# Asymptotic analysis of the parton branching equation at LHC energies

Wei Yang Wang
HEP Group
National University of Singapore (NUS)
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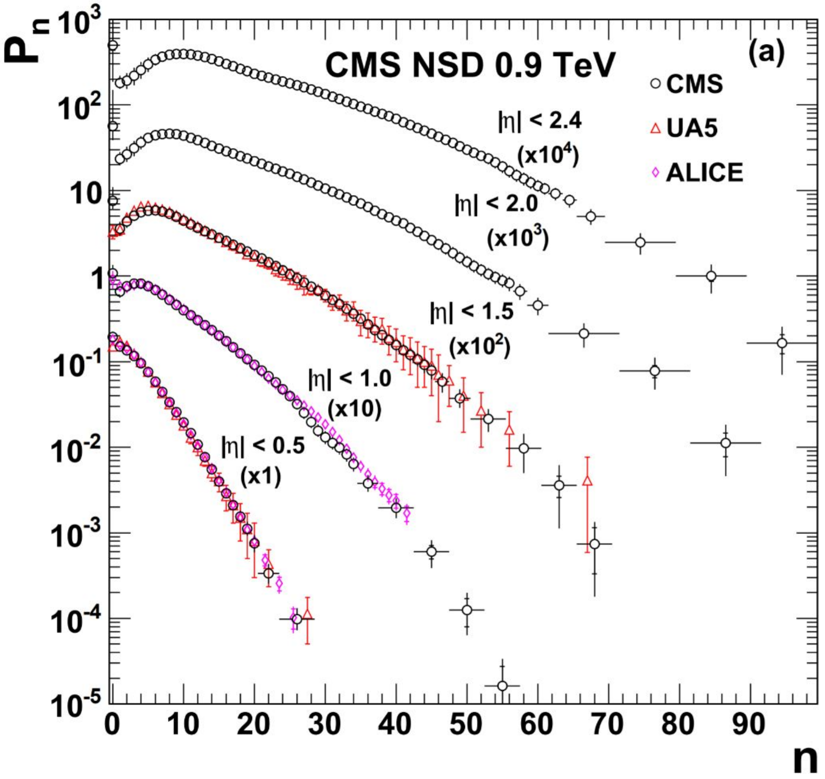
# Multiplicity Distributions

**Parton branching equation** describes the cascade of quarks and gluons through basic branching processes, eventually forming charged hadrons

Basic global observable ($P(n)$ probability of observing $n$ charged hadrons in an event)

Charged particle multiplicity distributions at the LHC:

- ALICE, ATLAS, CMS
- wide $|\eta|$ at various c.m. energies

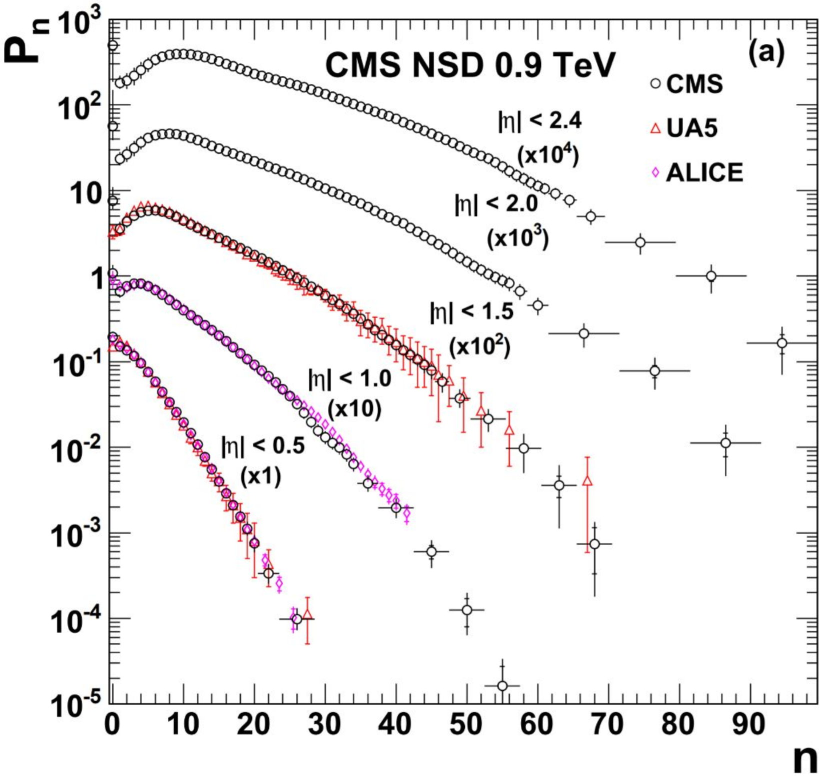



[CMS Collaboration, JHEP01 (2011) 079]

# Multiplicity Distributions

Reveals underlying correlations in particle production

- Independent emission of single particles ⟶ Poisson
- Thermal source ⟶ broader distribution e.g. Negative Binomial Distribution (NBD)
- Correlations mainly short range in rapidity (cluster decays…)
- Some long range correlations that increases with c.m. energy (much weaker in $e^+e^-$ collisions)

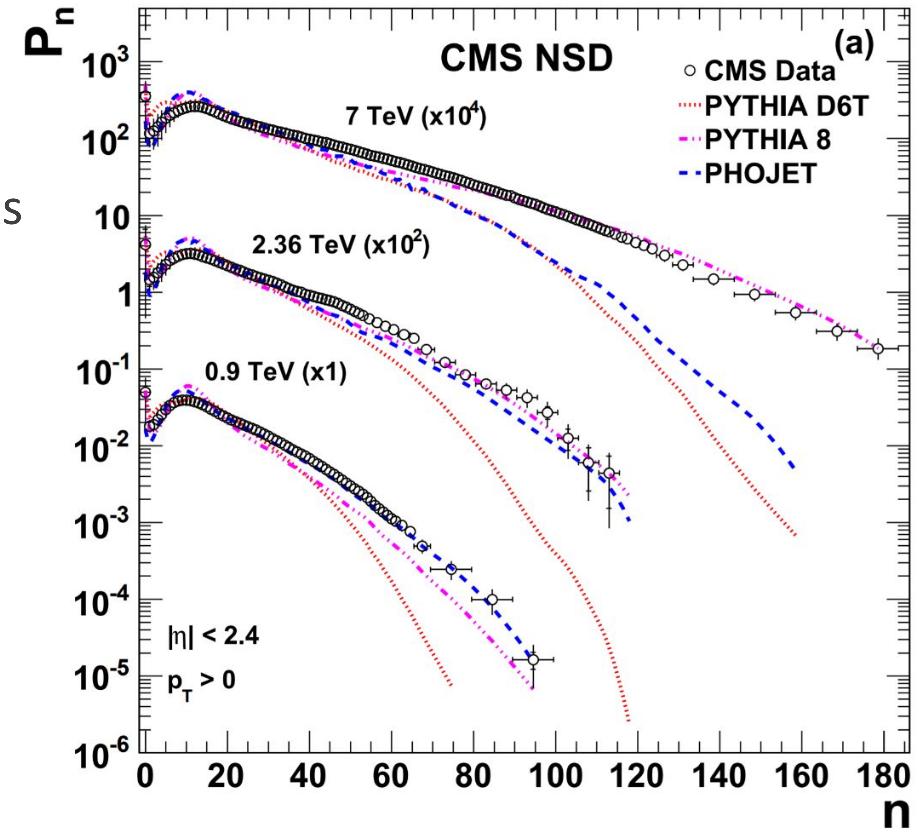



[CMS Collaboration, JHEP01 (2011) 079]

# Multiplicity Distributions

Popular models:

- Monte Carlo (QCD + QCD inspired phenomenological models e.g. MPI)
- Large multiplicity tails difficult to describe (especially without MPI)

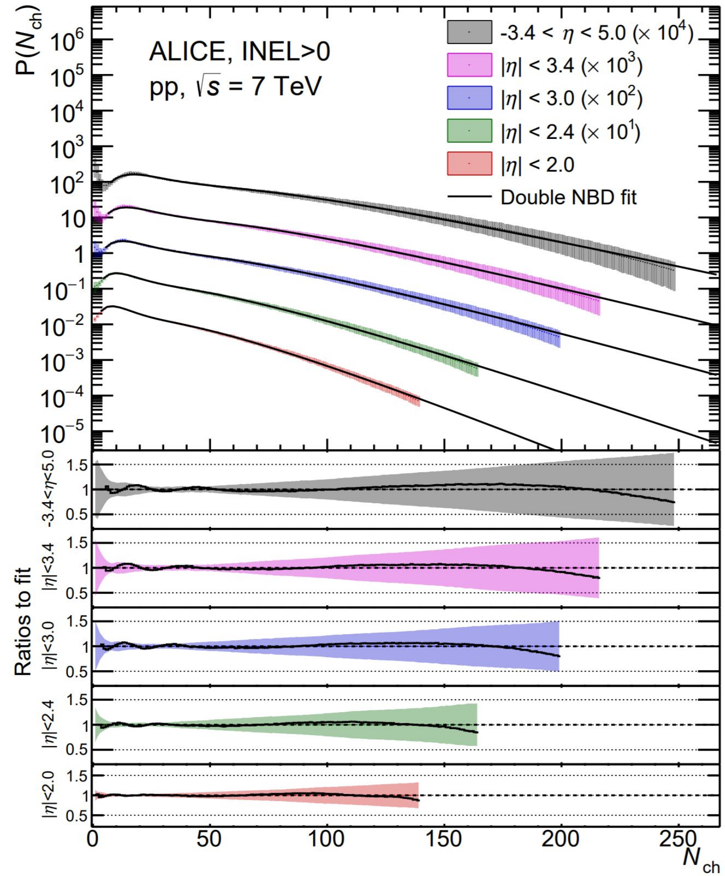



[CMS Collaboration, JHEP01 (2011) 079]

# Multiplicity Distributions

Popular models:

- Statistical models (e.g. NBD, double NBD)
- Single NBD does not reproduce the rise $n<10$ and "shoulder" $n{\sim}20$
  - Low n dominated by diffractive events
  - IP Glasma model (CGC) ⟶ NBD ($k \propto Q_s S_\perp$)
- Double NBD fits better but meaning not clear

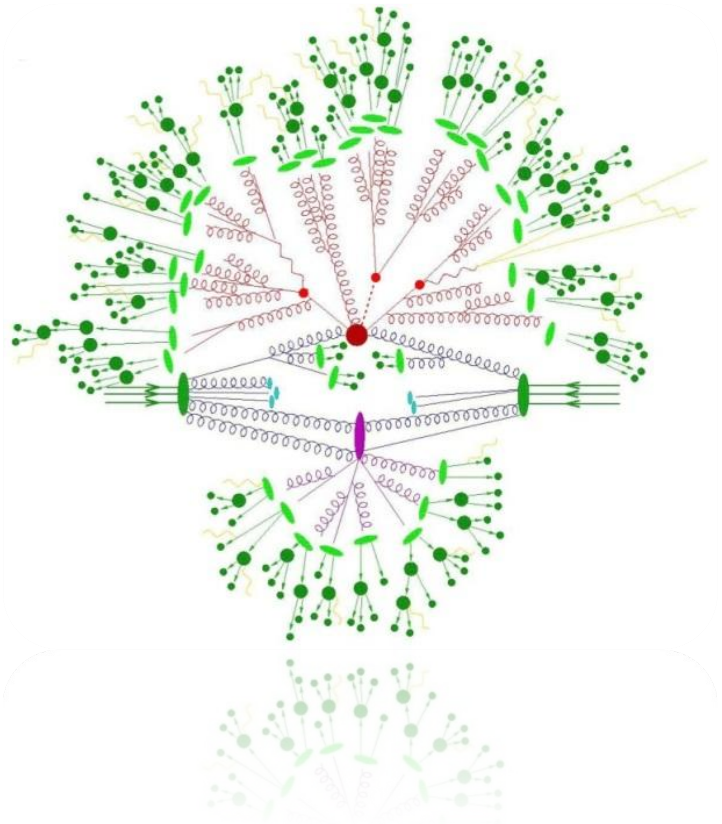



[ALICE Collaboration, arXiv:1708.01435, 2017]

# QCD-based parton branching models

Parton Branching Models
(Giovannini, 1979)

- Describes the cascade of quarks and gluons eventually forming charged hadrons
- NBD arises from the Markov branching process of quark bremsstrahlung
- Generalised Multiplicities Distribution (GMD)
- Asymptotic Multiplicity Distribution (AMD)



# QCD branching processes

$\tilde{A}$: Quark bremsstrahlung ($q \rightarrow q + g$)

$A$ : gluon fission ($g \rightarrow g + g$)

$B$ : quark pair production ($g \rightarrow q + \overline{q}$)

Giovannini, 1979

$$
\begin{aligned}
&P_{m,n}(t + \Delta t) \\
&= \left(1 - An\Delta t - \tilde{A}m\Delta t - Bn\Delta t\right)P_{m,n}(t) + A(n-1)\Delta t P_{m,n-1}(t) + \tilde{A}m\Delta t P_{m,m-1}(t) \\
&+ B(n+1)\Delta t P_{m-2,n+1}(t)
\end{aligned}
$$

Closed form solution difficult to obtain.
Assume quark pair production negligible ($B$ = $0$)

# Generalised Multiplicity Distribution (GMD)

$$P_{GMD}(n) = \frac{\Gamma(n+k)}{\Gamma(n-k'+1)\Gamma(k'+k)}\left(e^{-A}\right)^{k'+k}\left(1-e^{-A}\right)^{n-k'}$$

$n$: multiplicity
$k$: number of initial quarks $\times\ \tilde{A}/A$ ($k$=$m\tilde{A}/A$)
$k'$: number of initial gluons
$\mathrm{e}^{-At}$, where $t$: QCD evolution parameter

Mean:

$$\bar{n} = \frac{k'+k}{e^{-At}} - k$$

$k' \rightarrow 0 \Rightarrow$GMD $\rightarrow$ NBD

[chan, chew, Z. Phys. C 55 (1992) 503]
[Wang, Leong, Ng, Dewanto, Chan, Oh, Proceedings of the Conference in Honour of the 90th Birthday of Freeman Dyson (2014), 400]

# QCD branching processes

$\tilde{A}$: Quark bremsstrahlung $(q \rightarrow q + g)$

$A$ : gluon fission $(g \rightarrow g + g)$

$B$ : quark pair production $(g \rightarrow q + \bar{q})$

Giovannini, 1979

$C$ : 4-gluon vertex $(\mathrm{g} \rightarrow g + g + g)$ $\longrightarrow$ Expect to be more important at high energies like at LHC

$$P_{m,n}(t + \Delta t) \\ = \left(1 - An\Delta t - \tilde{A}m\Delta t - Bn\Delta t - Cn\Delta t\right)P_{m,n}(t) + A(n-1)\Delta t P_{m,n-1}(t) \\ + \tilde{A}m\Delta t P_{m,m-1}(t) + B(n+1)\Delta t P_{m-2,n+1}(t) + C(n-2)\Delta t P_{m,n-2}(t)$$

Difficult to get closed form solution.
Asymptotic solution possible.

# Asymptotic Multiplicity Distribution

Solution obtained by performing a Laplace transform ($n \sim x$)

$$
\begin{aligned}
sP^*(x,s) = &- AxP^*(x,s) + A(x-1)P^*(x-1,s) \\
&- A^\dagger P^*(x,s) + A^\dagger P^*(x-1,s) \\
&- BxP^*(x,s) + B(x+1)P^*(x+1,s) \\
&- CxP^*(x,s) + C(x-2)P^*(x-2,s)
\end{aligned}
$$

Defining

$$lnP^*(x,s) = L(x,s) = L(x)$$

Taylor expansion of $L(x+\delta x)$ about $x$

$$L_2(x,s) = -\int_{k'}^{x} ln\left(\frac{s+\alpha+\beta\omega}{\gamma+\beta\omega}\right) d\omega + ln\left[\frac{(\gamma+\beta x)^{\frac{1}{2}}(s+\alpha+\beta k')^{\frac{1}{2}}}{(\gamma+\beta k')^{\frac{1}{2}}(s+\alpha+\beta x)^{\frac{1}{2}}(s+A^\dagger+Dk')}\right]$$

# Asymptotic Multiplicity Distribution

Inverse Laplace transformation + saddle point approximation

$$P_2(x,t) = \frac{1}{2\pi i}\int_{C-i\infty}^{C+i\infty} g(s)e^{st-f(s)}ds$$

$$P_2(x,t) = \frac{g(s_0)e^{s_0 t-f(s_0)}}{\sqrt{-2\pi f''(s_0)}} \qquad \frac{\partial}{\partial s_0}[e^{s_0 t-\int_{k'}^{x} ln(\frac{s_0+\alpha+\beta\omega}{\gamma+\beta k'})d\omega}] = 0$$

$$g(s) = \frac{(\gamma+\beta x)^{\frac{1}{2}}(s+\alpha+\beta k')^{\frac{1}{2}}}{(\gamma+\beta k')^{\frac{1}{2}}(s+\alpha+\beta x)^{\frac{1}{2}}(s+A^{\dagger}+Dk')}$$

$$f(s) = \int_{k'}^{x} ln\left(\frac{s+\alpha+\beta\omega}{\gamma+\beta\omega}\right)d\omega$$

# Asymptotic Multiplicity Distribution

Solution

$$P_2(x,t) = \frac{e^{-t(\alpha+\beta k'+\beta)}(1-e^{-\beta t})^{(x-k')}}{\frac{B}{\beta}(1+k')(1-e^{-\beta t})+(x-k')e^{-\beta t}} \times (x-k')$$
$$\times \frac{(k+x-1)^{(k+x-\frac{1}{2})}}{(2\pi)^{(\frac{1}{2})}(x-k')^{(x-k'+\frac{1}{2})}(k+k'-1)^{(k+k'-\frac{1}{2})}}.$$

Stirling approximation $x! = \frac{x^{(x+\frac{1}{2})}}{e^x}(2\pi)^{\frac{1}{2}}$

+ replacing with original physical parameters

$$\boxed{P_{AMD}(n) = (n-k')\frac{\Gamma\left(n+\frac{A^\dagger - C}{A+C}\right)}{\Gamma(n-k'+1)\Gamma\left(k'+\frac{A^\dagger - C}{A+C}\right)}\,\frac{e^{-\left(A^\dagger + A - B + C\right)}e^{-(A+C)k'}\left(1-e^{-(A+C)}\right)^{n-k'}}{\frac{B}{A+C}(1+k')\left(1-e^{-(A+C)}\right)+(n-k')e^{-(A+C)}}}$$

# Asymptotic Multiplicity Distribution

$$P_{AMD}(n) = (n-k')\frac{\Gamma\left(n+\frac{A^{\dagger}-C}{A+C}\right)}{\Gamma(n-k'+1)\Gamma\left(k'+\frac{A^{\dagger}-C}{A+C}\right)}\frac{e^{-\left(A^{\dagger}+A-B+C\right)}e^{-(A+C)k'}\left(1-e^{-(A+C)}\right)^{n-k'}}{\frac{B}{A+C}(1+k')\left(1-e^{-(A+C)}\right)+(n-k')e^{-(A+C)}}$$

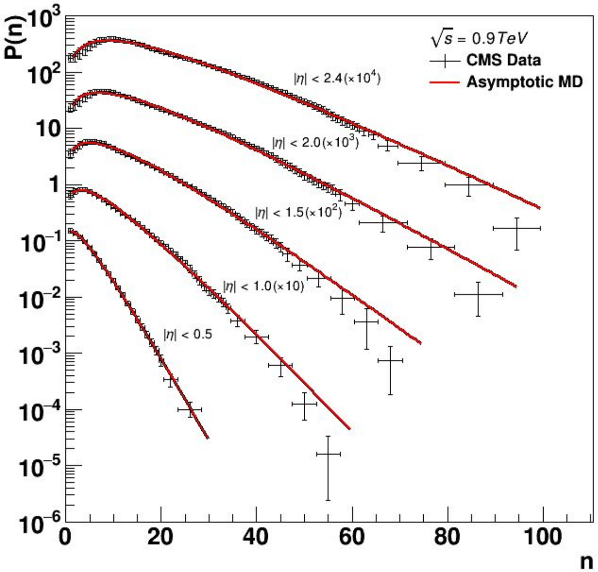
Reduces to GMD when $B$=$C$=$0$ (i.e. negligible quark pair production and 4-gluon vertex)

Further reduces to NBD when $k'$=$0$ (i.e. no initial gluons)

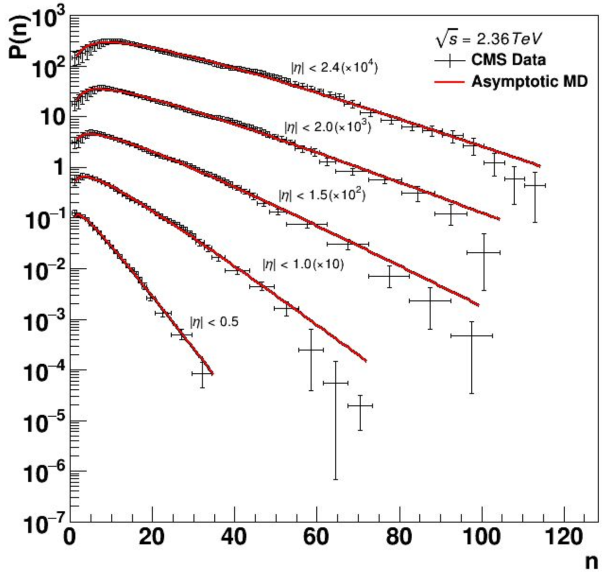
# LHC data

CMS charged particle multiplicity distribution

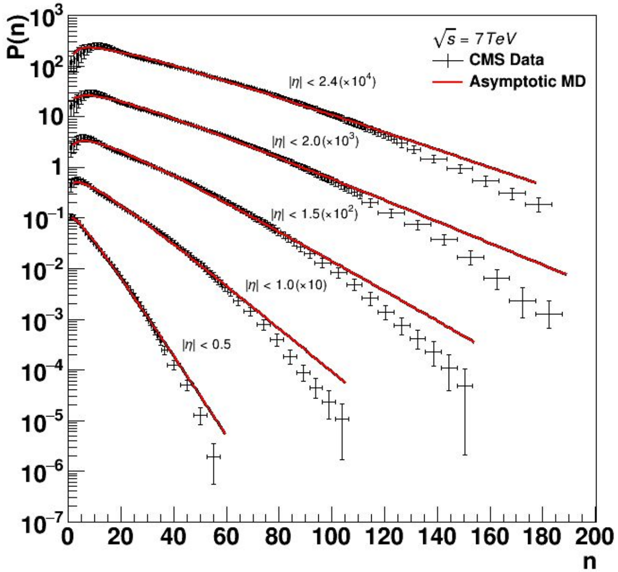
0.9 TeV




2.36 TeV

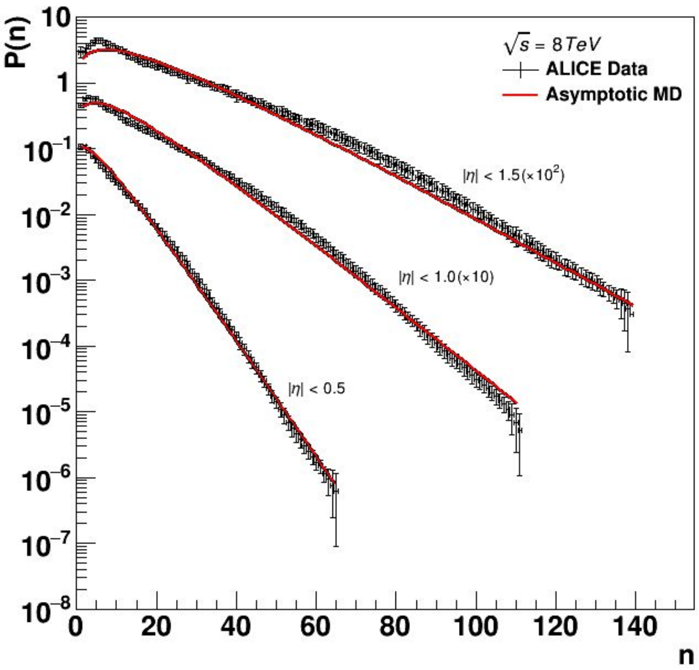



7 TeV

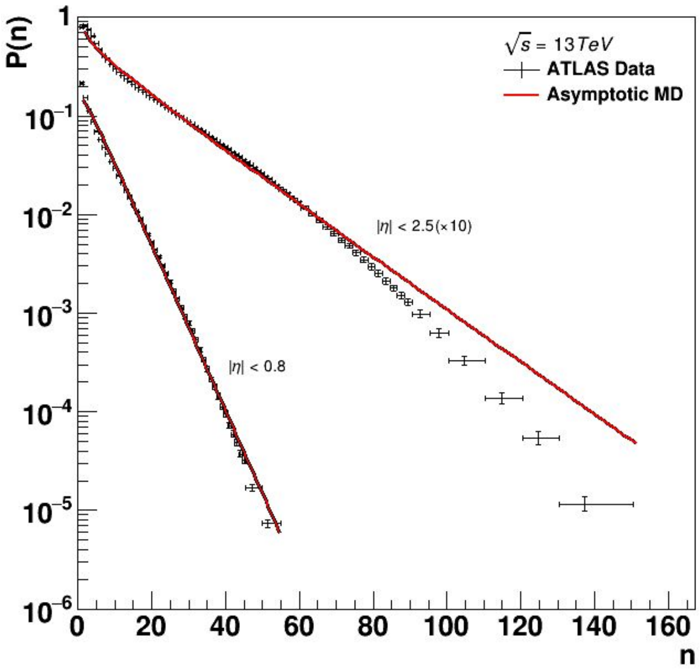


# LHC data

ALICE ($8\ \mathrm{TeV}$)




ATLAS ($13\ \mathrm{TeV}$)



# Summary

An asymptotic solution (AMD) to the parton branching equation including 4 branching processes can be derived using the Laplace transformation and saddle point approximation

- AMD is reducible to the GMD and NBD in the limits of parameter values
- Applied to 0.9, 2.36, 7, 8, 13 TeV data from ALICE, ATLAS, CMS

# Thank you for your attention!