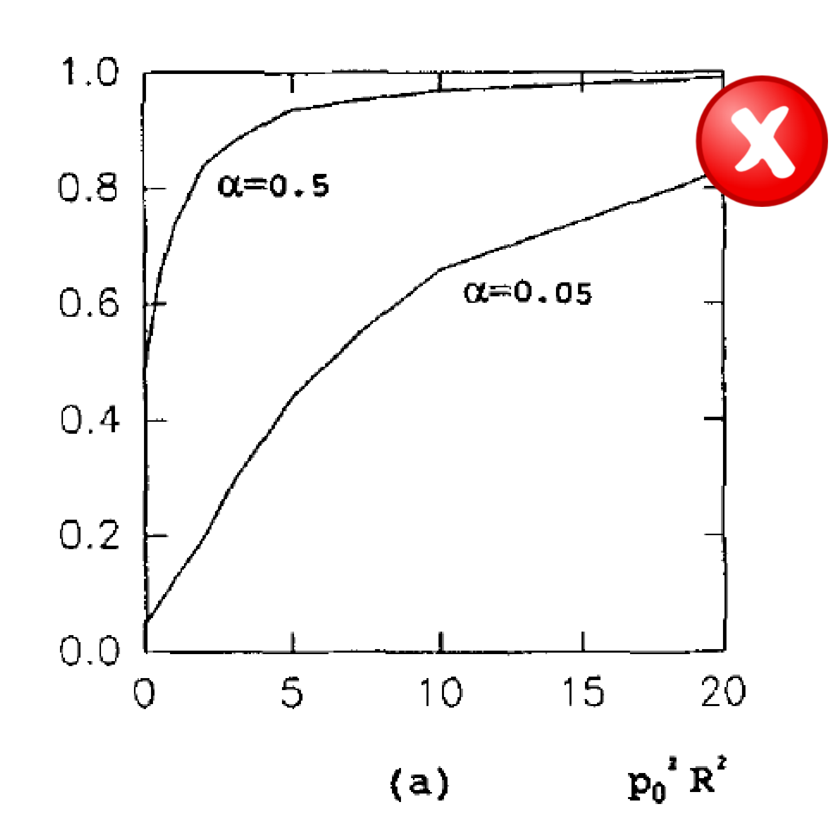
Yu, M. Sinyukov, A. Yu. Tolstykh,
Z. Phys. C 61, 593 (1994)

Model based on partial coherence

Static Gaussian source, size R

Thermal momentum spectrum,

Slope $p_0{}^2 = m\, T_{eff}$



Cross-check, pion laser model

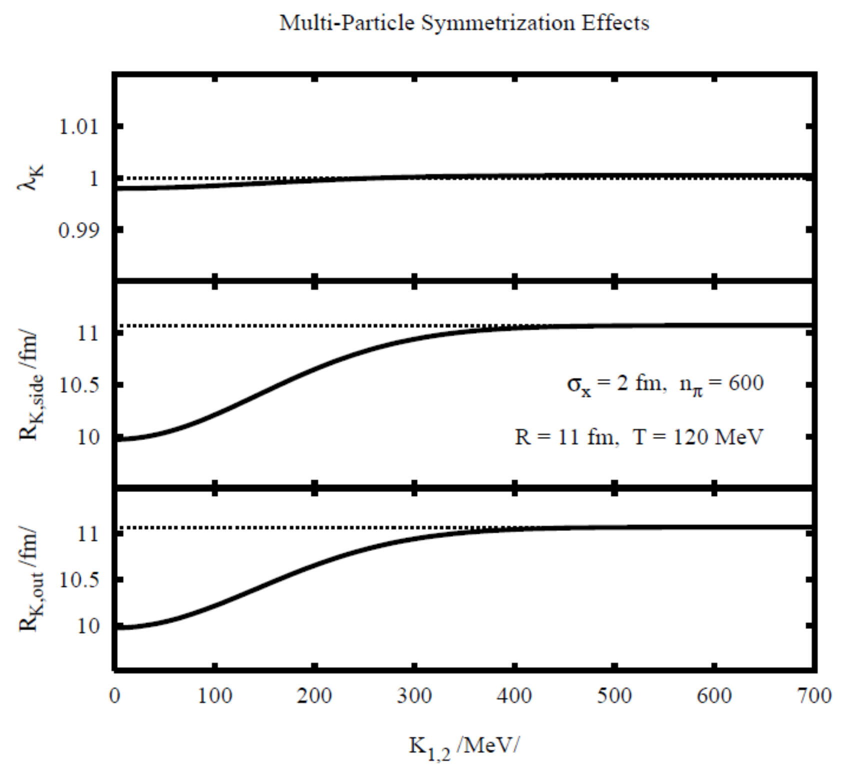


FIG. 1. Multi-particle symmetrization results at low **K** in a momentum-dependent reduction of the intercept parameter $\lambda_{\mathbf{K}}$, the side-wards and the outwards radius parameters, $R_{\mathbf{K},s}$ and $R_{\mathbf{K},o}$ from their static values of 1 and R_e, respectively. The enhancement of these parameters at high momentum is hardly noticeable for large and hot systems.

$$\lambda(m_T)/\lambda_{\max} = 1 - H \exp(-(m_T^2 - m_\pi^2)/(2\sigma^2))$$

T. Cs, J. Zimányi, Phys.Rev.Lett. 80 (1998) 916-918,

Pion-laser model solved exactly. NP hard.
Bose-Einstein condensation of wave-packets.

$$\lambda_{\mathbf{K}} = 1 + \frac{2}{(2x)^{\frac{3}{2}}}\left[1 - 2^{(5/2)} \exp\left(-\frac{\mathbf{K}^2}{\sigma_T^2}\right)\right]$$

$$x = R_e^2 \sigma_T^2$$

Analytic result for fixed multiplicity n.
N-particle Bose-Einstein symmetrization.
Suppression parameter H controlled
by HBT radii R_e and $\sigma_T{}^2 = 2\,\sigma^2$,
width of the depression in λ(mt)
What about numerical values ?
R_e: limiting value of R(K) at large K

$\lambda_{max} = 1+2/(2x)^{3/2}$ > 1 slightly
→ $H = 2^{5/2}/(1+ 2^{1/2}x^{3/2})$
→ $R_e \geq 4$ fm, $\sigma^2_T = 2\, m_\pi T$, T ≥ 170 MeV → x ≥ 16 → H ≤ 0.06, while H(preliminary) ~ 0.6
→ H, the size of the „hole" in $\lambda(m_T)/\lambda_{max}$ or is too small for PHENIX preliminary Au+Au

Cross-check, Akkelin-Sinyukov

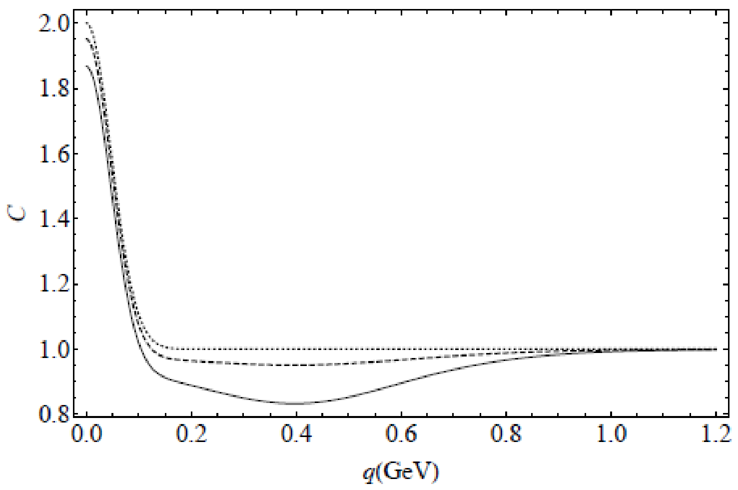


FIG. 1. The two-pion correlation functions $C(q_x = q, q_y = 0, q_z = 0; p_x = p, p_y = 0, p_z = 0)$, with $p = 0.2$ GeV/c, $R = 3$ fm, and $T = 0.06$ GeV in the case of the two-particle system: $N = 2$. The dotted line corresponds to the "standard" expression for the pure Bose-Einstein correlation function (CF) of the Gaussian source. The dashed line is related to the CF when one-boson spectra in the two-boson system is calculated from the two-particle spectra by integrating it over one of the momenta. The solid line corresponds to our approximation based on Eq. (42).

S.V. Akkelin, Yu.M. Sinyukov, arXiv:1603.02951 [nucl-th]

Multi-particle Bose-Einstein symmetrization
Quantum canonical ensemble, fixed multiplicity

$$\lambda(p) \simeq 1 - 4e^{-\beta E(p)+\beta m}(2\pi)^3 \frac{\lambda_T^3}{V}$$

$$R_G^2 = \frac{1+\lambda(p)}{2\lambda(p)} R^2.$$

$$\lambda_T = (2\pi m T)^{-1/2} \qquad R^3 = V(2\pi)^{-3/2}$$

Analytic result for fixed multiplicity n.
Suppression parameter H, in non-rel. appr. controlled by
HBT radii R_e and $\sigma_T^2 = 2\sigma^2 = 2$ m T,
width of the depression in λ(mt)

$$x = R_e^2 \sigma_T^2$$

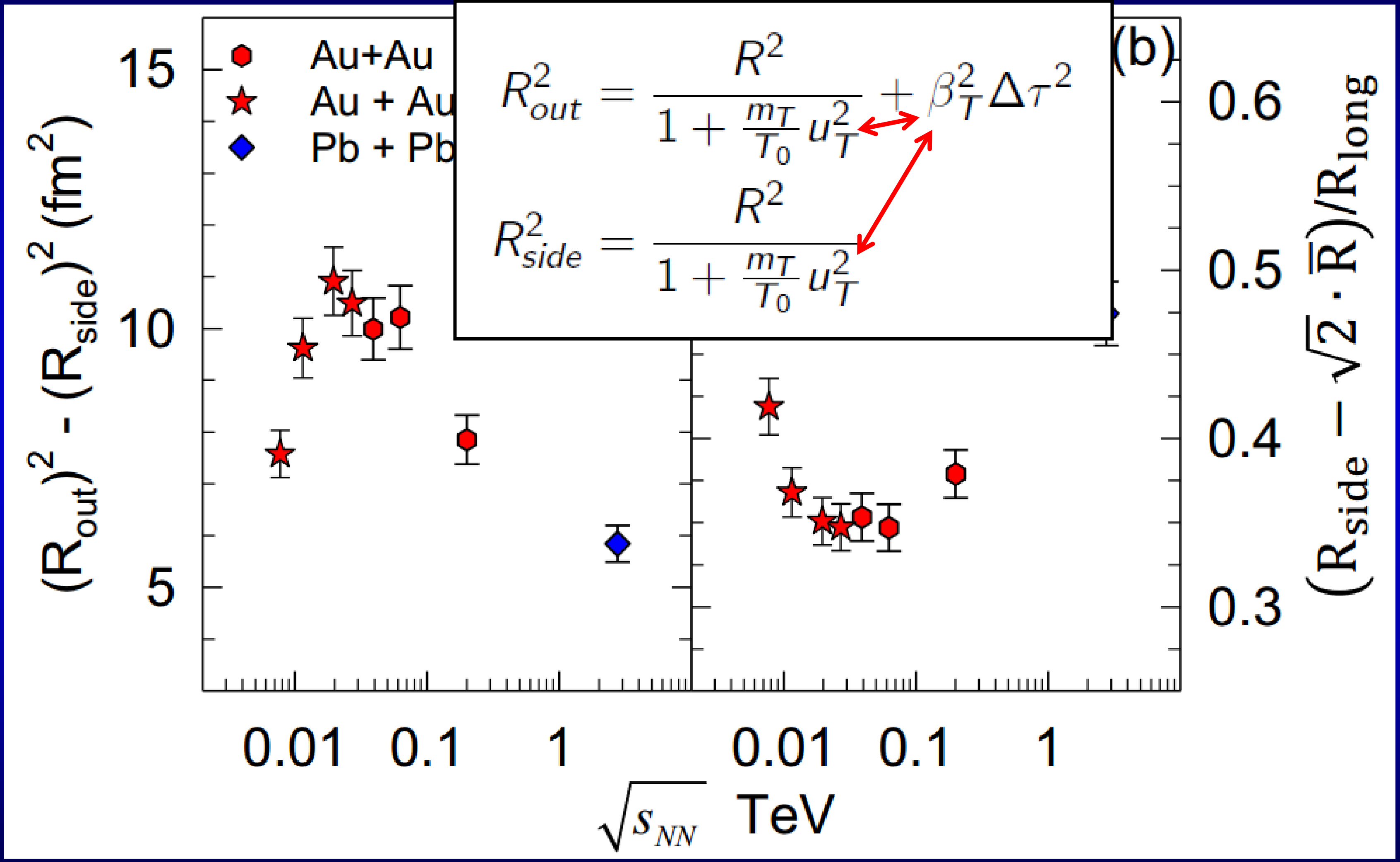
What about numerical values ?
R_e: limiting value of R(K) at large K

λ_{max} = 1 (increase not obtailed)

→ H = $2^{7/2}/x^{3/2}$ here
→ $R_e \geq 4$ fm, $\sigma^2_T = 2\, m_\pi$ T, T ≥ 170 MeV → x ≥ 16 → H ≤ 0.18, while H(preliminary) ~ 0.6

→ H, the size of the „hole" in $\lambda(m_T)/\lambda_{max}$ or is too small for PHENIX preliminary Au+Au

HBT: Signal of QCD Critical Point - or not ?



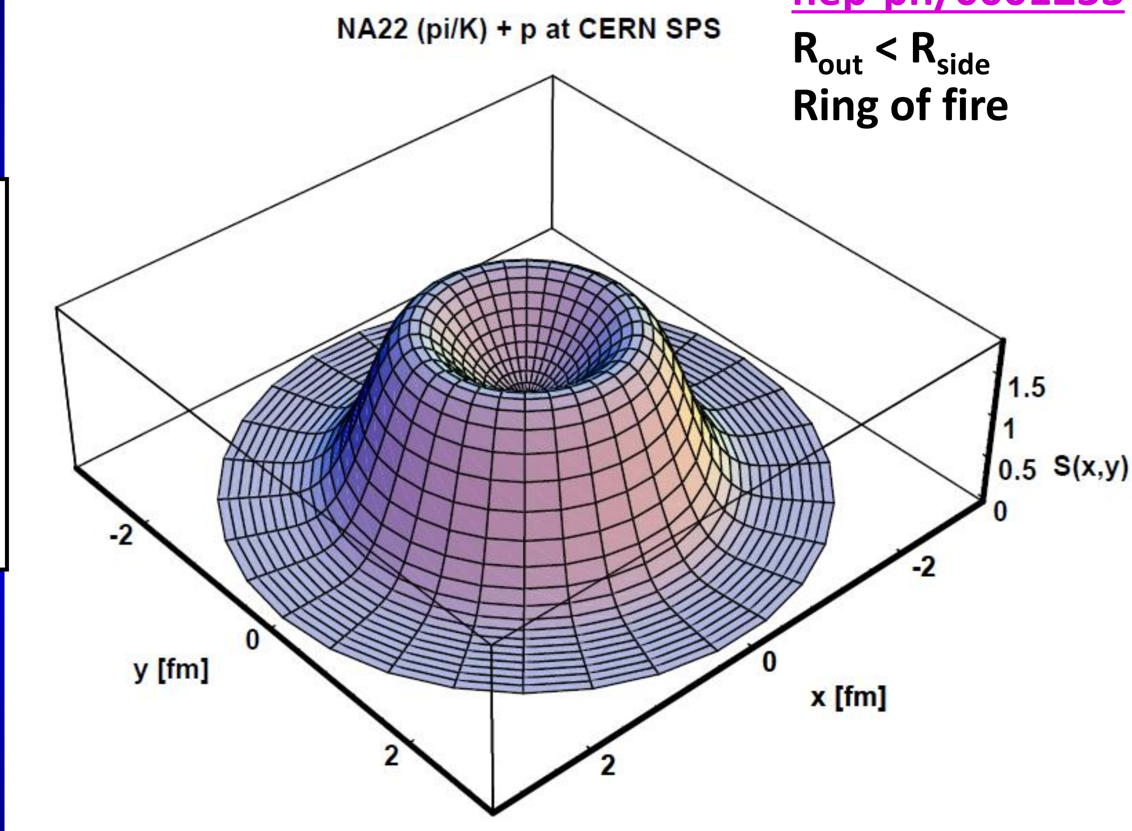
Clear indication of non-monotonic behavior in combined ALICE, STAR and PHENIX data

Roy Lacey: $\sqrt{s_{NN}}$ ~ 20-60 GeV → best value ~ 50 GeV. Needs further study!

HBT: Signal of QCD Critical Point - or not ?

$$R_{out}^2 = \frac{R^2}{1 + \frac{m_T}{T_0} u_T^2} + \beta_T^2 \Delta\tau^2$$

$$R_{side}^2 = \frac{R^2}{1 + \frac{m_T}{T_0} u_T^2}$$



hep-ph/0001233

$R_{out} < R_{side}$

Ring of fire

Roy Lacey: $\sqrt{s_{NN}}$ ~ 20-60 GeV → best value ~ 50 GeV.

Roy Lacey's 1st indication of QCD CEP needs further study!

Valid in a special m_T window: only if radial flow u_T ~ velocity of the pair β_T

Validity can be extended: also to $R_{out} < R_{side}$: use exact hydro solutions, allowing ring of fires !

HBT: New solutions of fireball hydro - triaxial, rotating and expanding

Coordinate-space ellipsoid at the beginning of time evolution

Final coordinate-space ellipsoid at freeze-out ϑ_f

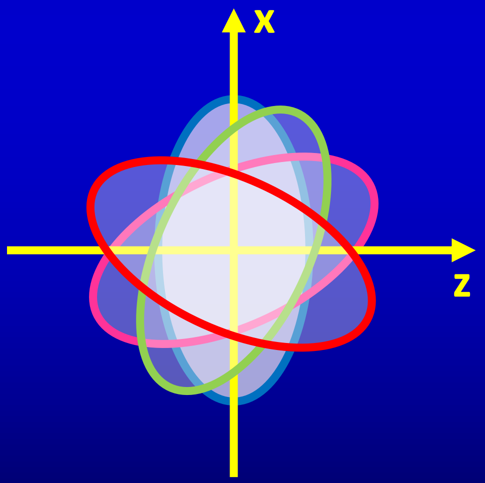
"Momentum-space ellipsoid" (eigenframe of single-particle spectrum) θ_p

"HBT-space ellipsoid" (eigenframe of HBT correlations) θ_q

$$\theta'_p = \frac{1}{2}\arctan\left(\frac{2T'_{xz}}{T'_{xx} - T'_{zz}}\right) = \frac{1}{2}\arctan\left(\frac{2\omega R}{\dot{X} + \dot{Z}}\right)$$

$$\theta_p = \vartheta_f + \theta'_p$$

$$\theta'_{q,i} = \frac{1}{2}\arctan\left(\frac{2XZT'_{xz,i}}{Z^2T'_{xx,i} - X^2T'_{zz,i}}\right)$$



Three angles of rotation: in momentum space p, in HBT's q-space, and in coordinate space r

$$\theta_p \neq \theta_q \neq \theta_r$$

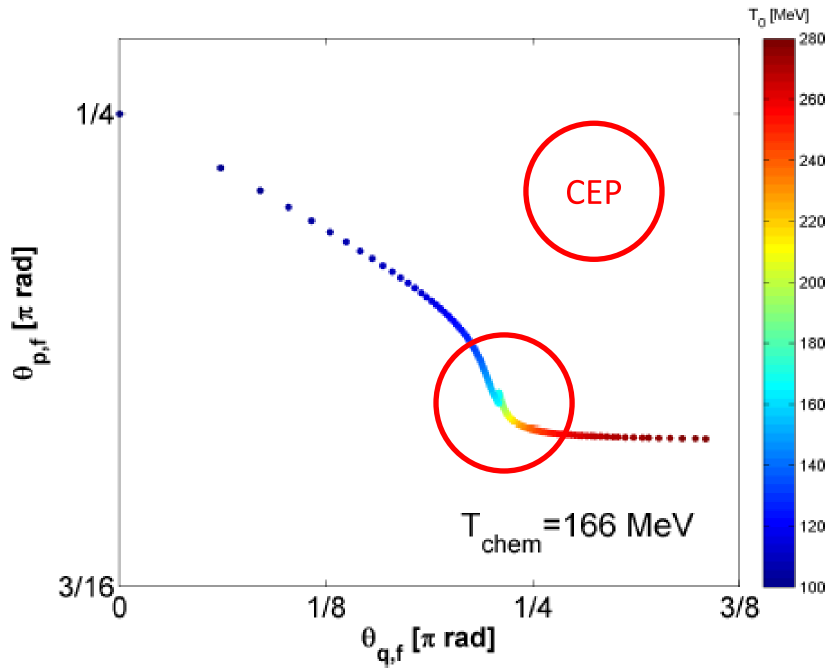
See Gábor Kasza's talk for more details

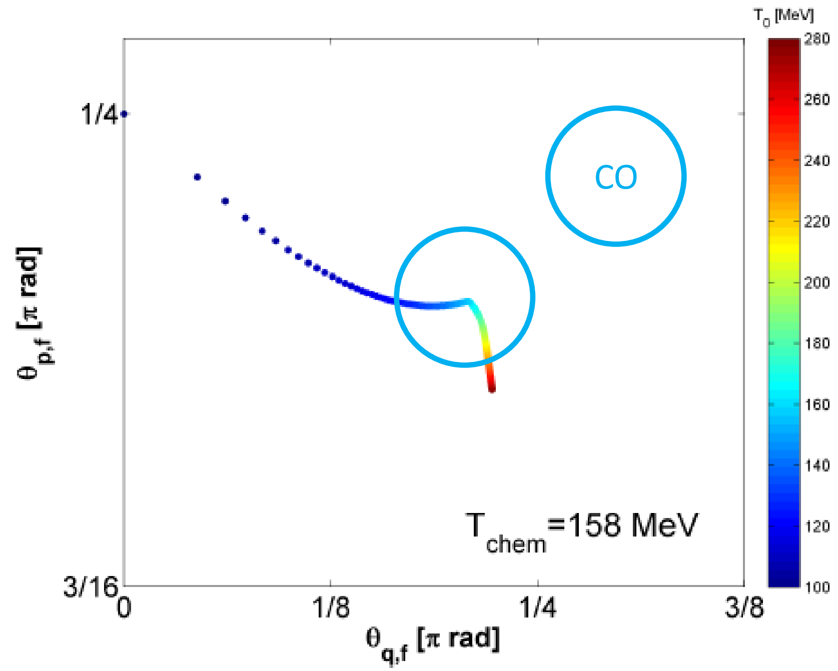
arXiv:1610.02197

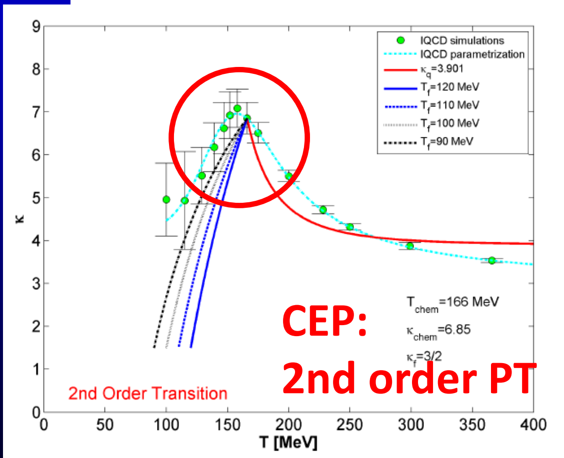
M.I. Nagy and T. Cs: arXiv:1606.09160

New, triaxial and rotating solutions

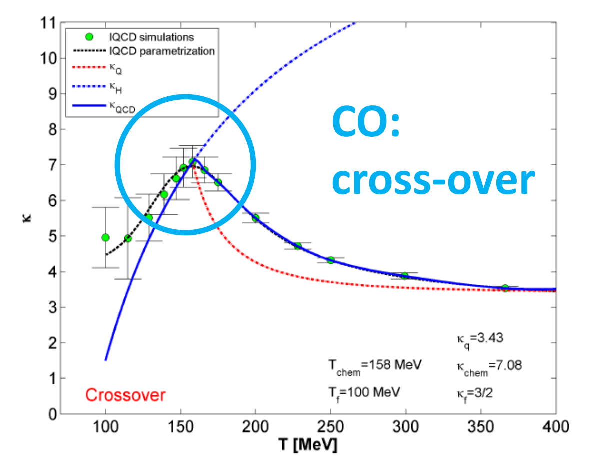
lattice QCD EoS for a 2nd order QCD transition







See G. Kasza's talk for more details



Summary and conclusions

Positive definitene form – or not?

Has to be checked:

L3 and CMS data show anticorrelated region

in e^+e^- at LEP and in pp at LHC

Two-particle symmetrization effect – or not?

ALICE data indicates possible partial coherence

higher order symmetrization effects in system of charged particles

Seems to be consistent with theoretical expectations

Bose-Einstein condensation of charged particles never seen before!

MUST be cross-checked

Gaussian shape – or not?

Has to be checked:

PHENIX preliminary data indicates non-Gaussian structure

Levy index of stability α < 2 (Gaussian) significantly

New possibilities

- **model independent shape analysis**
- **to measure η' modification**
- **to identify QCD CEP from angles of rotation**

Backup slides

Questions?

Hanbury Brown: a compound family name

A SURVEY OF 23 LOCALIZED RADIO SOURCES IN THE NORTHERN HEMISPHERE

R. Hanbury Brown and C. Hazard

Mon. Not. R. astr. Soc. (1971) **151**, 161–176.

A STUDY OF α VIRGINIS WITH AN INTENSITY INTERFEROMETER

D. Herbison-Evans, R. Hanbury Brown, J. Davis and L. R. Allen

(Received 28 August 1970)

R. Hanbury Brown, J. Davis and L. R. Allen

(Received 1973 November 5)

Hanbury Brown: a *family* name

1971MNRAS.151..161H

Mon. Not. R. astr. Soc. (1971) **151,** 161–176.

A STUDY OF α VIRGINIS WITH AN INTENSITY INTERFEROMETER

D. Herbison-Evans, R. Hanbury Brown, J. Davis and L. R. Allen

(Received 28 August 1970)

Grandfather: Sir Robert Hanbury Brown, K.C.M.G., a notable irrigation engineer (Wiki link)

Father: Basil Hanbury Brown

Twin sons:

- **Robert Hanbury Brown**
- **Jordan Hanbury Brown**

Daughter:

- **Marion Hanbury Brown**

„It is not all that unusual that an English last name is a compound one, with or without a hyphen.”

Wes Metzger

Thank you Wes!

- **For private communications on the family of Sir Robert *Hanbury Brown***

HBT: 1 + positive definite term: - how to check ?

Model-independent method, to analyze Bose-Einstein correlations
IF experimental data satisfy

- The measured data tend to a constant for large values of the observable Q.
- There is a non-trivial structure at some definite value of Q, shift it to Q = 0.

Model-independent, but experimentally testable:

- t = Q R
- dimensionless scaling variable
- approximate form of the correlations *w(t)*
- *Identify w(t)* with a measure in an abstract Hilbert-space

$$\int dt w(t) h_n(t) h_m(t) = \delta_{n,m},$$

$$f(t) = \sum_{n=0}^{\infty} f_n \, h_n(t),$$

$$f_n = \int dt w(t) \, f(t) \, h_n(t).$$

e.g. $t = Q_I R_I$

T. Csörgő and S: Hegyi, hep-ph/9912220, T. Csörgő, hep-ph/001233

HBT: 1 + positive definite term: How to check ?

$$C_2(\mathbf{k}_1, \mathbf{k}_2) = \frac{N_2(\mathbf{k}_1, \mathbf{k}_2)}{N_1(\mathbf{k}_1)\, N_1(\mathbf{k}_2)},$$

$$C_2(t) = \mathcal{N}\left\{1 + \lambda_w\, w(t) \sum_{n=0}^{\infty} g_n h_n(t)\right\}$$

Model-independent AND experimentally testable:

- method for any approximate shape $w(t)$
- the core-halo intercept parameter of the CF is
- coefficients by numerical integration (fits to data)
- condition for applicability: experimentally testabe
- Nearly Gauss correlations, (- ∞, ∞) → Edgeworth
- Nearly Gauss correlations, (0, ∞) → Gauss New!
- Nearly exponential correlations, (0, ∞) → Laguarre
- Nearly Levy correlations, (0, ∞) → Levy expansion New!

$$\lambda_* = \lambda_w \sum_{n=0}^{\infty} g_n h_n(0)$$

$$g_n = \int dt\, R_2(t) h_n(t)$$

$$\int dt\, \left[R_2^2(t)/w(t)\right] < \infty$$

Edgeworth expansion method

Gaussian w(t), -∞ < t < ∞

$$t = \sqrt{2} Q R_E,$$

$$w(t) = \exp(-t^2/2),$$

$$\int_{-\infty}^{\infty} dt \, \exp(-t^2/2) H_n(t) H_m(t) \propto \delta_{n,m},$$

$$H_1(t) = t,$$
$$H_2(t) = t^2 - 1,$$
$$H_3(t) = t^3 - 3t,$$
$$H_4(t) = t^4 - 6t^2 + 3, \; \ldots$$

$$H_n(t) = \exp(t^2/2) \left(-\frac{d}{dt} \right)^n \exp(-t^2/2).$$

$$C_2(Q) = \mathcal{N} \left\{ 1 + \lambda_E \exp(-Q^2 R_E^2) \times \left[1 + \frac{\kappa_3}{3!} H_3(\sqrt{2} Q R_E) + \frac{\kappa_4}{4!} H_4(\sqrt{2} Q R_E) + \ldots \right] \right\}.$$

3d generalization straightforward

- **Applied by NA22, L3, STAR, PHENIX, ALICE, CMS (LHCb)**

Gauss expansion method

Gaussian w(t), 0 < t < ∞

$$L_0(t\,|\,\alpha=2) = \frac{\sqrt{\pi}}{2},$$
$$L_1(t\,|\,\alpha=2) = \frac{1}{2}\left\{\sqrt{\pi}t - 1\right\},$$
$$L_2(t\,|\,\alpha=2) = \frac{1}{32}\left\{(\pi-2)t^2 - \sqrt{\pi}t + 2 - \frac{\pi}{2}\right\}.$$

Provides a new expansion around a Gaussian shape that is defined for the non-negative values of *t* only.

Edgeworth expansion different, its around two-sided Gaussian, includes non-negative values of *t* also.

arXiv:1604.05513 [physics.data-an]

Laguerre expansion method

Model-independent but experimentally tested:

- **w(t): Exponential**
- **$0 < t < \infty$**
- **Laguerre polynomials**

$$t = QR_L,$$
$$w(t) = \exp(-t)$$
$$\int_0^\infty dt\, \exp(-t) L_n(t) L_m(t) \propto \delta_{n,m},$$

$$L_n(t) = \exp(t)\frac{d^n}{dt^n}(-t)^n \exp(-t).$$

$$L_0(t) = 1,$$
$$L_1(t) = t - 1,$$

$$C_2(Q) = \mathcal{N}\left\{1 + \lambda_L \exp(-QR_L)\left[1 + c_1 L_1(QR_L) + \frac{c_2}{2!}L_2(QR_L) + \ldots\right]\right\}$$

First successful tests

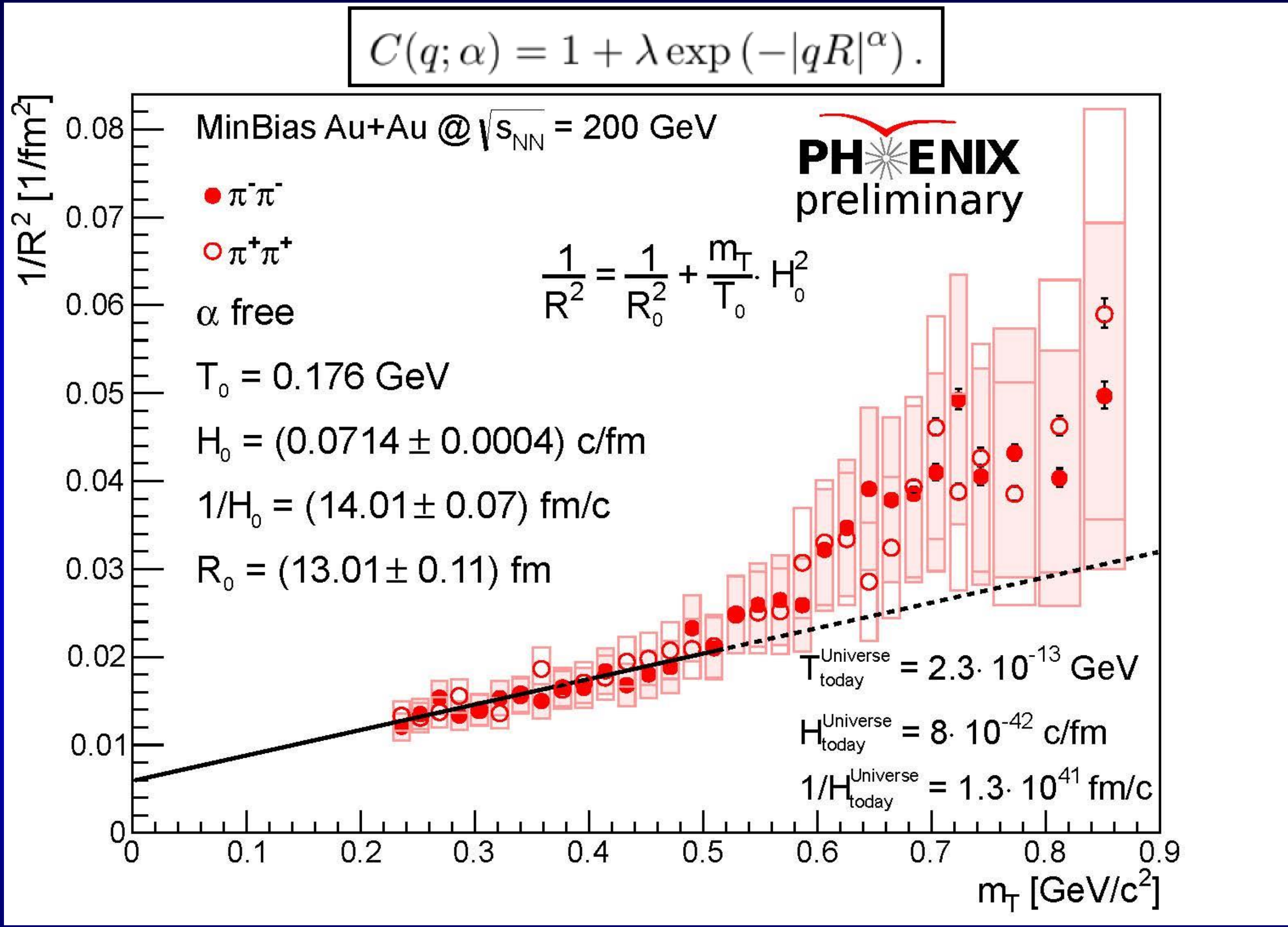
on NA22, UA1 data , convergence criteria satisfied

$$\int_0^\infty dt\, R_2^2(t) \exp(+t) < \infty,$$

Intercept: $\lambda_* \sim 1$

$$\lambda_* = \lambda_L[1 - c_1 + c_2 - \ldots],$$
$$\delta^2\lambda_* = \delta^2\lambda_L\left[1 + c_1^2 + c_2^2 + \ldots\right] + \lambda_L^2\left[\delta^2 c_1 + \delta^2 c_2 + \ldots\right]$$

HBT: Interpretation of R



Possibility: hydro scaling behaviour of R at low m_T

Hubble ratio of Big Bang and Little Bangs ~ 10^{40} (needs centrality dependence, α = 2 …)

M. Csanád, T. Cs, B. Lörstad, A. Ster, nucl-th/0403074

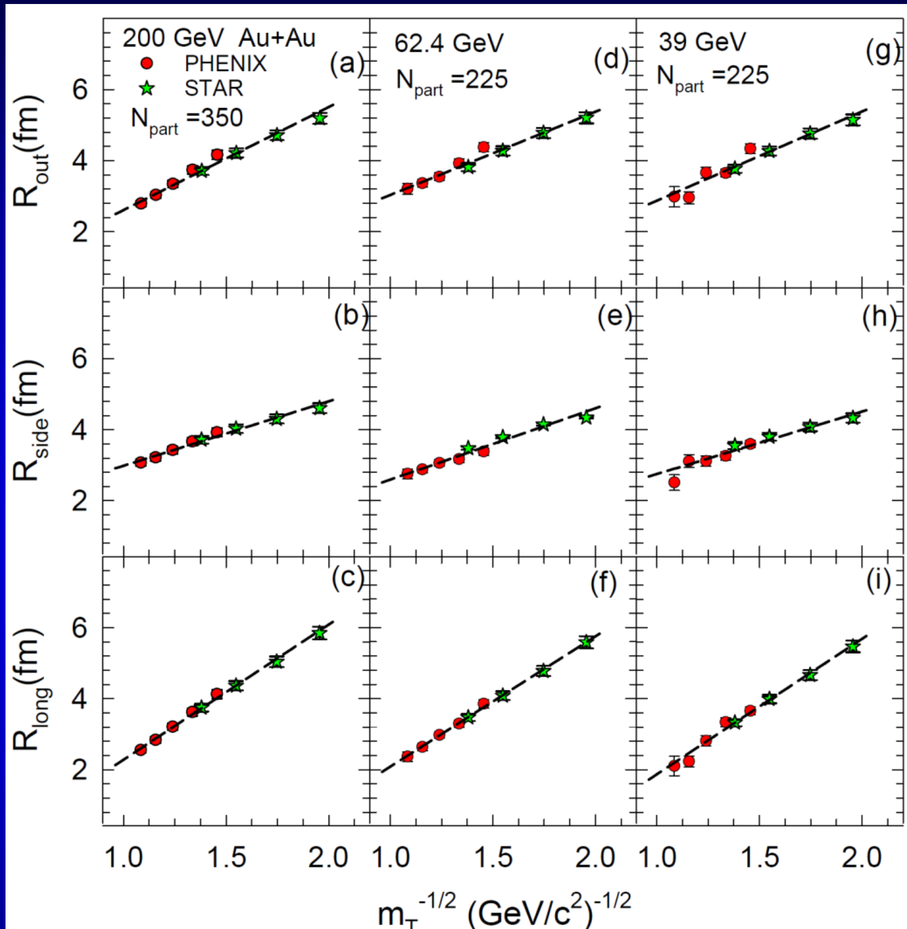
HBT: New solutions of fireball hydro - but academic ?

$\mathbf{QM}\ (T_i \geq T \geq T_{chem})$	$\mathbf{HM}\ (T_{chem} > T \geq T_f)$
$\partial_t \sigma + \nabla(\sigma \mathbf{v}) = 0$	$\partial_t n_i + \nabla(n_i \mathbf{v}) = 0, \quad \forall i$
$T\sigma(\partial_t + \mathbf{v}\nabla)\mathbf{v} = -\nabla p$	$\sum_i m_i n_i (\partial_t + \mathbf{v}\nabla)\mathbf{v} = -\nabla p$
$\frac{1+\kappa}{T}\left[\frac{d}{dT}\frac{\kappa T}{1+\kappa}\right](\partial_t + \mathbf{v}\nabla)T + \nabla\mathbf{v} = 0$	$\frac{1}{T}\left[\frac{d(\kappa T)}{dT}\right](\partial_t + \mathbf{v}\nabla)T + \nabla\mathbf{v} = 0$
$p = \sigma T/(1+\kappa)$	$p = \sum_i p_i = T\sum_i n_i$

$$
\begin{aligned}
\varepsilon + p &= \sum_i \mu_i n_i + T\sigma, \\
\varepsilon + p &\approx T\sigma, & (T_i \geq T \geq T_{chem}), \\
\varepsilon + p &\approx \sum_i m_i n_i & (T_{chem} > T \geq T_f).
\end{aligned}
$$

See the talk of G. Kasza for details

HBT: Signals of 3d hydro flow



$$\frac{1}{\Delta\overline{\eta}^2} = \frac{1}{\Delta\eta^2} + \frac{M_t}{T_0},$$

$$\overline{R}_\perp^2 = \frac{R_G^2}{1 + \frac{M_t}{T_0}\left(\langle u_t\rangle^2 + \langle\frac{\Delta T}{T}\rangle_r\right)},$$

$$R_l^2 = \overline{\tau}^2\Delta\overline{\eta}^2,$$
$$R_o^2 = \overline{R}_\perp^2 + \beta_t^2\Delta\overline{\tau}^2,$$
$$R_s^2 = \overline{R}_\perp^2$$

$$R_s^2 = R_{\cdot\cdot}^2,$$
$$R_o^2 = R_{:}^2 + \beta_t^2[\cosh^2(\overline{\eta})R_=^2 + \sinh^2(\overline{\eta})R_\parallel^2],$$
$$R_{ol}^2 = -\beta_t\sinh(\overline{\eta})\cosh(\overline{\eta})(R_=^2 + R_\parallel^2),$$
$$R_l^2 = \cosh^2(\overline{\eta})R_\parallel^2 + \sinh^2(\overline{\eta})R_=^2,$$

Indication of hydro scaling behaviour of R(side,out,long) at low m_T

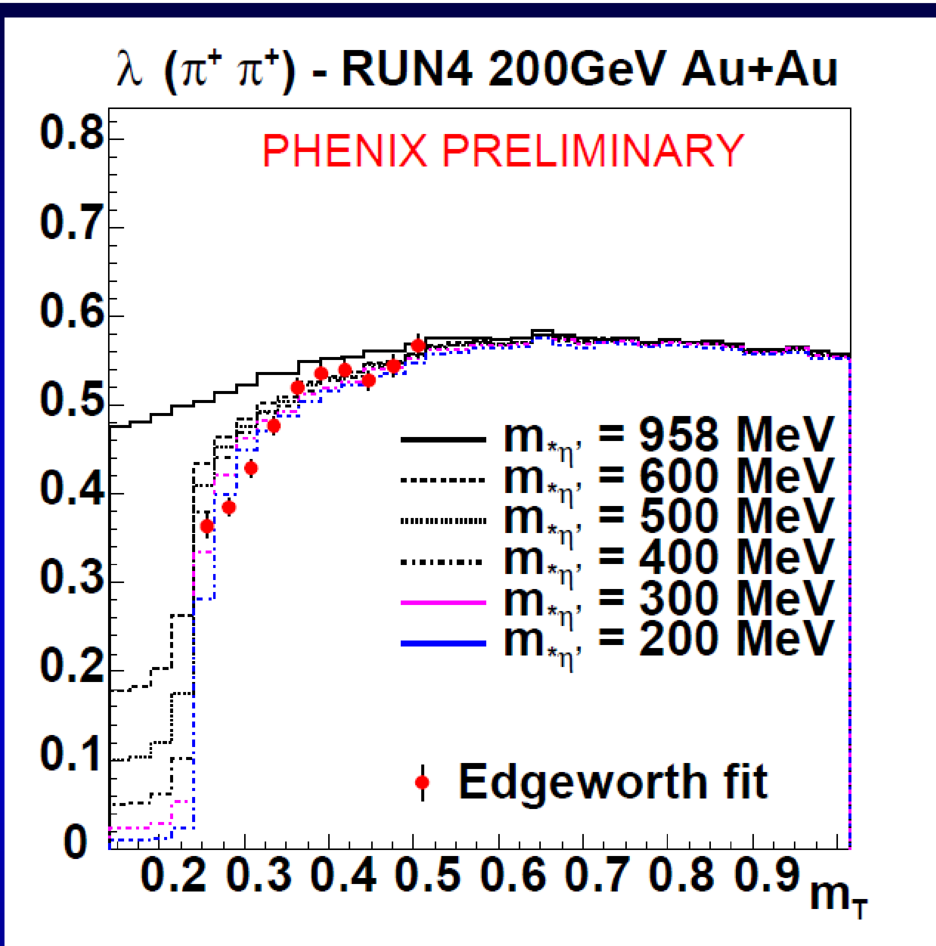
$$\frac{1}{R^2} = \frac{1}{R_0^2} + \frac{m_T}{T_0} \cdot H_0^2$$

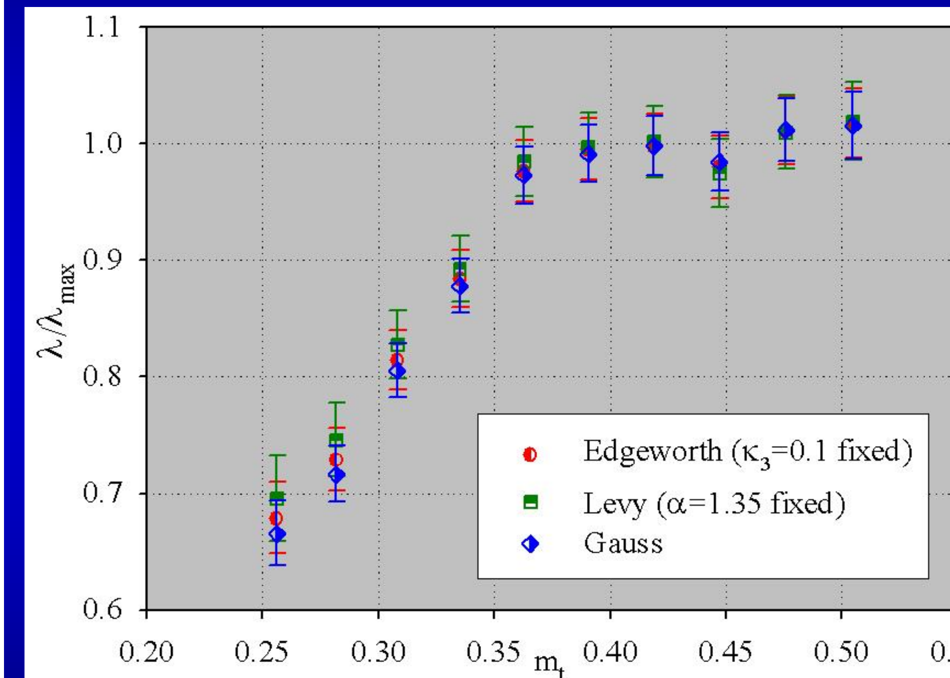
R_{long} m_t-scaling: Yu. Sinyukov and A. Makhlin: Z.Phys. C39 (1988) 69

R_{side} , R_{out} , R_{long} m_t-scaling: T. Cs, B. Lörstad, hep-ph/9509213 (shells of fire vs fireballs)

S. Chapman, P. Scotto, U. W. Heinz, hep-ph/9408207

HBT: Interpretation of λ, α and R

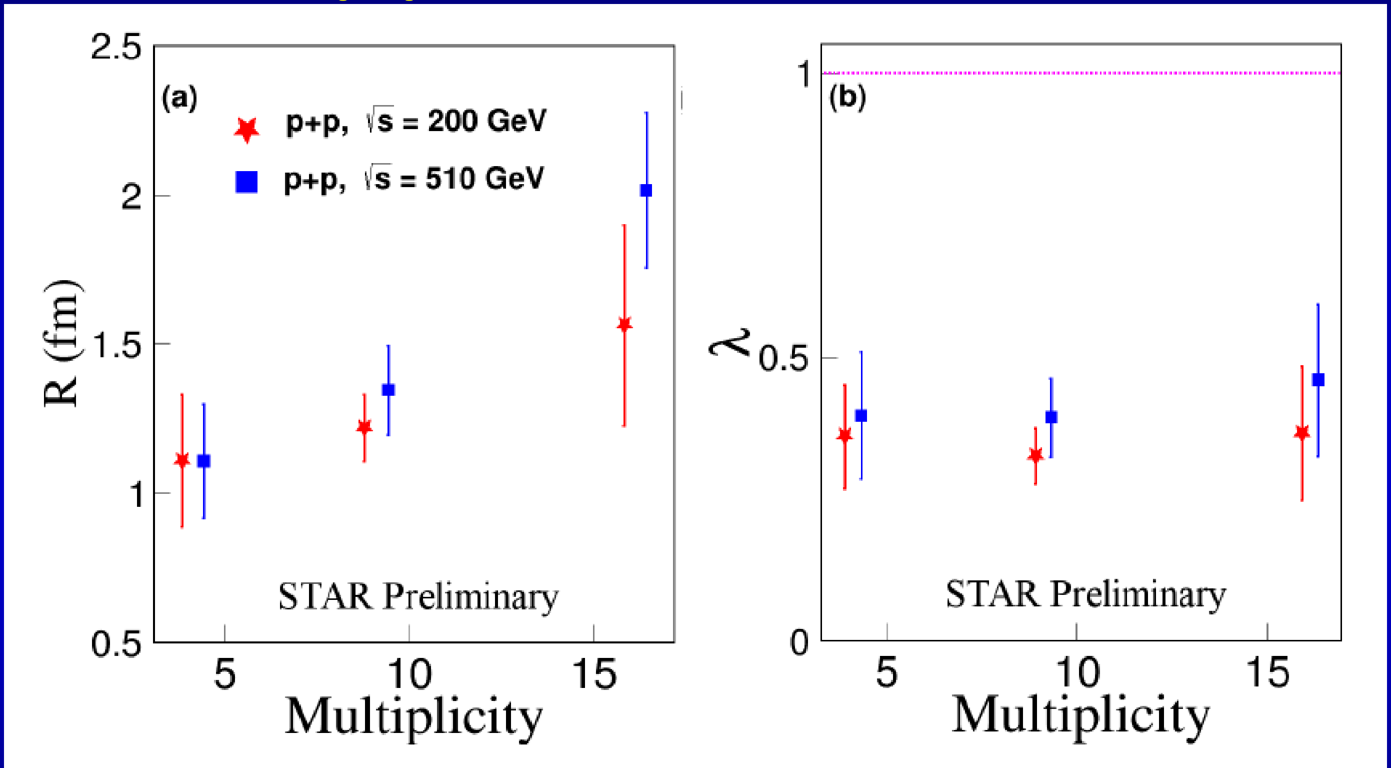




PHENIX preliminary data from
arXiv:nucl-ex/0509042

Cross-check: partial coherence for pions only - or not ?

STAR p+p → $K^{\pm} K^{\pm}$ + X $\sqrt{s}$ = 200, 510 GeV



Fewer long lived resonances expected to decay to K (but ϕ)

Partial coherence not expected either: if λ_2 (Kaons) < 2 (f_c, p_c) ≠ (1,0) ?

STAR preliminary result: λ_2(Kaons) < 2 in p+p . Halo from ϕ ? Cross-checks, implications ?

For details, see G. Nigmatkulov for STAR, Proc. SQM 15

For heavy ions: G. Nigmatkulov's talk at HDNM 2017 and WPCF 2017