Transverse Momentum Dependent Functions: Challenges and Future Prospects

ISMD2017

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&
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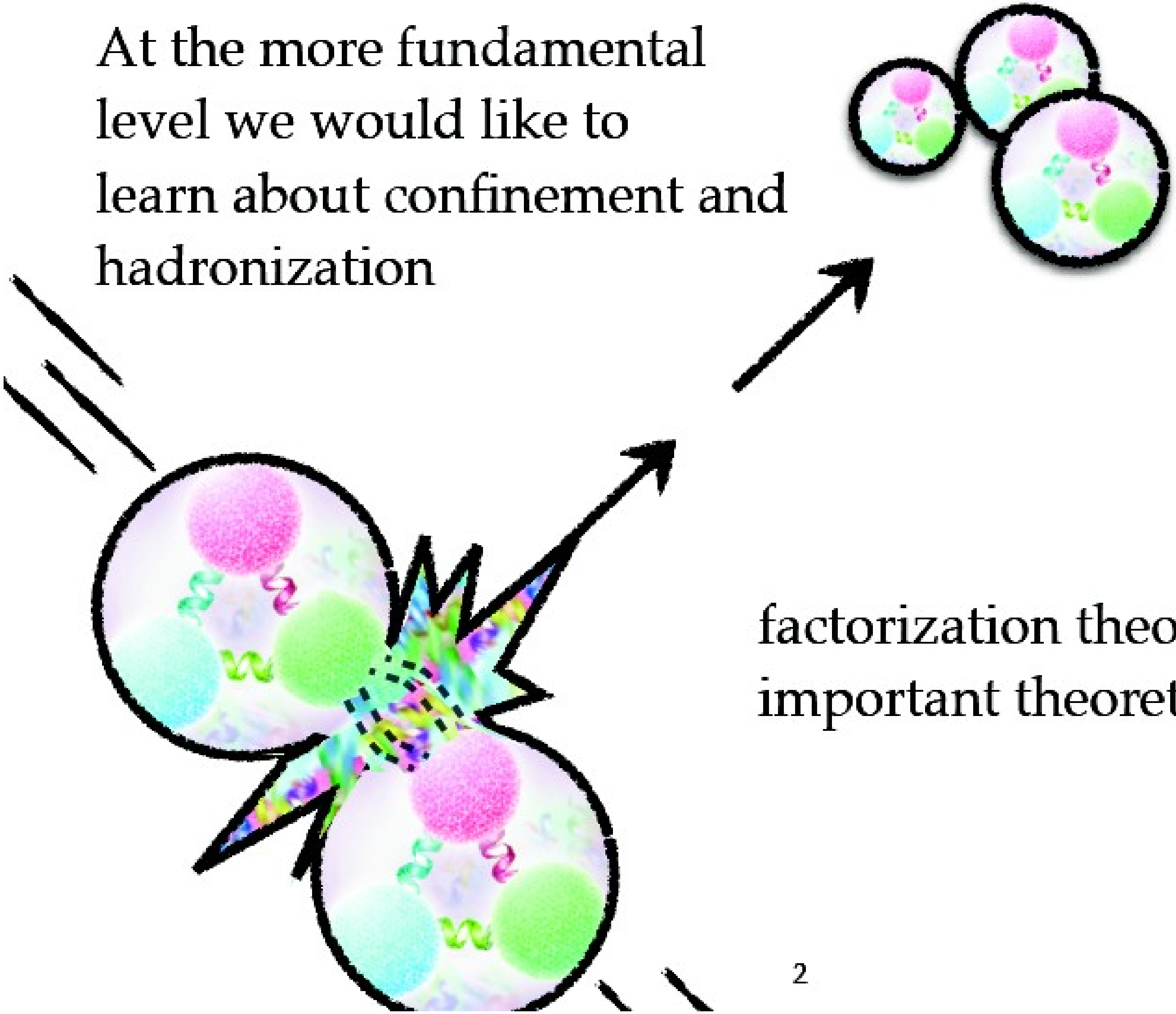
Motivation

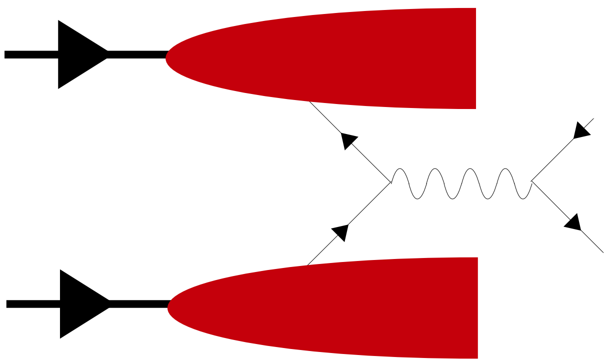
At the more fundamental level we would like to learn about confinement and hadronization

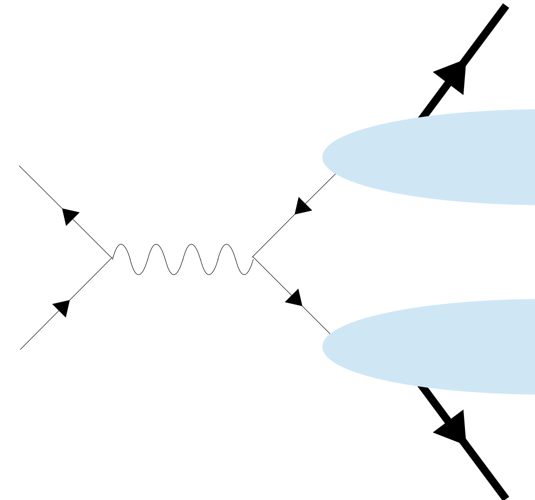


factorization theorems, important theoretical tool

2

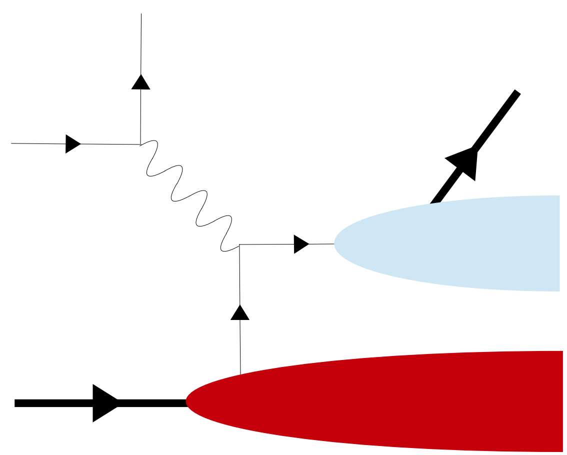
Drell Yan





e+e-

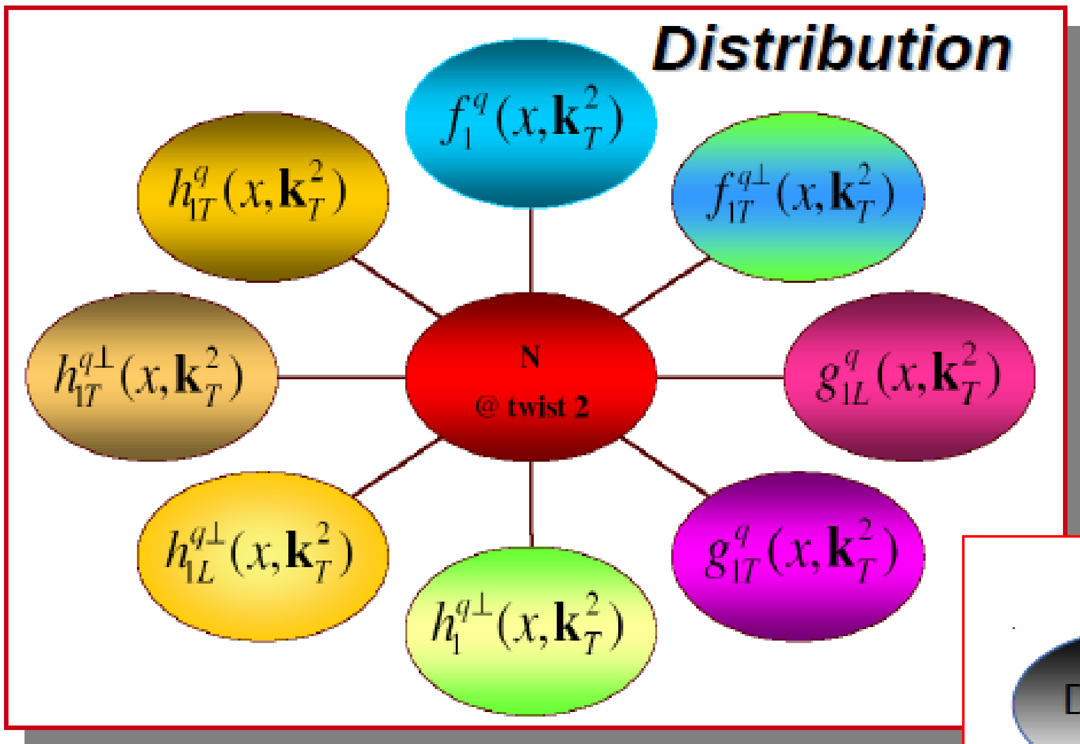
PDFs



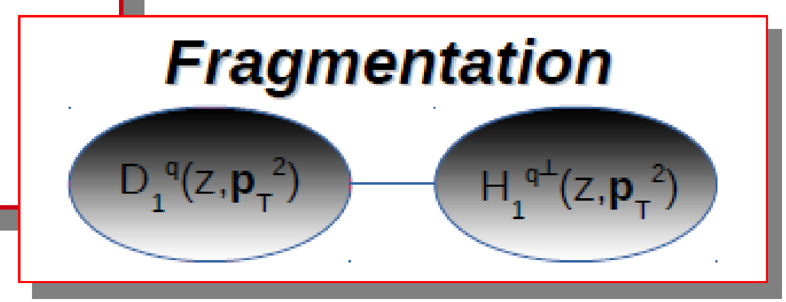
Fragmentation Functions

SIDIS

Beyond the Collinear Picture



Correlations between spin and transverse momentum



Source of Errors?

Example:Unpolarized SIDIS cross section (current region)

$$\frac{d\sigma^{\ell+p\to\ell' hX}}{dx_B\, dQ^2\, dz_h\, dP_T^2} = \frac{2\,\pi^2\alpha^2}{(x_B\, s)^2}\,\frac{\left[1+(1-y)^2\right]}{y^2}\,F_{UU}$$

$$F_{UU} = \sum_q \mathcal{H}_q\ \text{F.T.}\left\{\boxed{\tilde{D}_{h/q}(z, z\,\boldsymbol{b}_\perp; Q)\ \tilde{f}_{q/P}(x, \boldsymbol{b}_\perp; Q)}\right\}$$

$$\boxed{+\ \text{large}\ q_T\ \text{corrections}} + \boxed{\text{power suppressed terms}}$$

Perturbation Theory **Factorization**

Global Fits?

A. Bacchetta, F. Delcarro, C. Pisano,
M. Radici , A. Signori
arXiv:1703.10157

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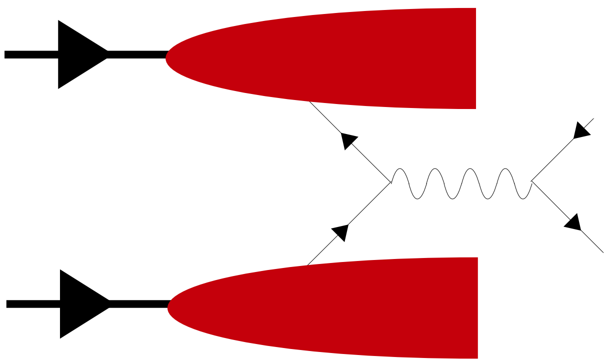
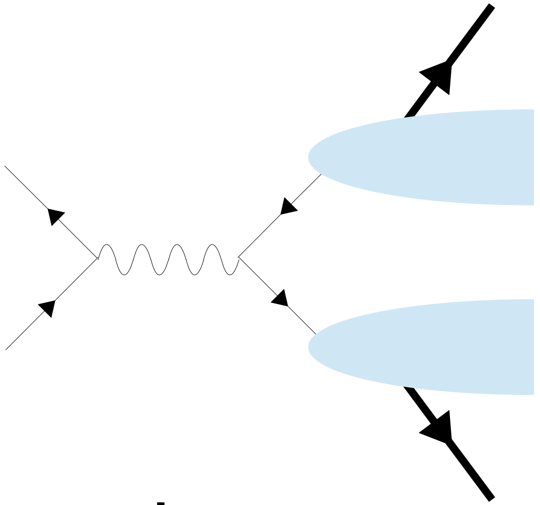
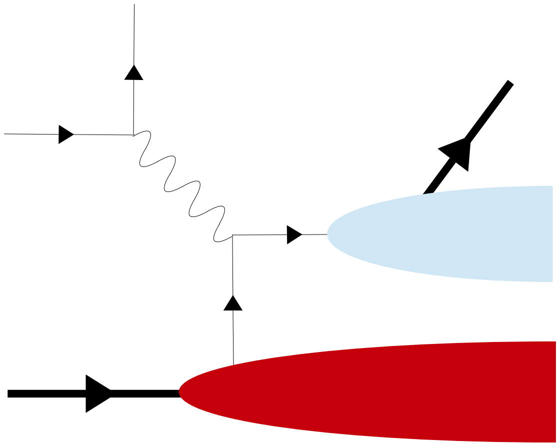




e+e-



PDFs

Fragmentation Functions

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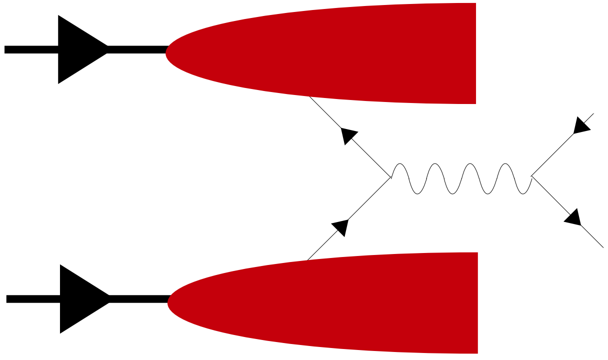
Drell Yan



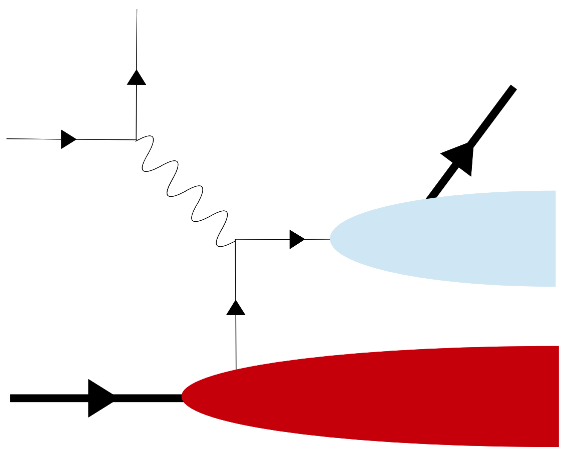
Under control, high precision phenomenology:

See for example:
arXiv:1706.01473
Ignazio Scimemi, Alexey Vladimirov

Must still address some issues.

Delicate kinematics of available multidimensional data

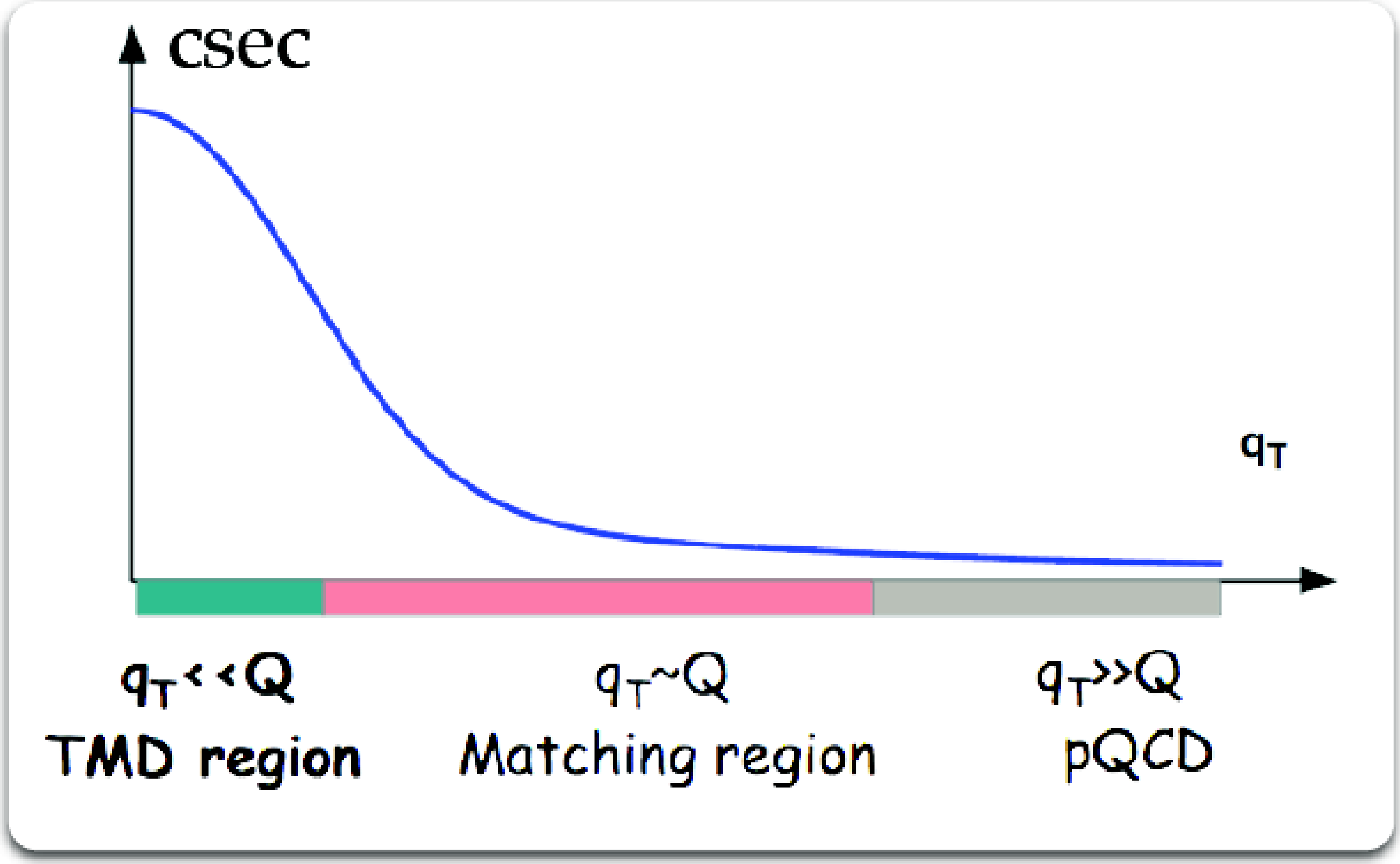
The matching between low and large transverse momentum



SIDIS

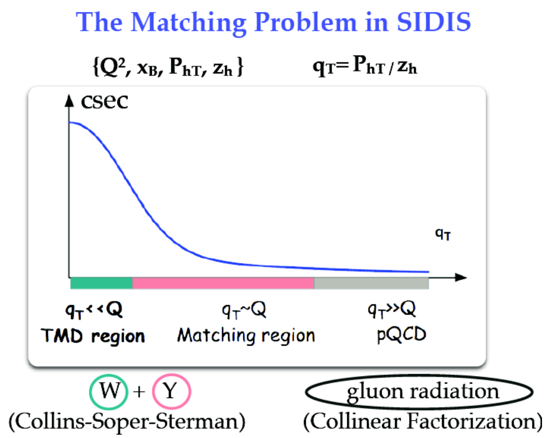
The Matching Problem in SIDIS

$\{Q^2, x_B, P_{hT}, z_h\}$ $q_T = P_{hT} / z_h$



W + Y
(Collins-Soper-Sterman)

gluon radiation
(Collinear Factorization)



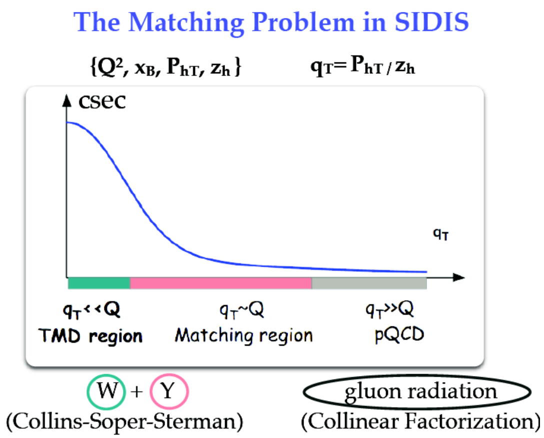
Works for SIDIS at high enough, $Q^2 > 10$ GeV^2, energy flow (**integration over z_h**)

Nadolsky, Stump, Yuan
DOI: 10.1103/PhysRevD.64.059903

However, information about z-dependence gets washed out. Also, integration over z mixes TMD and collinear factorization effects.

TMD Fragmentation Function definition

$$\tilde{D}_{h/q}(z, \boldsymbol{b}_\perp; Q) = \sum_j \left[\left(\tilde{C}_{j/q} \otimes \frac{d_{h/j}}{z^2} \right) e^{\Gamma_D(Q)} \right] \exp\left\{ g_{j/P}(x, b_\perp) + g_K(b_\perp) \log\left(\frac{Q}{Q_0} \right) \right\}$$



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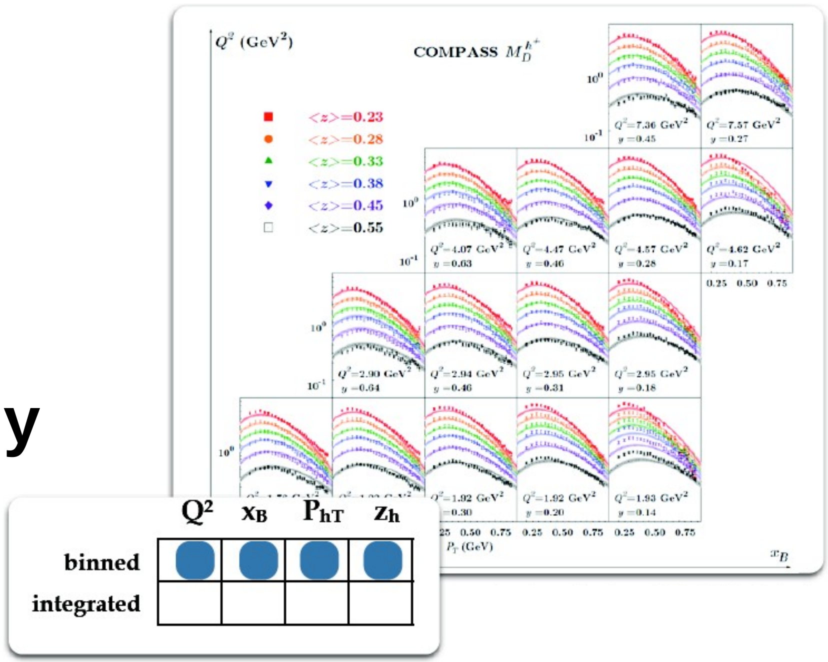
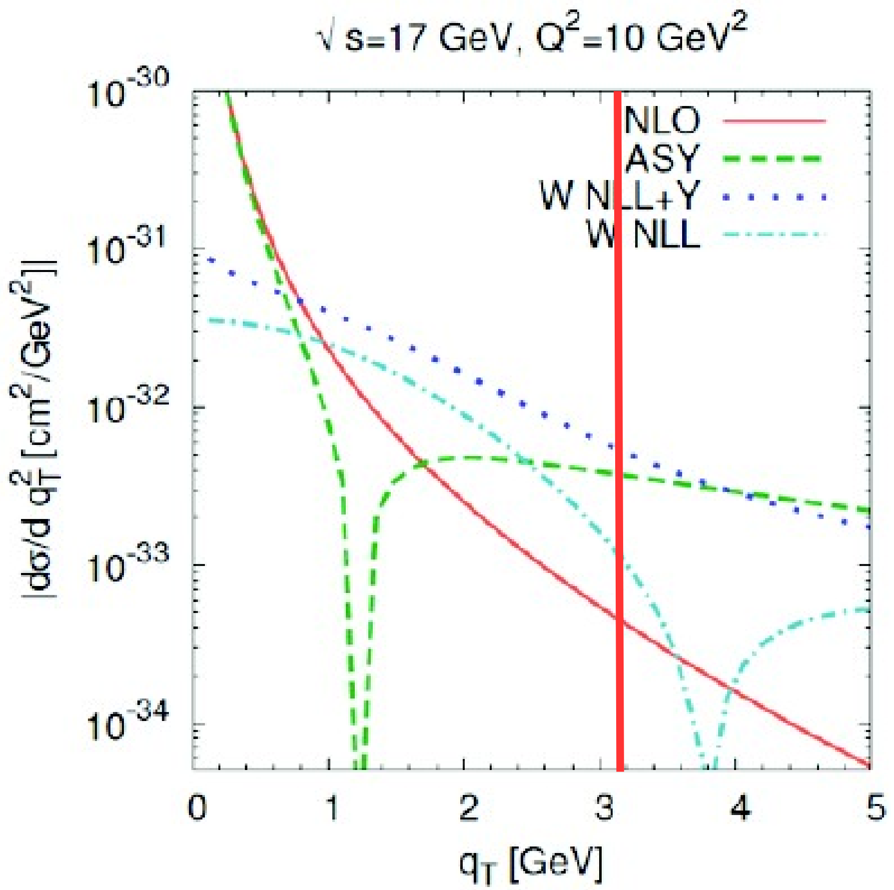
Multidimensional data are ideal.

Can CSS be successfully Implemented?



M. Anselmino, M. Boglione, J.O.G.H., S. Melis , A. Prokudin: Published in JHEP 1404 (2014) 005

Large qT corrections are hard to implement.



- Large Y-term at small q_T
- Small cross section at large q_T
- No smooth matching
- Delicate kinematics

Delicate kinematics

Source of Errors?

Unpolarized SIDIS cross section (current region)

$$\frac{d\sigma^{\ell+p\to\ell' hX}}{dx_B\, dQ^2\, dz_h\, dP_T^2} = \frac{2\,\pi^2\alpha^2}{(x_B\, s)^2}\,\frac{\left[1+(1-y)^2\right]}{y^2}\, F_{UU}$$

$$F_{UU} = \sum_q \mathcal{H}_q \text{ F.T.}\left\{\boxed{\tilde{D}_{h/q}(z, z\,\boldsymbol{b}_\perp; Q)\ \tilde{f}_{q/P}(x, \boldsymbol{b}_\perp; Q)}\right\}$$

$$\boxed{+\ \text{large } q_T \text{ corrections}} + \boxed{\text{power suppressed terms}}$$

Perturbation Theory **Factorization**

(Re)Calculation of large qT SIDIS cross section

Work in progress:

J.O.G.H., T. Rogers, N. Sato, A. Signori, B. Wang

$$F_{UU} = \sum_q \mathcal{H}_q \text{ F.T.} \left\{ \boxed{\tilde{D}_{h/q}(z, z\,\boldsymbol{b}_\perp; Q)\ \tilde{f}_{q/P}(x, \boldsymbol{b}_\perp; Q)} \right\}$$

$$\boxed{+ \text{ large } q_T \text{ corrections}} + \text{power suppressed terms}$$

Perturbation Theory

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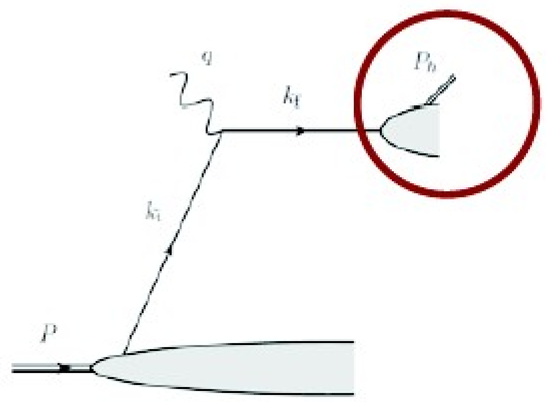
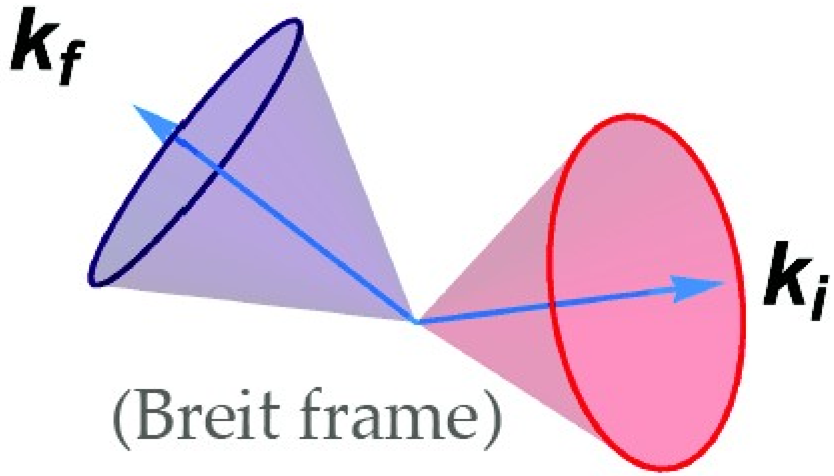
$$+ \text{ large } q_T \text{ corrections} + \boxed{\text{power suppressed terms}}$$

Factorization

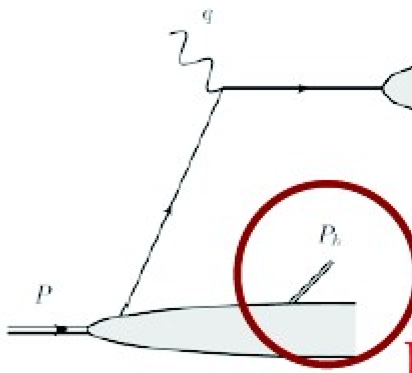
Which Region?



TMDs

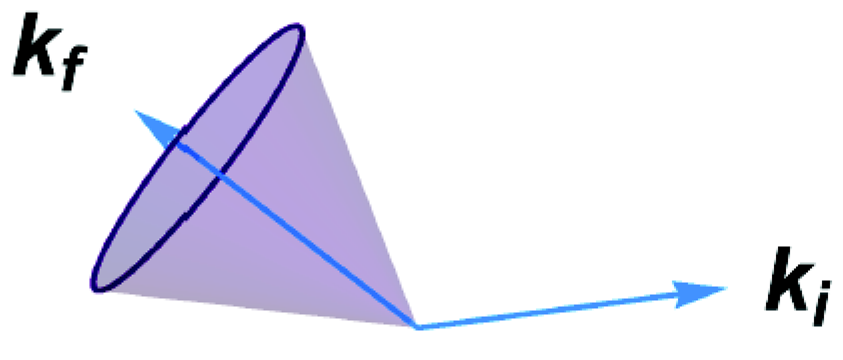


factorization theorems for different leading regions



Fracture Functions

Power counting and kinematics of the current region



small masses

$$P_h \cdot k_{\mathrm{f}} = O\left(\boxed{m^2}\right)$$

$$P_h \cdot k_{\mathrm{i}} = O\left(\boxed{Q^2}\right)$$

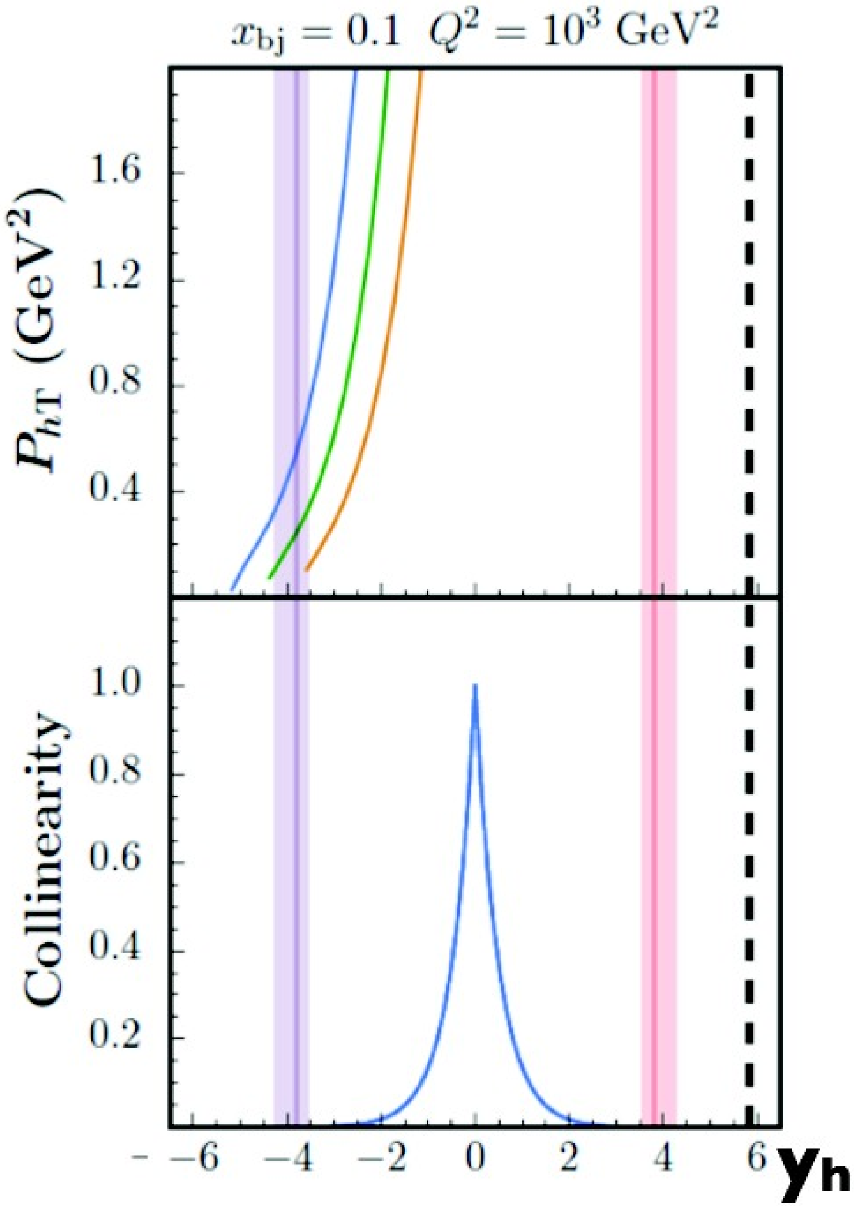
hard scale

require small values for

$$\boxed{R \equiv \frac{P_h \cdot k_{\mathrm{f}}}{P_h \cdot k_{\mathrm{i}}}}$$

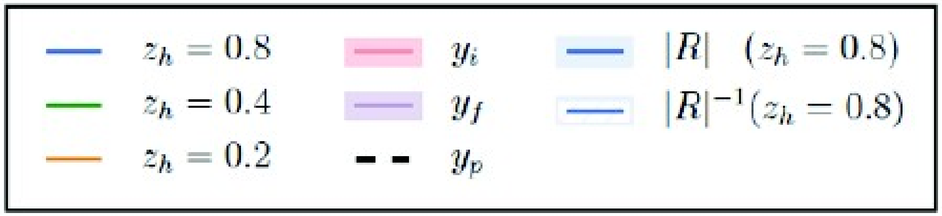
notice quark momenta have to be estimated

M. Boglione, J. Collins, L. Gamberg, JOGH, T. C. Rogers,
and N. Sato,Phys. Lett.B766, 245 (2017),1611.10329.



$$y_h \equiv \frac{1}{2} \log \frac{P_h^+}{P_h^-}$$

$$\boxed{R \equiv \frac{P_h \cdot k_\mathrm{f}}{P_h \cdot k_\mathrm{i}}}$$



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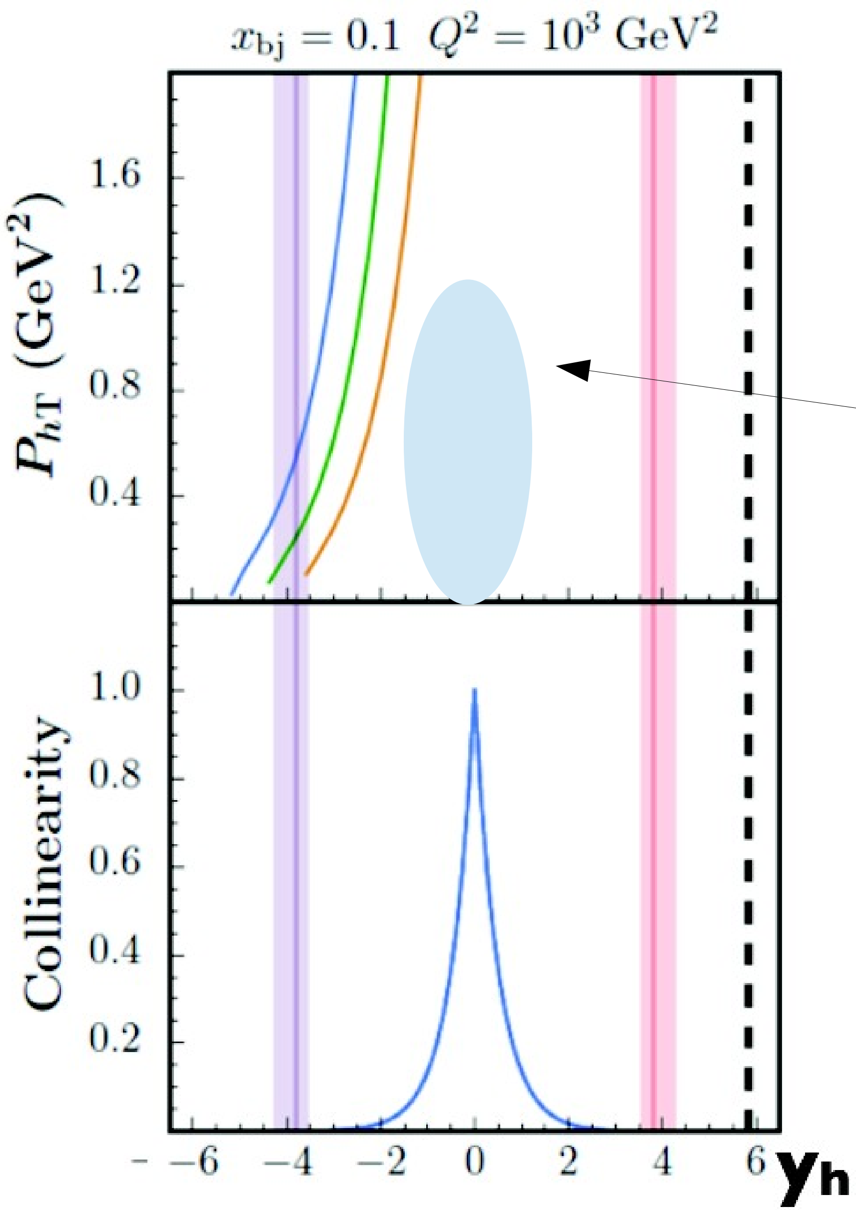
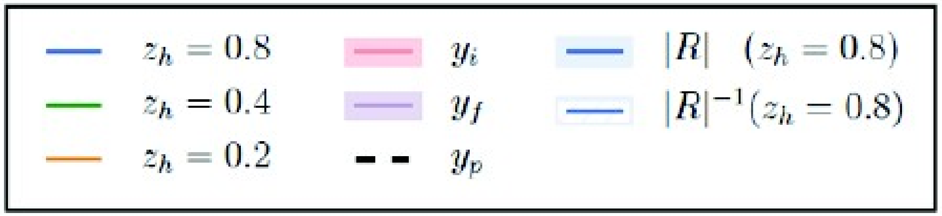


EIC projected energies.

Avoid region of central rapidity, soft non-TMD effects

$$\boxed{R \equiv \frac{P_h \cdot k_f}{P_h \cdot k_i}}$$



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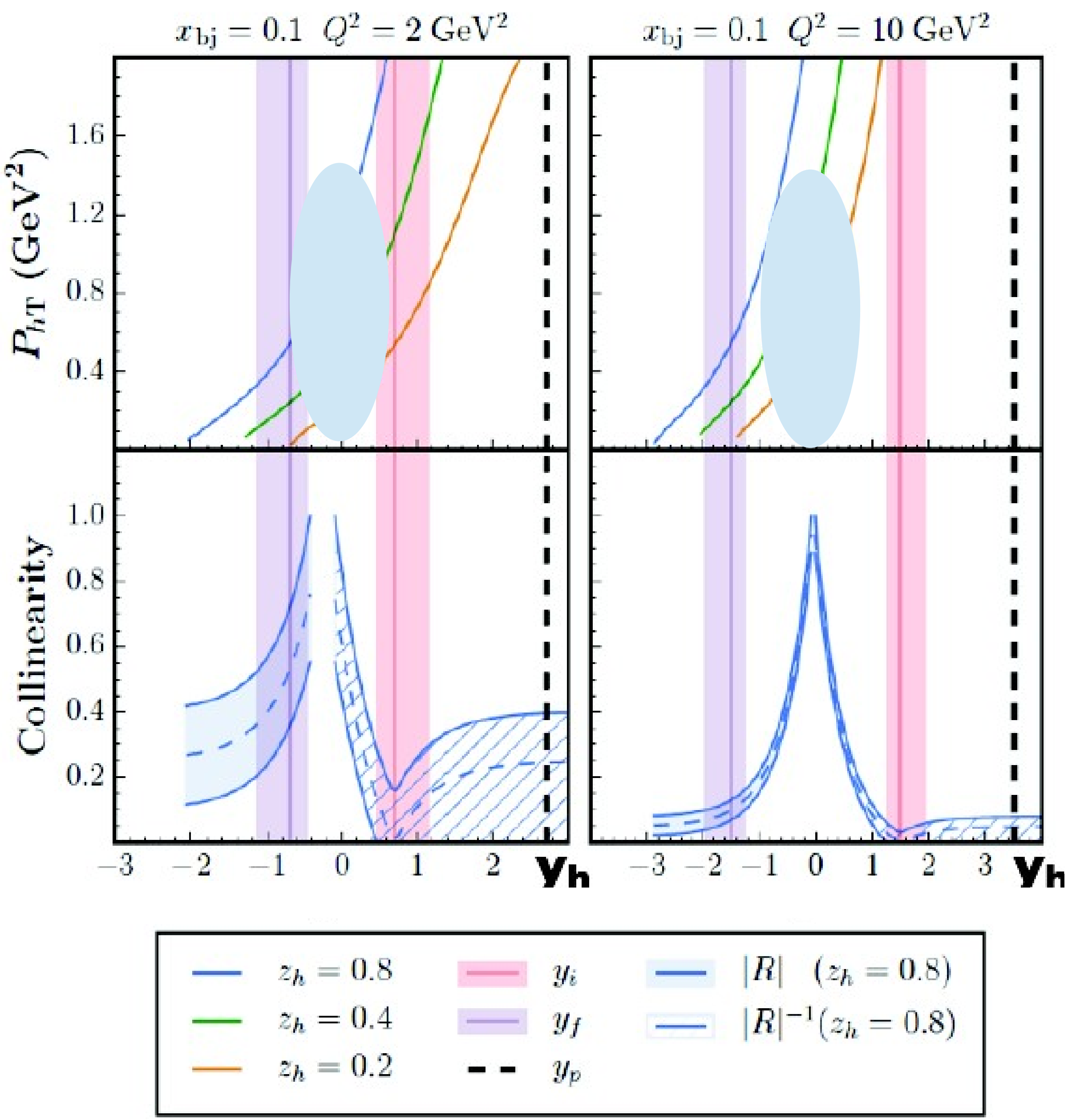
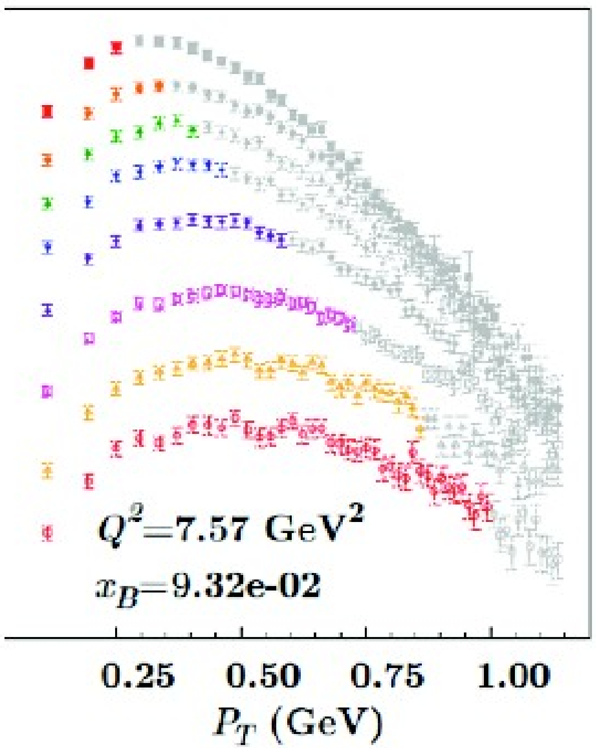


Available data is likely to receive contributions from non-TMD physics.

$$\boxed{R \equiv \frac{P_h \cdot k_f}{P_h \cdot k_i}}$$

M. Boglione, J. Collins, L. Gamberg, JOGH, T. C. Rogers, and N. Sato,Phys. Lett.B766, 245 (2017),1611.10329.



precise implementation of the R criterion on data is work in progress

a better set of variables?

$\{Q^2, x_B, P_{hT}, z_h\}$

$q_T = P_{hT} / z_h \qquad y_h$

***ONLY AN EXAMPLE**

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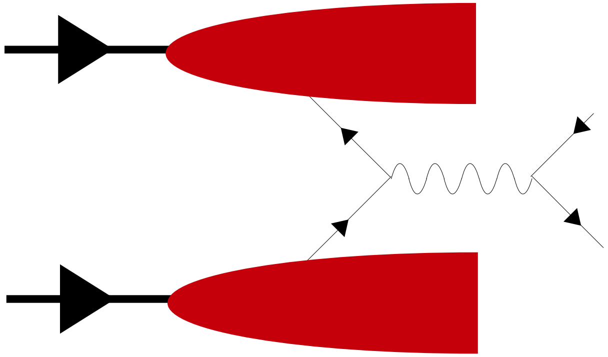
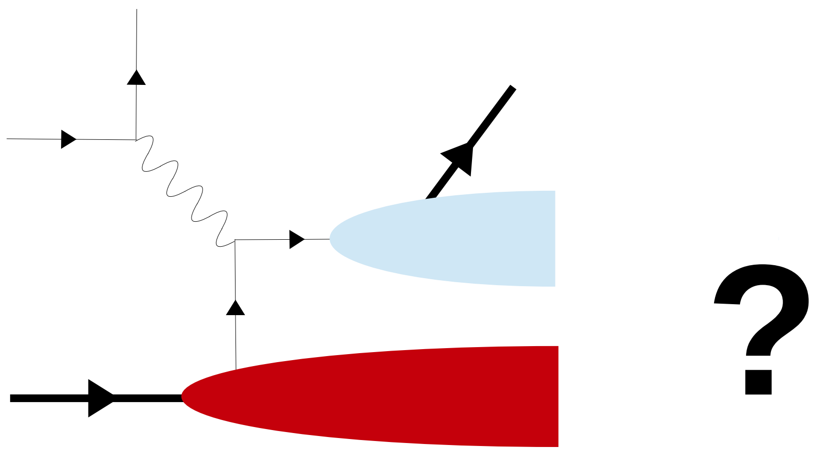
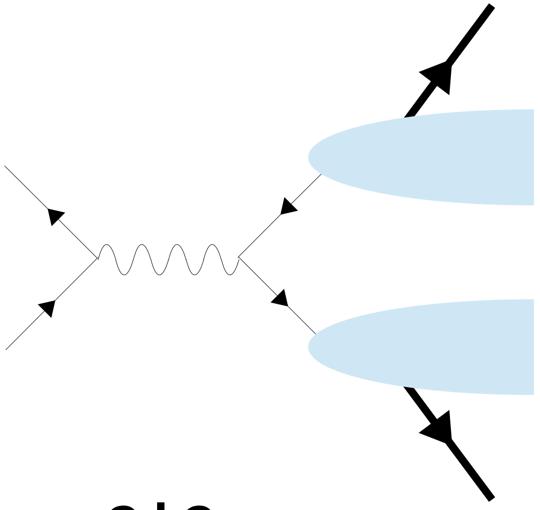




SIDIS

Recently, BELLE, BaBar, BES III Collins asymmetries.

No modern unpolarized measurements are available.



e+e-

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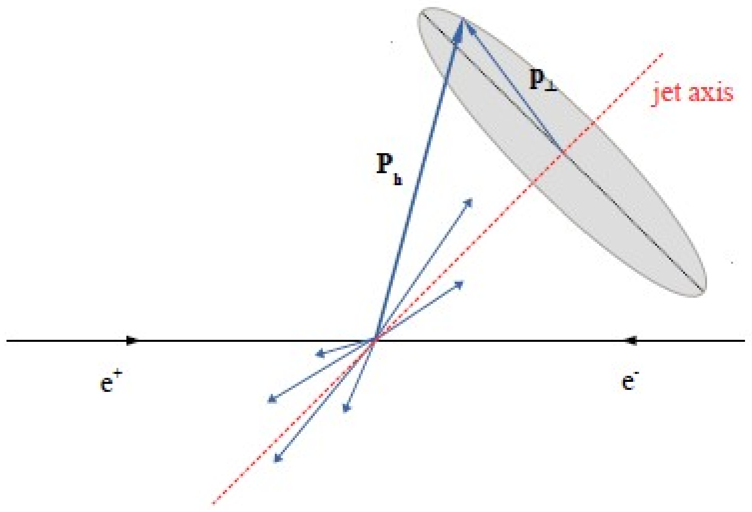
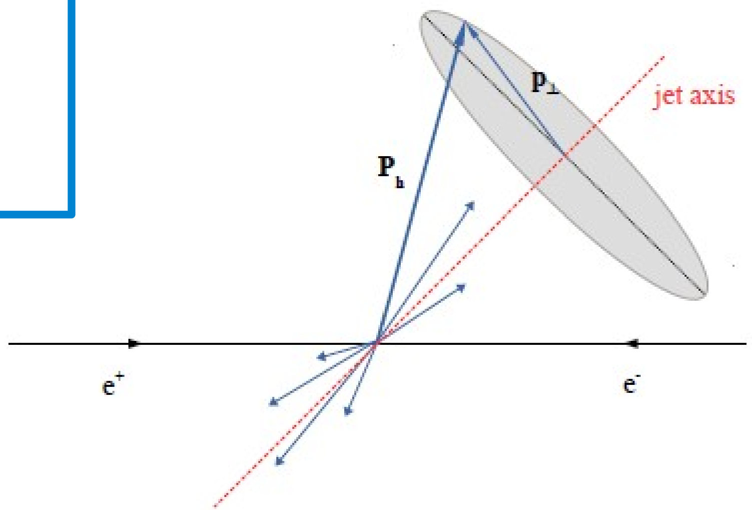
No modern unpolarized measurements are available.

TASSO, MARK II available for
e+e- → X h

- **pT** distributions
- different energies
- integrated over **z**



Boglione, JOGH, R. Taghavi
Phys.Lett. B772 (2017) 78
arXiv:1704.08882



TASSO, MARK II available for

e+e- → X h

- **pT** distributions
- different energies
- integrated over **z**

Big Limitation

New analysis:

how much information about the **unpolarized TMD FF** can we get from these data sets?

Use this...

Assuming factorization

$$D_{h/q}(z, p_\perp) = d_{h/q}(z)\, h_d(p_\perp)$$

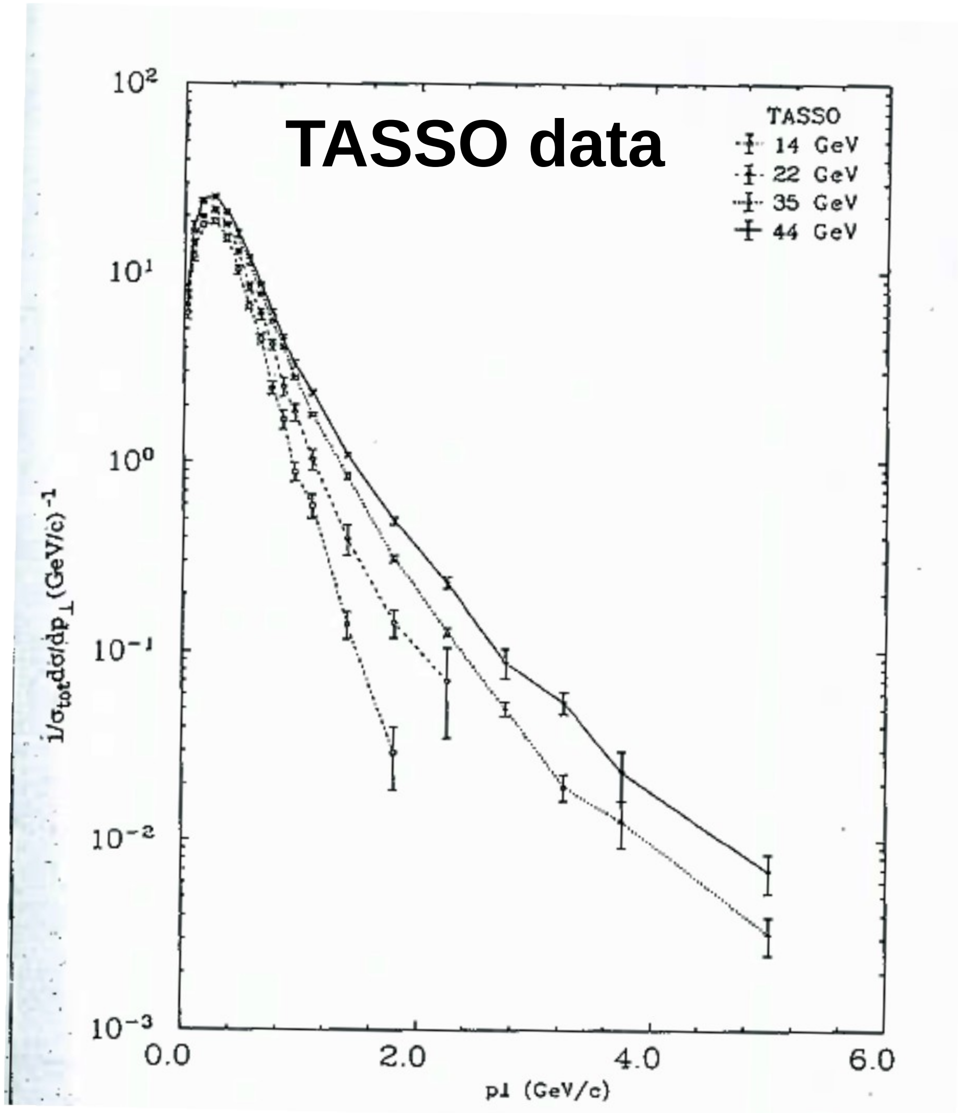
To get information about this

QCD picture

$$\tilde{D}_{h/q}(z, \boldsymbol{b}_\perp; Q) = \sum_j \left[\left(\tilde{C}_{j/q} \otimes \frac{d_{h/j}}{z^2} \right) e^{\Gamma_D(Q)} \right] \exp\left\{ g_{j/P}(x, b_\perp) + g_K(b_\perp) \log\left(\frac{Q}{Q_0} \right) \right\}$$

Things to investigate:

- appropriate functional form for $\mathbf{g}_{j/P}$
- scale evolution regulated by $\mathbf{g}_K$



$$\tilde{D}_{h/q}(z, \boldsymbol{b}_\perp; Q) = \sum_j \left[\left(\tilde{C}_{j/q} \otimes \frac{d_{h/j}}{z^2} \right) e^{\Gamma_D(Q)} \right] \boxed{\exp\left\{ g_{j/P}(x, b_\perp) + g_K(b_\perp) \log\left(\frac{Q}{Q_0} \right) \right\}}$$

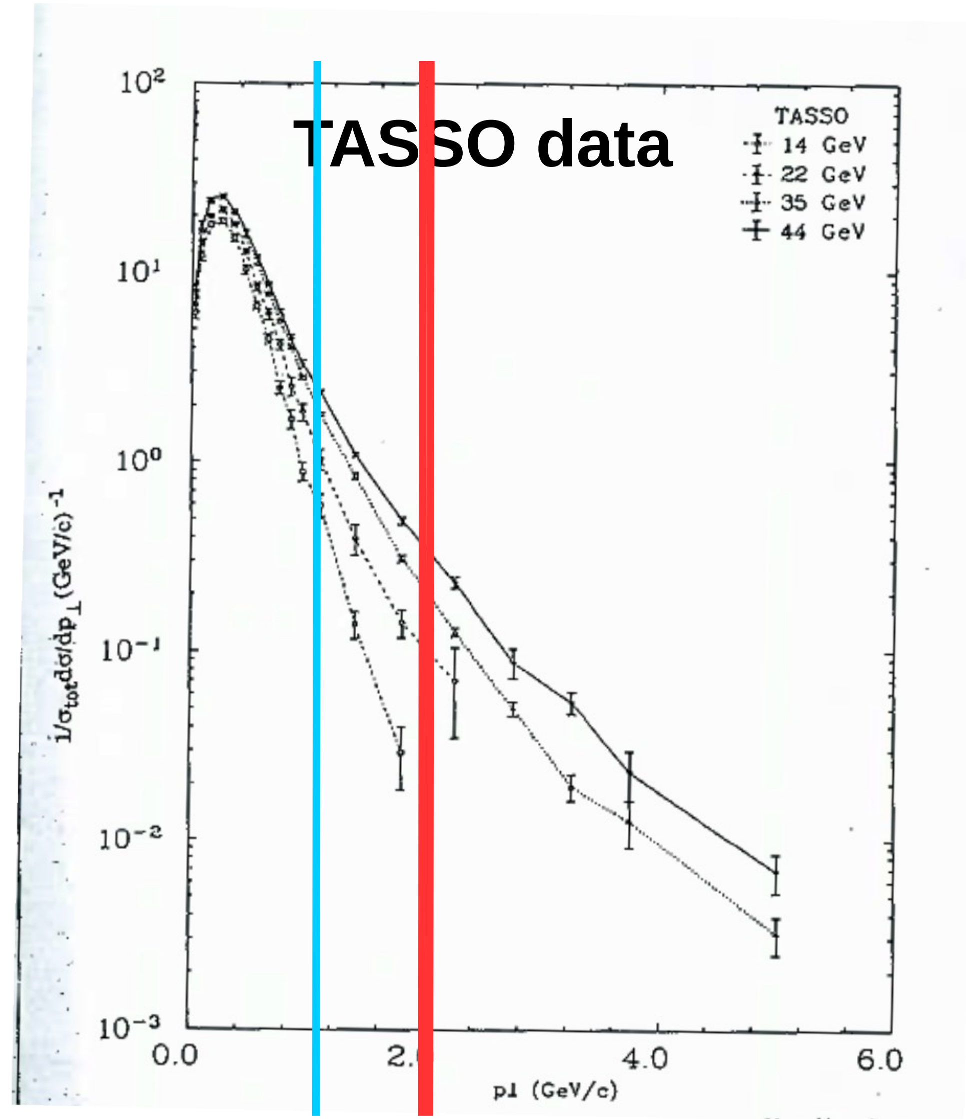
Identify region where TMD Effects dominate:

For fully differential cross sections, matching region is Expected to be at

$$p_\perp \sim zQ$$

Use experimental **<z>** to make an estimate

$$p_\perp \sim 2\,\mathrm{GeV}$$



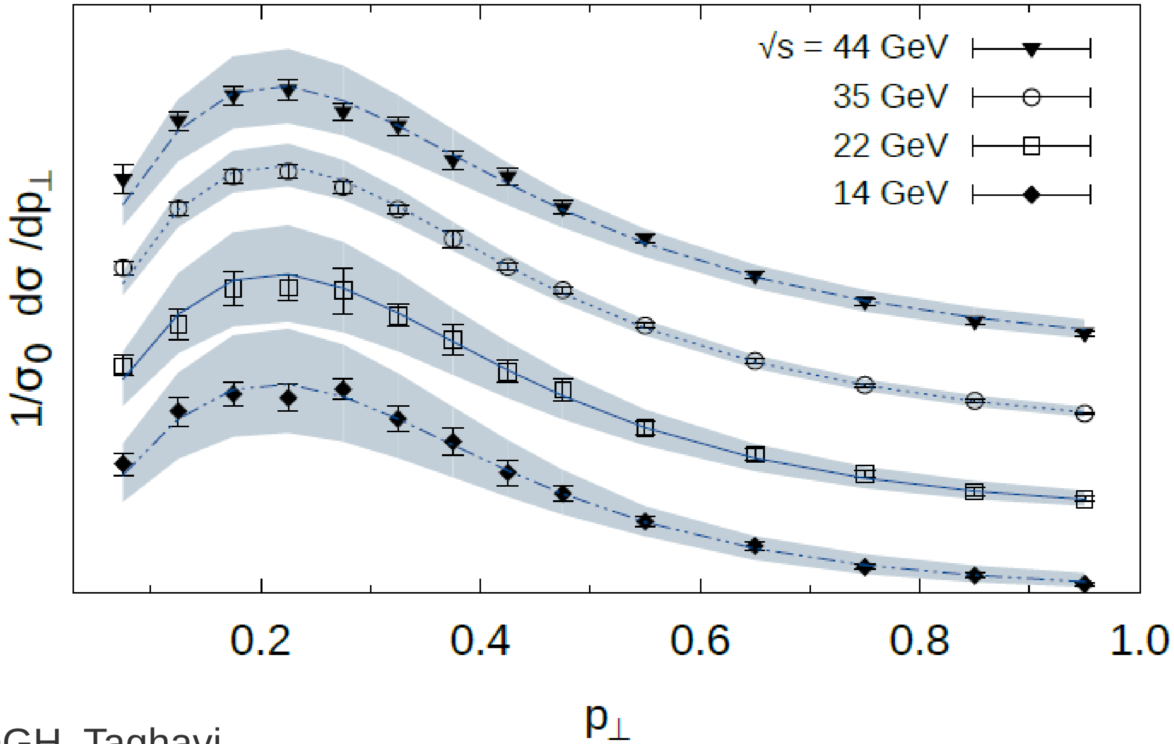
We looked at a restricted range: $p_\perp < 1\ \mathrm{GeV}$

Power law to model transverse momentum dependence

$$D_{h/q}(z, p_\perp) = d_{h/q}(z)\; h_d(p_\perp)$$

$$h(p_\perp) = 2(\alpha - 1)\mathrm{M}^{2\,(\alpha-1)} \frac{1}{\left(p_\perp^2 + \mathrm{M}^2\right)^\alpha}$$

FIT TO TASSO

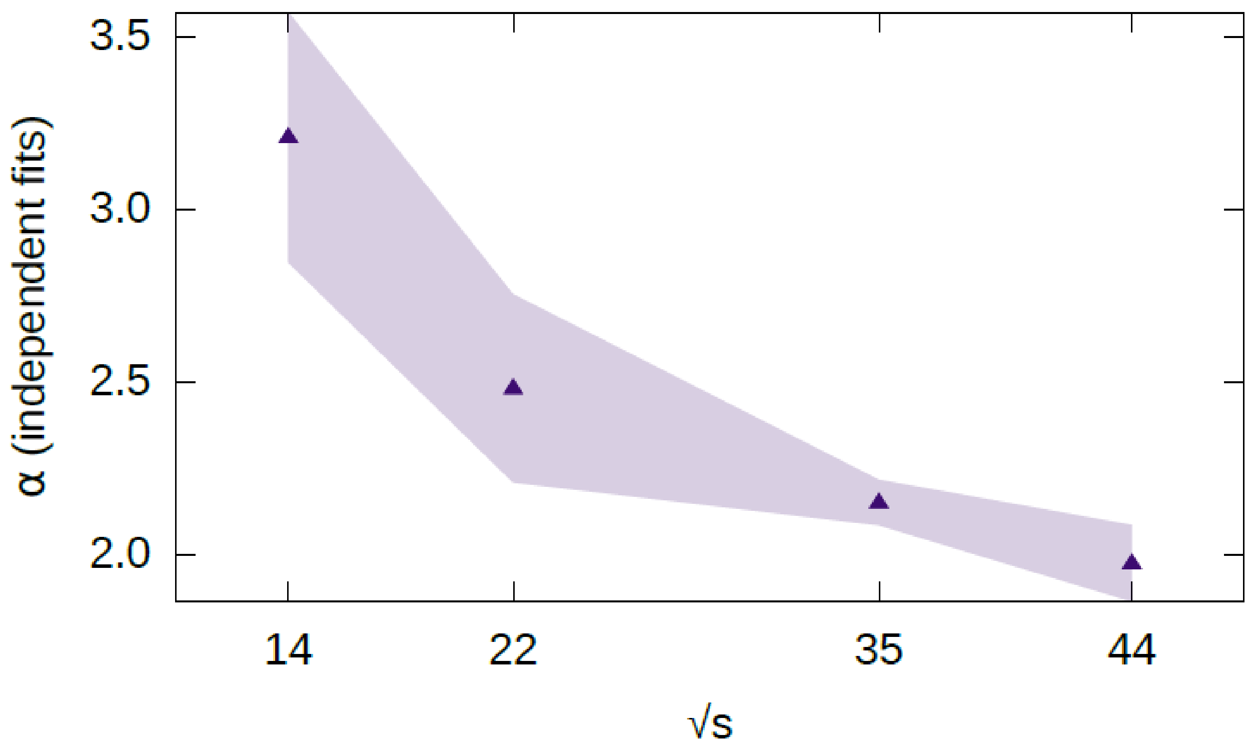


Boglione, JOGH, Taghavi

Phys.Lett. B772 (2017) 78-86

Power law parameters follow a logarithmic trend

$$h(p_\perp) = 2(\alpha - 1)\mathrm{M}^{2\,(\alpha-1)} \frac{1}{\left(p_\perp^2 + \mathrm{M}^2\right)^{\alpha}}$$



Boglione, JOGH, Taghavi

Phys.Lett. B772 (2017) 78-86

TMD

$$\mathcal{F}^{-1}\left\{\frac{d\sigma^h}{dz\, d^2\boldsymbol{p}_\perp}\right\} \propto \exp\left\{\left(\lambda_\Gamma(b_*) + g_K(b_\perp)\right)\log\left(\frac{Q}{Q_0}\right)\right\}\Bigg|_{b_\perp \to z\, b_\perp}$$

$$\lambda_\Gamma(b_*) \equiv \frac{32}{27}\log\left(\log\frac{2e^{-\gamma_E}}{\Lambda_{QCD}\, b_*}\right)$$

MODEL

$$h(p_\perp) = 2(\alpha-1)\mathrm{M}^{2\,(\alpha-1)}\frac{1}{\left(p_\perp^2 + \mathrm{M}^2\right)^\alpha}$$

$$\mathcal{F}^{-1}\left\{\frac{1}{\left(p_\perp^2 + \mathrm{M}^2\right)^\alpha}\right\} \xrightarrow{\text{large } b_\perp} \frac{1}{2^\alpha\,\pi\,\Gamma(\alpha)}\left(\frac{b_\perp}{\mathrm{M}}\right)^{\alpha-1}\sqrt{\frac{\pi}{2}}\,\frac{e^{-b_\perp \mathrm{M}}}{\sqrt{b_\perp \mathrm{M}}}\left[1 + \mathcal{O}\left(\frac{1}{b_\perp \mathrm{M}}\right)\right]$$

TMD

$$\mathcal{F}^{-1}\left\{\frac{d\sigma^h}{dz\,d^2\boldsymbol{p}_\perp}\right\} \propto \exp\left\{\Big(\lambda_\Gamma(b_*) + g_K(b_\perp)\Big)\log\left(\frac{Q}{Q_0}\right)\right\}\Bigg|_{b_\perp \to z\, b_\perp}$$

$$\lambda_\Gamma(b_*) \equiv \frac{32}{27}\log\left(\log\frac{2e^{-\gamma_E}}{\Lambda_{QCD}\, b_*}\right)$$

Logarithmic behavior of alpha may be interpreted as a consequence of the **Log** in the definition of the **TMD FF**.

$$\alpha = \alpha_0 + \tilde{\alpha}\log\left(\frac{Q}{Q_0}\right)$$

$$g_K(b_\perp) \xrightarrow{\text{large } b_\perp} \tilde{\alpha}\log(\nu\, b_\perp)$$

TMD

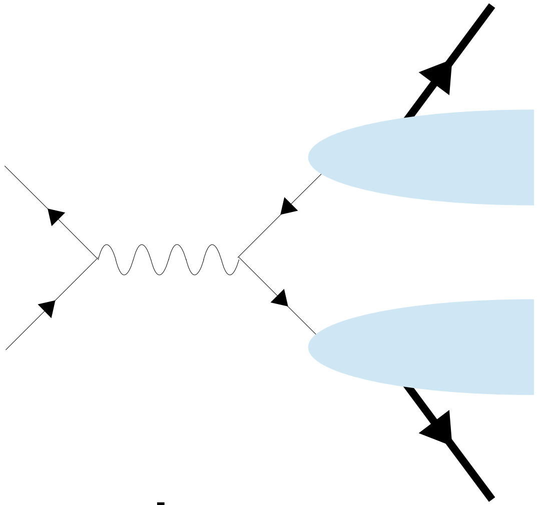
There are caveats on this interpretation, while consistent with theoretical expectations, it's not the only possibility.
(loss of information through z-integration)

Logarithmic behavior of alpha may be interpreted as a consequence of the **Log** in the definition of the **TMD FF**.

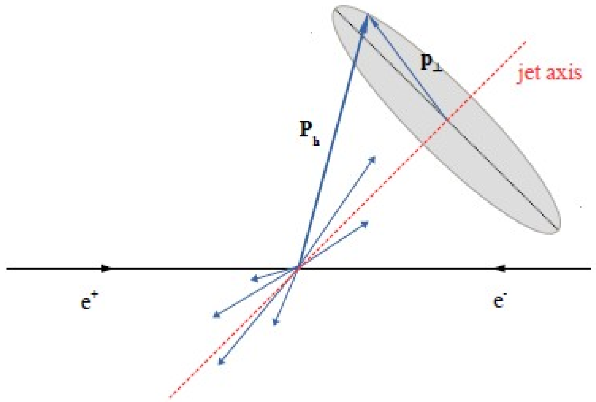
$$\boxed{\begin{gathered} \alpha = \alpha_0 + \tilde{\alpha} \log\left(\frac{Q}{Q_0}\right) \\ g_K(b_\perp) \xrightarrow{\text{large } b_\perp} \tilde{\alpha} \log(\nu\, b_\perp) \end{gathered}}$$

The lack of information about **z** hinders a full TMD extraction of the FF.

Future upcoming data by BELLE on unpolarized one-hadron production may allow for a combined analysis with TASSO and MARK II data.



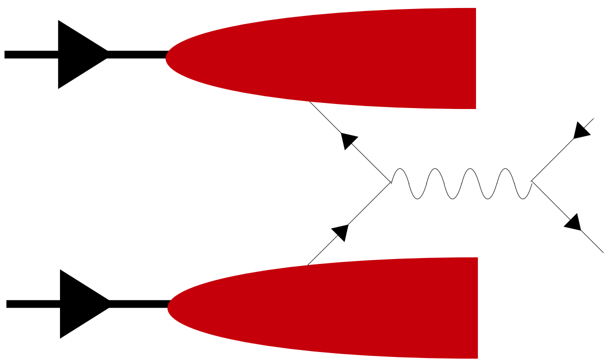
e+e-

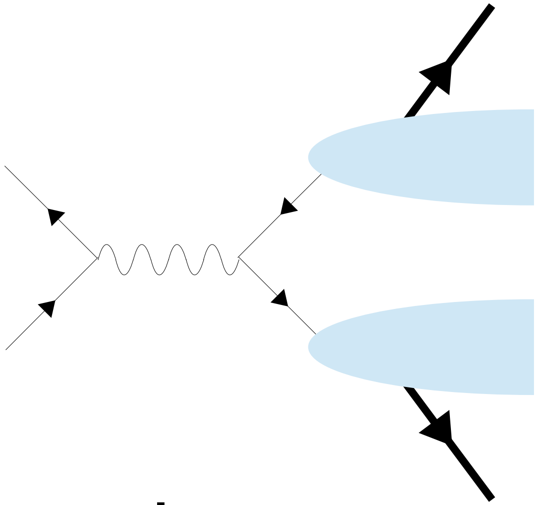


Phenomenological Test for factorization

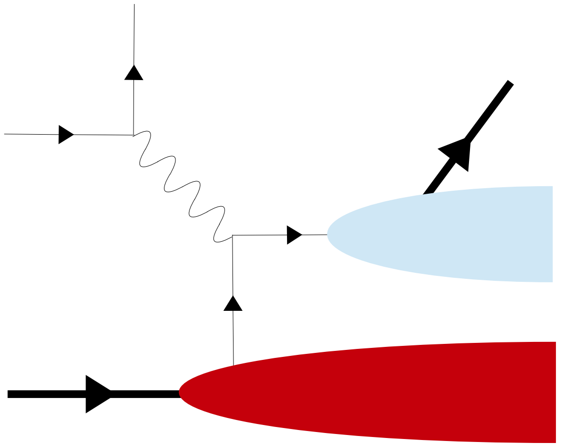
Extraction from data?

Drell Yan





e+e-



SIDIS

Fragmentation Functions

Let's keep working, but let us be careful with interpretations

Final Remarks

Currently, we are attempting to do phenomenology within **full QCD picture**.

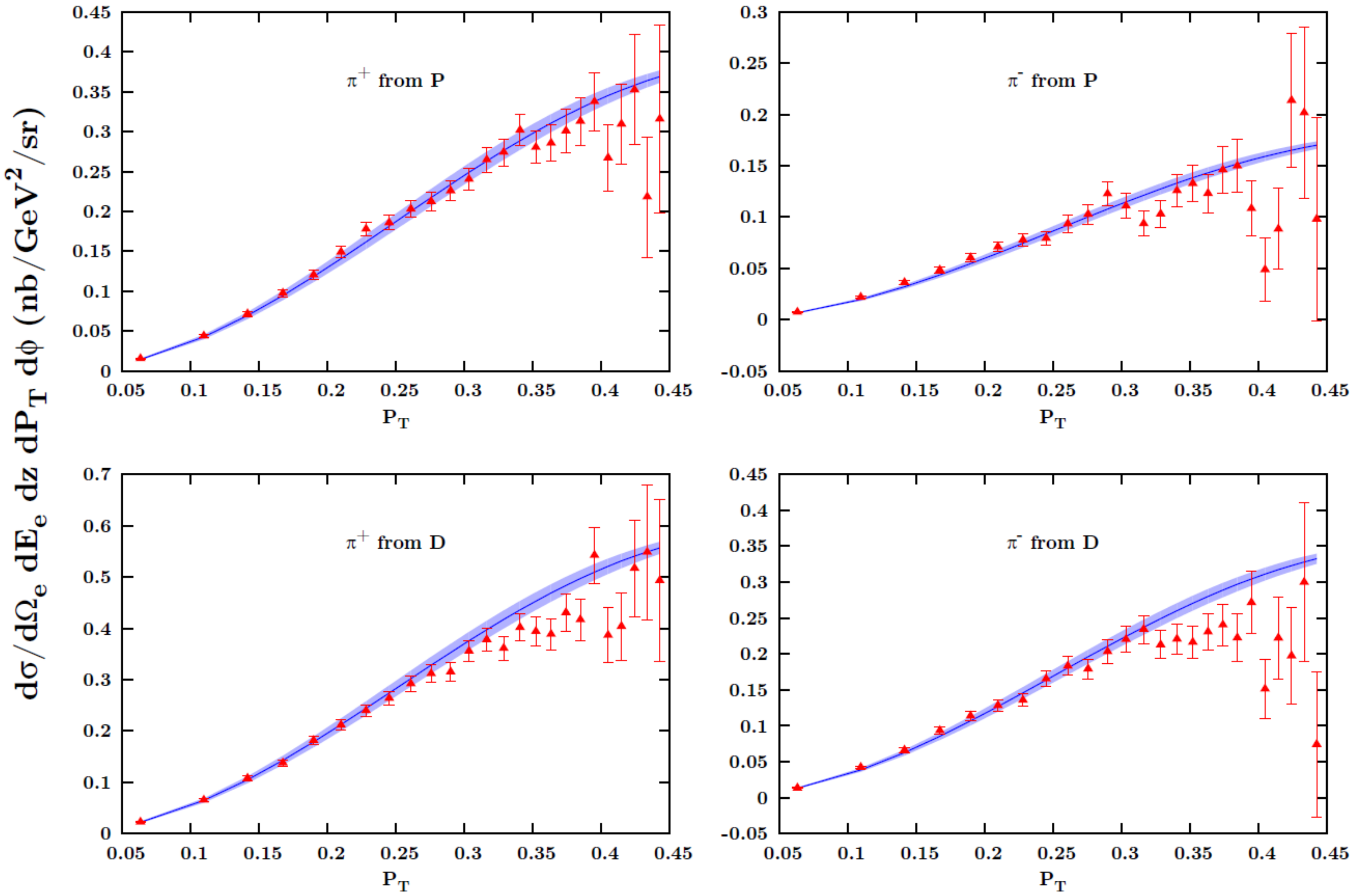
Recent SIDIS multidimensional data is so far the most suitable way to access information about the unpolarized TMD FF. Must solve some *theoretical issues*.

On the side of e+e- one hadron production, in the near future unpolarized cross sections by BELLE may allow for an analysis of the older sets, TASSO MARKII within a full TMD picture.

TMD Factorization for e+e- one hadron production?

Thank you.

Jlab SIDIS data (2012) (Parameters from HERMES extraction).



Ingredients for extraction of Collins function.

$e^+e^- \to \pi\,\pi\,X$

Unpolarized TMDFF **Collins TMDFF**

$$\frac{d\sigma^{e^+e^-\to h_1h_2X}}{dz_1\,dz_2\,d^2\boldsymbol{P}_{1T}\,d\cos\theta_2} = \frac{3\pi\alpha^2}{2s}\left\{\boxed{D_{h_1\,h_2}} + \boxed{N_{h_1\,h_2}}\cos 2\phi_1\right\}$$

$$P_0^{U,L,C} = \frac{N^{U,L,C}}{D^{U,L,C}}$$

Ratio

$$D^U = D_{\pi^+\pi^-} + D_{\pi^-\pi^+} \qquad N^U = N_{\pi^+\pi^-} + N_{\pi^-\pi^+}$$
$$D^L = D_{\pi^+\pi^+} + D_{\pi^-\pi^-} \qquad N^L = N_{\pi^+\pi^+} + N_{\pi^-\pi^-}$$
$$D^C = D^U + D^L \qquad N^C = N^U + N^L\,,$$

$$\frac{A_0^U}{A_0^{L(C)}} \equiv 1 + \cos(2\phi_1)A_0^{UL(C)}$$

Double Ratio

SIDIS

$$\frac{d\sigma^{\ell(S_\ell)+p(S)\to\ell' hX}}{dx_B dQ^2 dz_h d^2\boldsymbol{P}_T d\phi_S} =$$

$$\frac{2\alpha^2}{Q^4}\left\{\frac{1+(1-y)^2}{2}F_{UU} + \quad \cdots\right.$$

$$\left. + S_T(1-y)(\sin(\phi_h+\phi_S)F_{UT}^{\sin(\phi_h+\phi_S)}\right\}.$$

$$A_{UT}^{\sin(\phi_h+\phi_S)} \sim \frac{F_{UT}^{\sin(\phi_h+\phi_S)}}{F_{UU}}$$

Ratio

Unpolarized TMDFF &TMDPDF

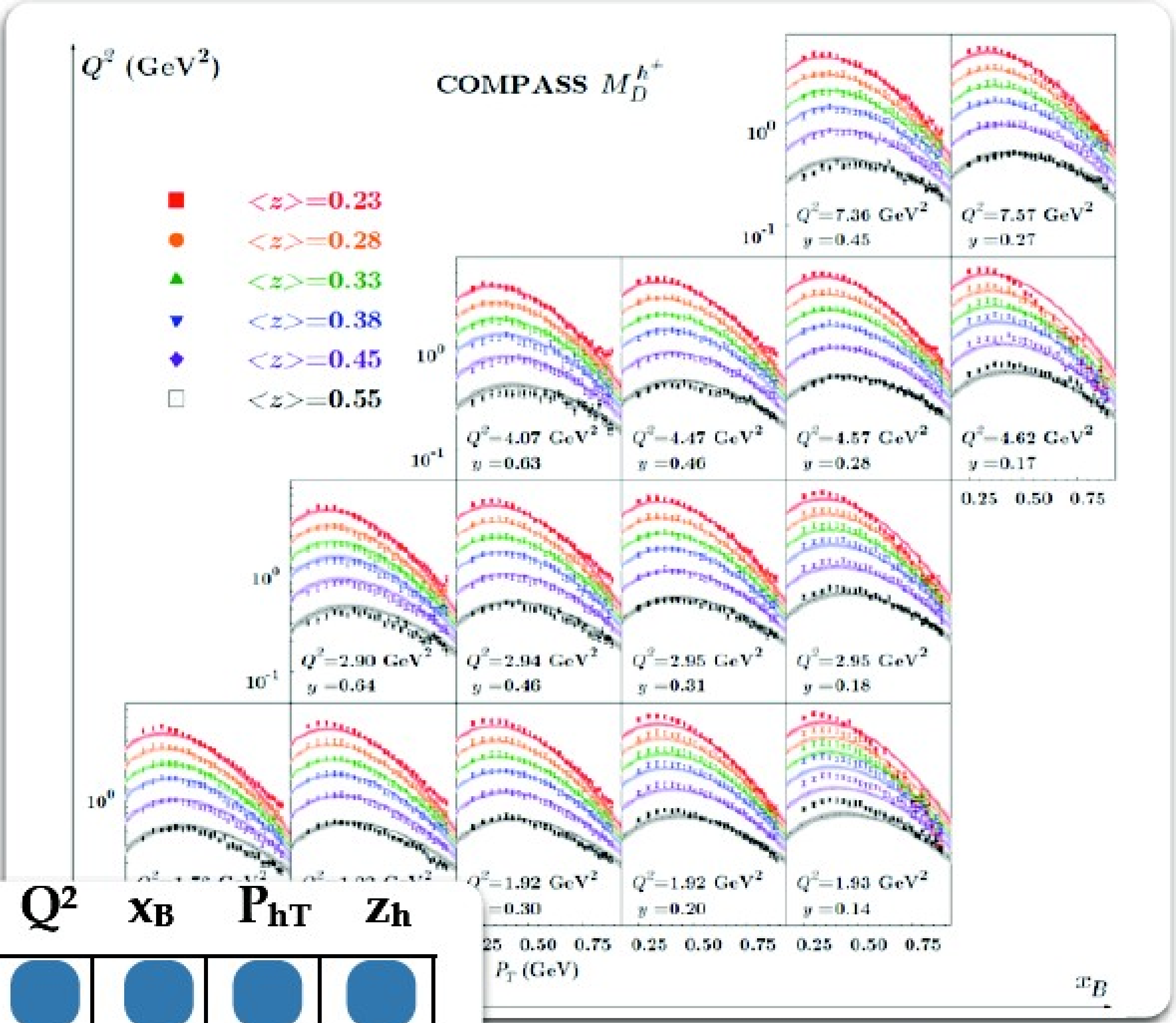
TMD Transversity & Collins function

Unpolarized SIDIS cross section (current region)

$$\frac{d\sigma^{\ell+p\to\ell' hX}}{dx_B\, dQ^2\, dz_h\, dP_T^2} = \frac{2\,\pi^2\alpha^2}{(x_B\, s)^2}\,\frac{\left[1+(1-y)^2\right]}{y^2}\,F_{UU}$$

$$F_{UU} = \sum_q \mathcal{H}_q \text{ F.T.}\left\{\tilde{D}_{h/q}(z, z\,\boldsymbol{b}_\perp; Q)\ \tilde{f}_{q/P}(x, \boldsymbol{b}_\perp; Q)\right\}$$

$$+ \text{ large } q_T \text{ corrections} + \text{power suppressed terms}$$



	Q^2	x_B	P_{hT}	z_h
binned	●	●	●	●
integrated				