

# Baryon properties from DSEs/BSEs

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**NAWI Graz**  
Natural Sciences

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# Contents

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- **The framework and its goals**
- **Spectrum**
- **Electromagnetic structure of baryons**
- **Future (Outlook)**

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- **The framework and its goals**
- **Spectrum**
- **Electromagnetic structure of baryons**
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**DISCLAIMER:**

**This talk reflects only part of the work of our group.**

**Many other groups use DSE/BSEs in different systems**

**and/or at different approximation levels**

**(e.g. El-Bennich's talk on Tuesday)**

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# Motivation. First principles

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- Strong interactions are described by QCD. QCD is a theory of **quarks** and **gluons**. But the **only observable** particles are **Hadrons** (quarks and gluons are confined within bound states). Can we understand their structure from QCD?

## **Ultimate Goal:**

- Using only QCD input, (propagators, vertices, etc.) extract hadron properties, and do it directly in a **continuum QFT** formulation.
- In a DSE/BSE framework we could add/remove interaction terms and study their effect on hadron properties (example: what is the effect of the different components of the quark-gluon vertex in the spectrum?)

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# Motivation. Useful phenomenology

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- The spectrum of hadrons is not completely understood.
- Perhaps **more interesting**: Form factors contain information about the internal structure of hadrons.
- They also tell us how the hadron couples to external fields (e.g. photons). Important for other research fields.
- Very little is known experimentally about hadron FFs, with the exception of pion and nucleon and some static properties of other hadrons.
- We aim at **providing reliable information on properties of hadrons**. How reliable they are, one infers from comparison with known data.

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In **this talk** I will focus on **baryons** (mostly as three-quark objects)

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# Framework (Covariant BSEs in a nutshell)

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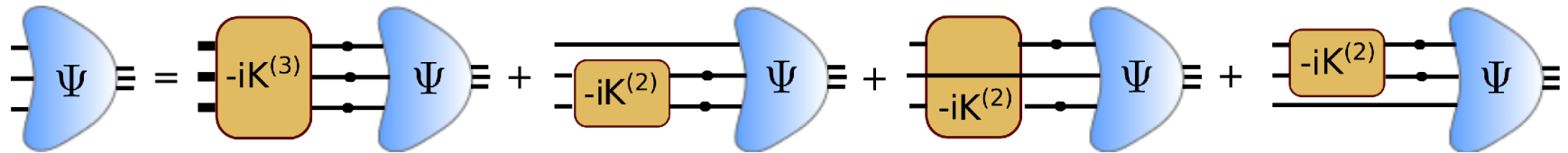
Further details:  
Eichmann, HSA, Williams, Alkofer,  
Fischer -- PPNP 91 (2016) 1-100

HSA, Williams  
To appear in Comp. Phys. Comm.

# Framework (Covariant BSEs in a nutshell)

Baryon spectrum

(**Three-body Bethe-Salpeter eq.**  $\sim$  Faddeev eq.):



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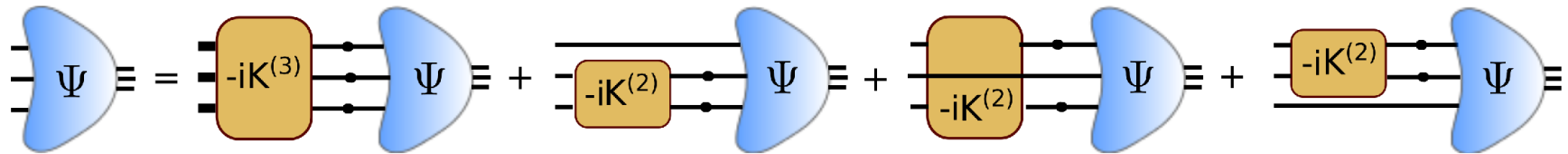
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# Framework (Covariant BSEs in a nutshell)

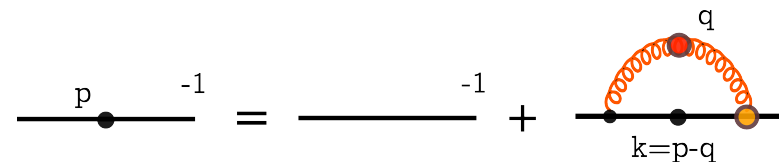
Baryon spectrum

(**Three-body Bethe-Salpeter eq.** ~ Faddeev eq.):



Elements needed:

- Interaction kernels  $K$
- **Quark propagator. We obtain this by solving the quark Dyson-Schwinger eq.**



- i.e. additionally we need the quark-gluon vertex and the gluon propagator

Further details:

**Eichmann, HSA, Williams, Alkofer, Fischer -- PPNP 91 (2016) 1-100**

**HSA, Williams**

**To appear in Comp. Phys. Comm.**

# Framework (Covariant BSEs in a nutshell)

Coupling to external current:

$$\begin{aligned}
 J^\mu = \sum_{\text{perm.}} & \equiv \text{Diagram 1} + \text{Diagram 2} \\
 & + \sum_{\text{perm.}} \text{Diagram 3} - \sum_{\text{perm.}} \text{Diagram 4}
 \end{aligned}$$

Additional elements needed:

- **Quark-photon vertex. We obtain this by solving the vertex (inhomogeneous) BSE**

$$\Gamma^\mu = \gamma^\mu + \text{Diagram with Kernel K}$$

- Additionally, we need to know how does the current couple to the interaction kernels

Further details:

**Eichmann, HSA, Williams, Alkofer, Fischer -- PNP 91 (2016) 1-100**

**HSA, Williams**

**To appear in Comp. Phys. Comm.**

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# Symmetries and Truncations

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Clearly, the equations are not exactly solvable, since they are an **infinite system** of coupled equations.

They must be **truncated to a finite system** (curse of DSEs/BSEs)

What do we demand to a «**good truncation**» for QCD phenomenology?

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What do we demand to a «**good truncation**» for QCD phenomenology?

- Preserve chiral symmetry in the chiral limit
- Implement a mechanism for dynamical chiral symmetry breaking

Those are essential features for hadron phenomenology

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What do we demand to a «**good truncation**» for QCD phenomenology?

- Preserve chiral symmetry in the chiral limit
- Implement a mechanism for dynamical chiral symmetry breaking

Those are essential features for hadron phenomenology

- Respect (electromagnetic) charge conservation.

Obviously important for, e.g., calculation of form factors

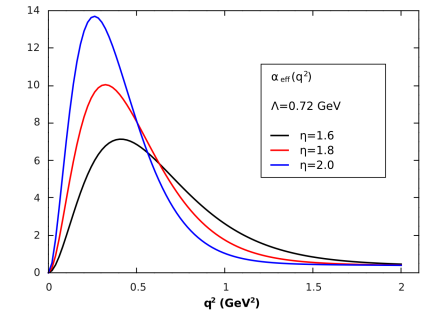
# Symmetries and Truncations

The results we will show in what follows are obtained using the **Rainbow-Ladder truncation** of the DSE/BSE system:



Effective coupling

**2-parameter model.**  
We fit to pion physics  
(pion mass and decay constant)  
once and for all!



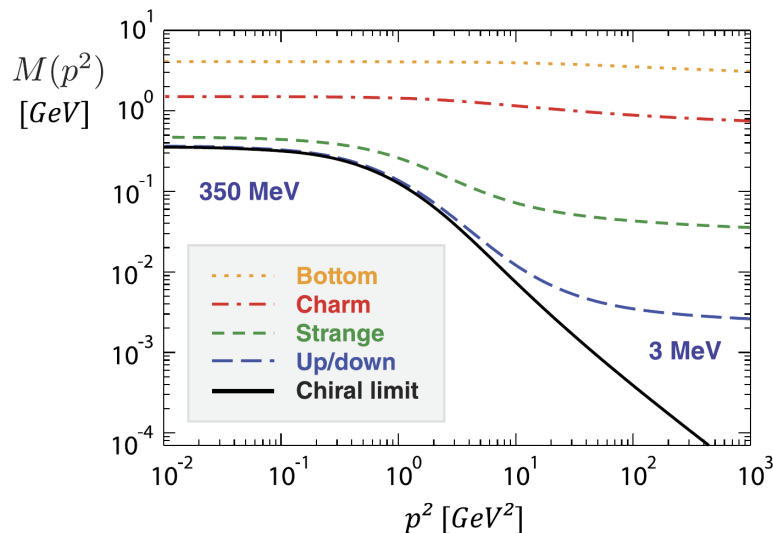
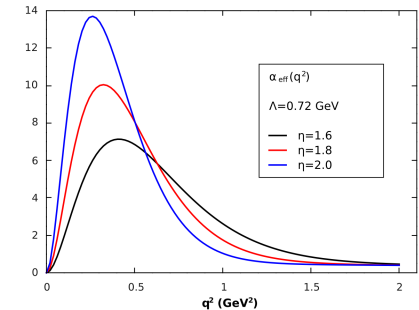
# Symmetries and Truncations

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Effective coupling

**2-parameter model.**  
We fit to pion physics  
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- Simplest truncation fulfilling the previous requirements
- As we will see, **performs surprisingly well for ground-state phenomenology**
- From symmetry requirements only, we cannot enlarge this truncation systematically (**more on truncations later**)

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# Selected RL results. Baryon spectrum

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(LIGHT) BARYON MASSES

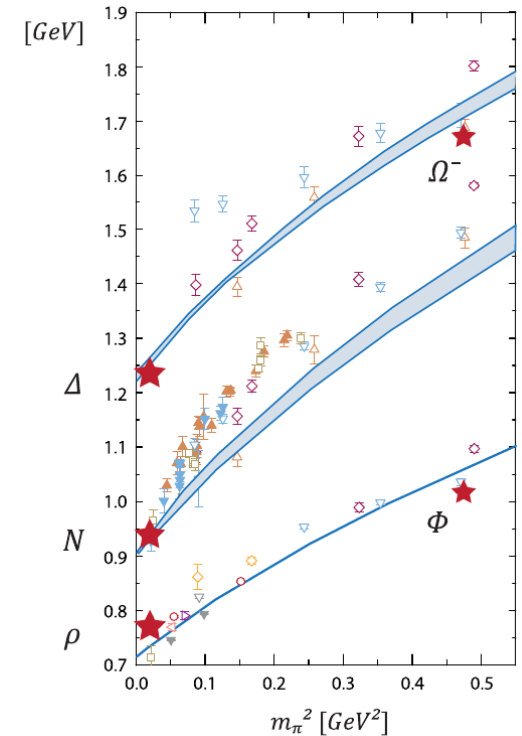
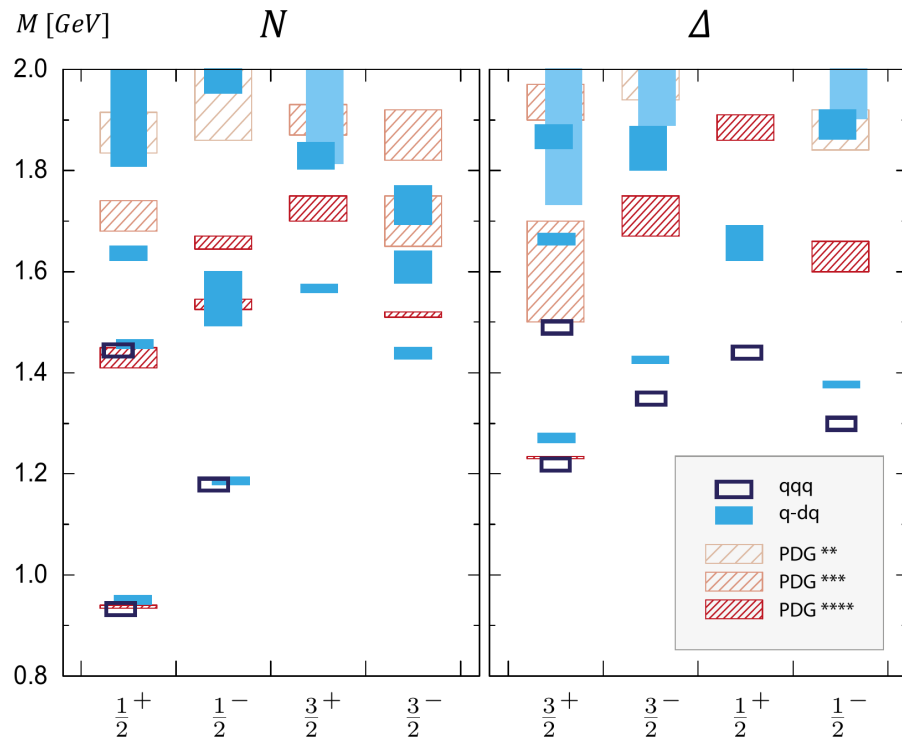


# Selected RL results. Baryon spectrum

Eichmann, HSA, Fischer Phys.Rev. D94 (2016)

Eichmann, HSA, Williams, Alkofer, Fischer PPNP 91 (2016) 1-100

- Ground-state positive-parity masses well reproduced



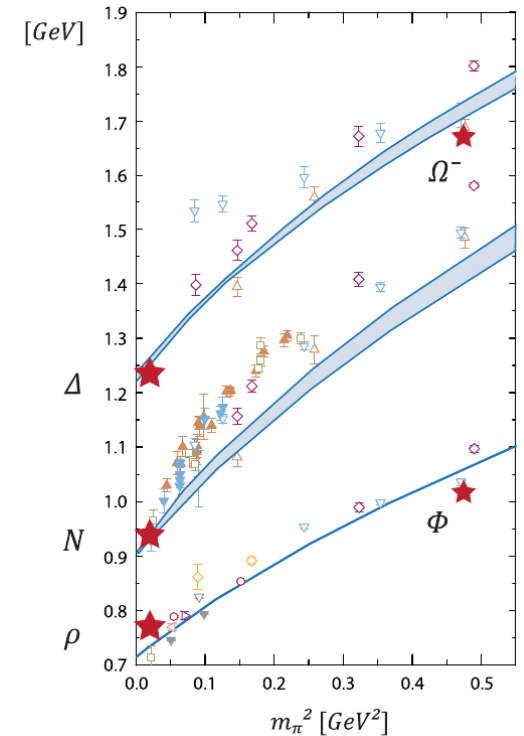
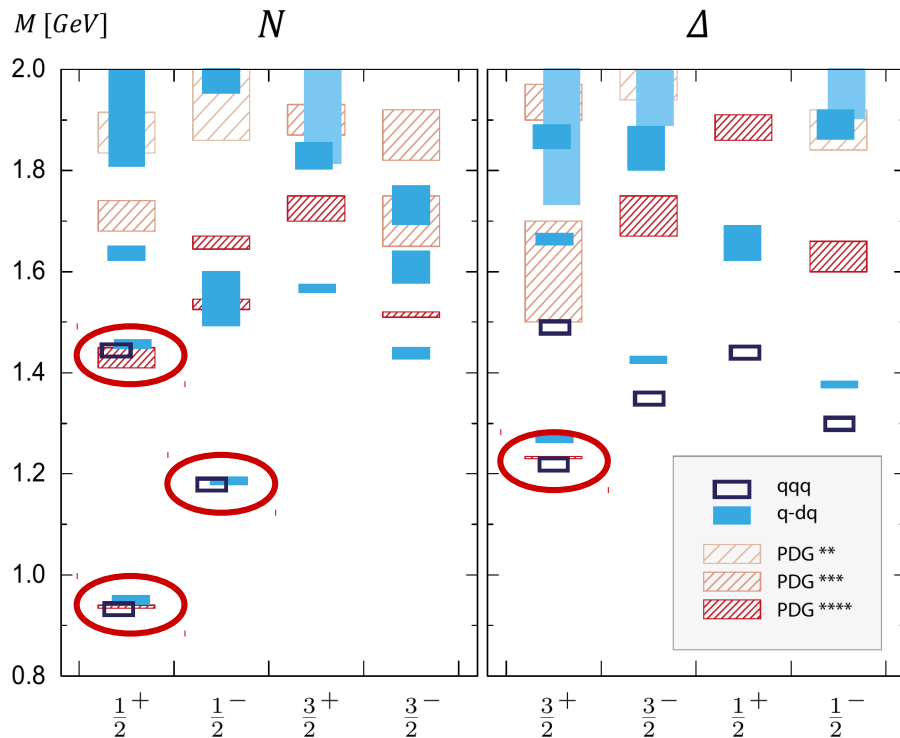
- Baryon-mass evolution with the quark mass allows to understand explicit chiral-symmetry breaking
- It also allows to compare with lattice QCD; there one can work with unphysical quark masses

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# Selected RL results. Baryon spectrum

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HSA, Fischer      Phys.Rev. D90 (2014)  
Eichmann, HSA, Williams, Alkofer, Fischer      PPNP 91 (2016) 1-100

- Ground-state strange baryons slightly underestimated. Reason: flavour independence of RL truncation
- Still, agreement reasonably good, given the simplicity of the model

$1/2^+$	$N$	$\Sigma$	$\Lambda$	$\Xi$
Faddeev	0.930 (3)	1.073 (1)	1.073 (1)	1.235 (5)
Experiment	0.938	1.189	1.116	1.315
Relative difference	< 1 %	10 %	4 %	6 %

$3/2^+$	$\Delta$	$\Sigma^*$	$\Xi^*$	$\Omega$
Faddeev	1.21 (2)	1.33 (2)	1.47 (3)	1.65 (4)
Experiment	1.232 (1)	1.385 (2)	1.533 (2)	1.672
Relative difference	2 %	4 %	4 %	1 %

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# Selected RL results. Baryon spectrum

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Take-away message

- The simplest truncation possible is capable of reproducing positive-parity ground-state masses surprisingly well.
- Other parity channels and excited states have to wait for more sophisticated truncations (more on this later)

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# Selected RL results. Baryon structure

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## BARYON FORM FACTORS

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# **(spacelike) Electromagnetic FFs**

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# (spacelike) Electromagnetic FFs

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## What about electromagnetic structure?

- Experiment: Nucleon elastic and Nucleon-Delta transition.
- Lattice QCD (first-principles computer simulation): Delta and Octet hyperons
- What can we do for phenomenology?

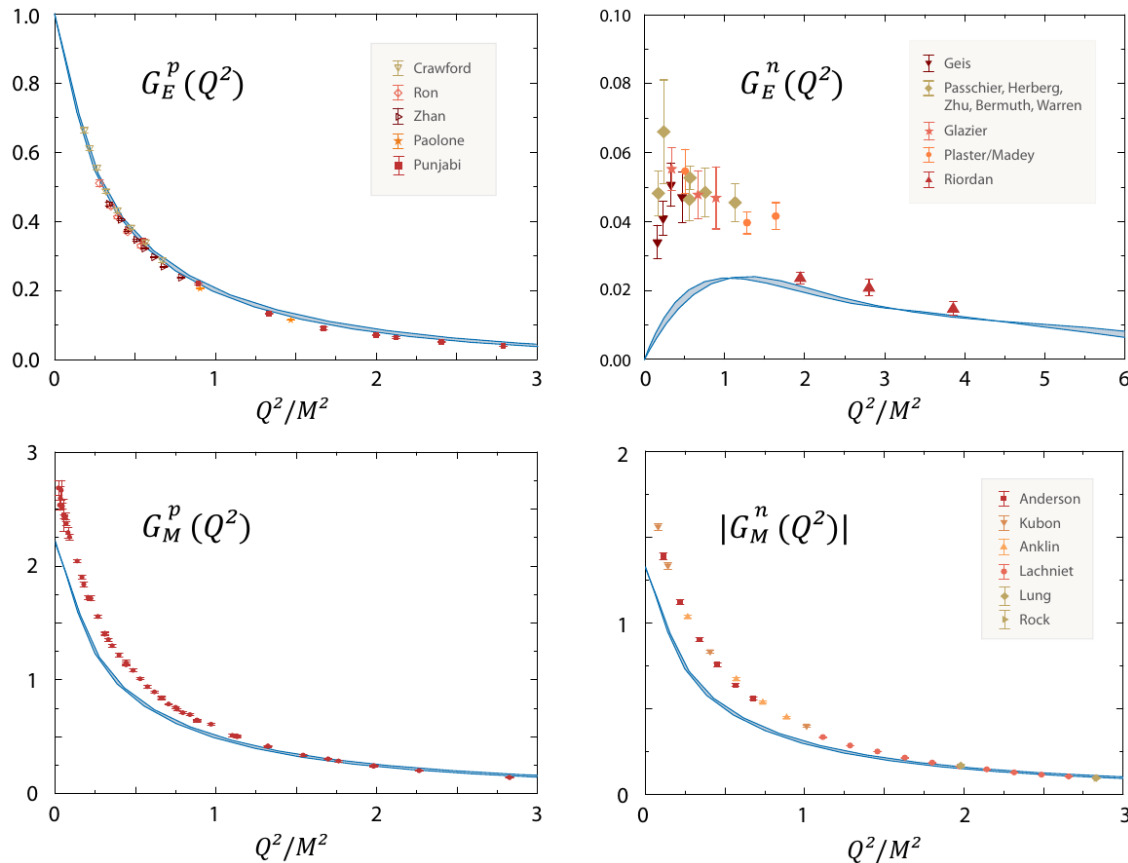
## Strategy:

- Where experiment or lattice QCD data exists: compare and learn **where does our model show deficiencies and where is it reliable.**
- Where no data available: From what we learned above, we can make **predictions in some momentum regimes.**

# Octet electromagnetic FFs. Nucleon

## Nucleon electromagnetic form factors

Eichmann *Phys.Rev. D84 (2011) 014014*



- Effect of **pion cloud** expected to be **sizable at low photon momentum ( $Q^2$ )**, especially for neutron.
- This appears as a discrepancy of our result with experiment at low- $Q^2$
- Where the influence of pion cloud is small (moderate to high  $Q^2$ ), the calculation is in **excellent agreement with experiment.**

Also calculated (baryons):

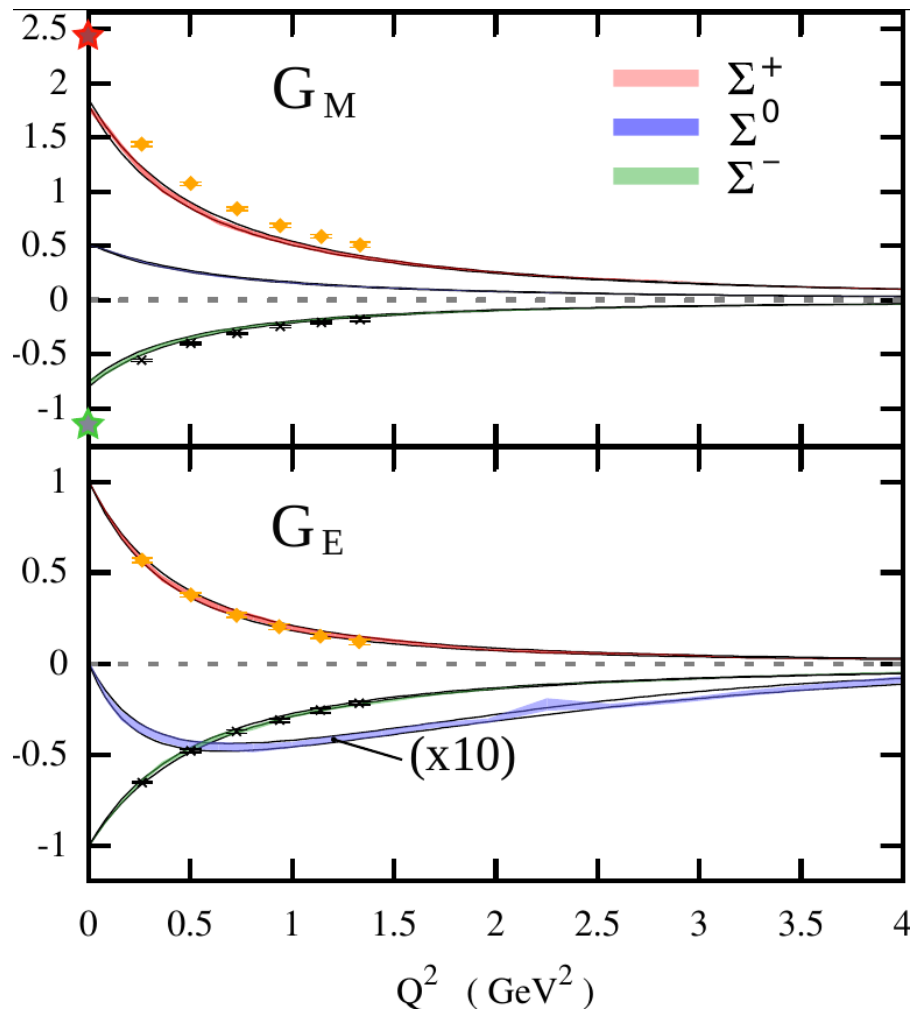
- Nucleon Axial FFs. **Eichmann, Fischer**  
**Eur.Phys.J. A48 (2012) 9**  
Same pattern



# Octet electromagnetic FFs. Sigma

LATTICE: Shanahan et al. PRD89 (2014) PRD90 (2014)

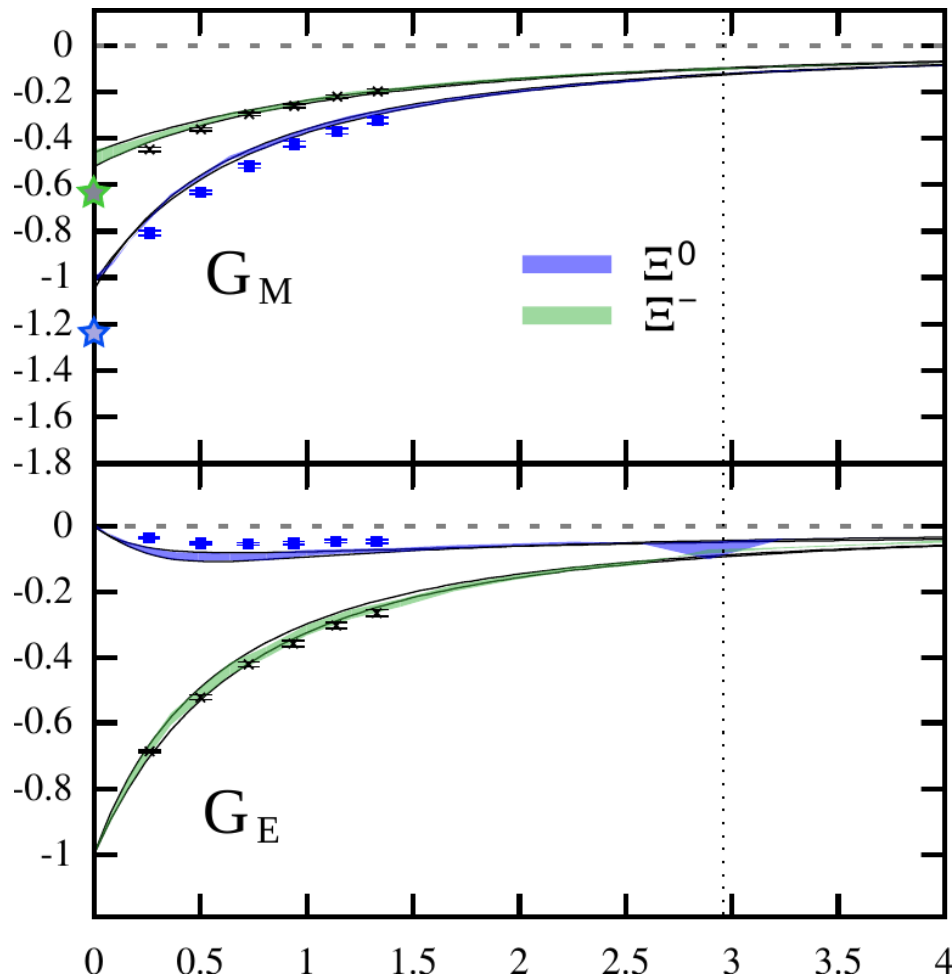
HSA, Fischer Eur.Phys.J. A52 (2016) no.2, 34



- Here, pion but also strange-meson cloud (e.g. Kaon cloud) play a role.
- Electric FF (GE) in excellent agreement with lattice QCD. They are «protected» by charge conservation.
- Kaon cloud stronger for  $\Sigma^+$  than for  $\Sigma^-$  (see  $\chi$ -PT calculation [Boinepalli et al. Phys. Rev. D74 \(2006\)](#), [Leinweber Phys. Rev. D69 \(2004\)](#)), thus better agreement for  $\Sigma^+$  at low  $Q^2$
- No other data at high  $Q^2$ . **Prediction?**
- No other data for  $\Sigma^0$ . **Prediction?**
- Static values ( $Q=0$ ) always underestimated.

# Octet electromagnetic FFs. Xi

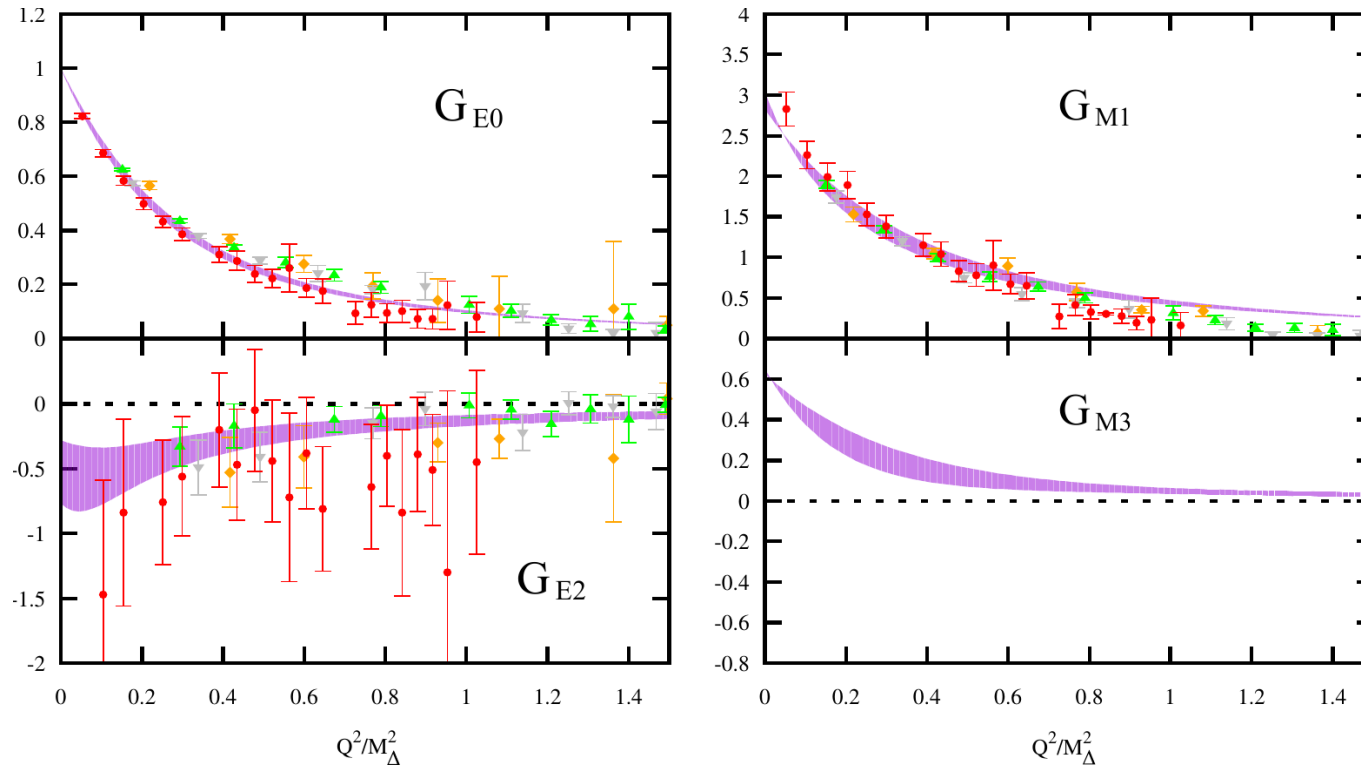
HSA, Fischer Eur.Phys.J. A52 (2016) no.2, 34



- Pion and Kaon cloud generally smaller than for  $\Sigma$ 's (see again [Boinepalli et al. Phys. Rev. D74 \(2006\)](#), [Leinweber Phys. Rev. D69 \(2004\)](#) ), and even smaller for  $\Xi^-$
- Agreement with lattice QCD improved wrt. Nucleon and  $\Sigma$ 's.
- Again, no data at high  $Q^2$ .  
**Prediction?**
- Static values underestimated.

# Decuplet electromagnetic FFs. Delta

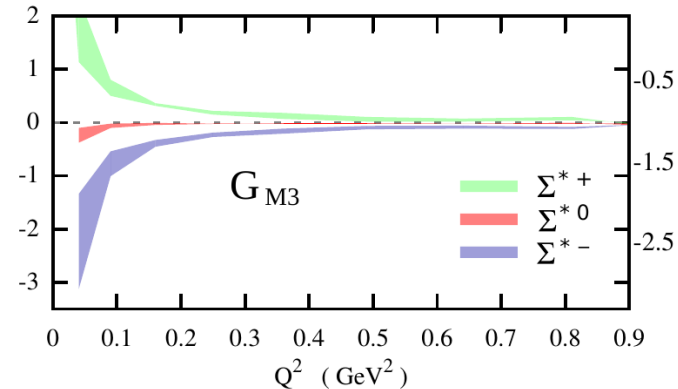
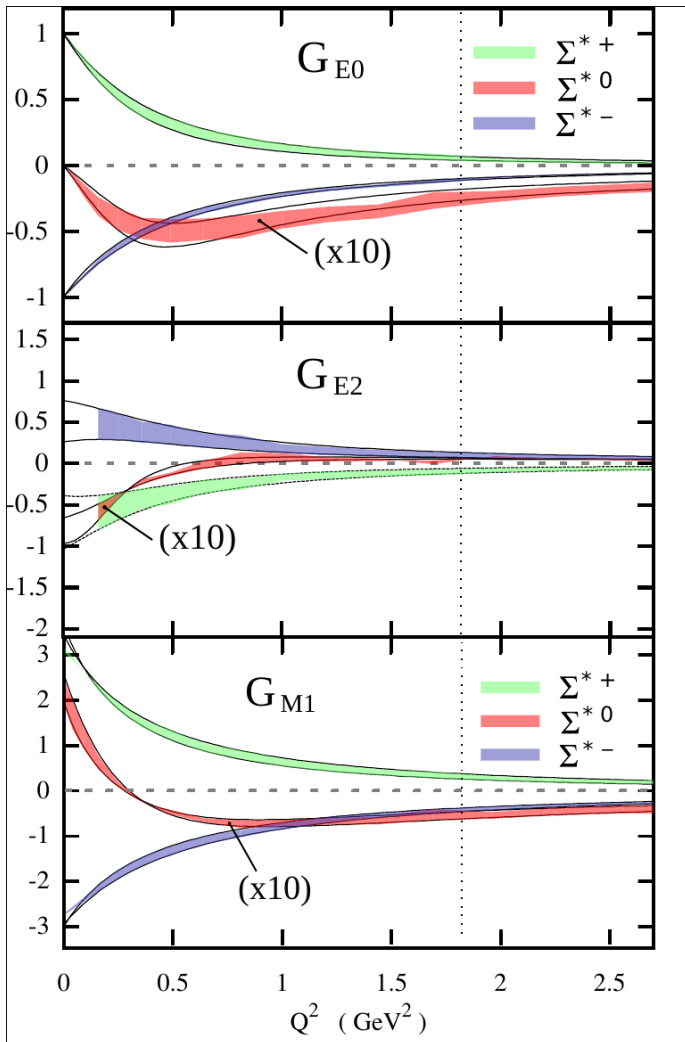
HSA, Williams, Alkofer *Phys.Rev. D87 (2013)*



- **Similar pattern as with the octet FFs** (here compared with lattice data at unphysical pion mass. Thus, absence of meson cloud less apparent)
- For spin-3/2 baryons we have **direct access to their shape**:
  - Deformation of electric charge distribution  $G_{E2}$ : **+/- Oblate/ Prolate**
  - Deformation of magnetic moment distribution  $G_{M4}$ :

# Decuplet electromagnetic FFs. Sigma\*

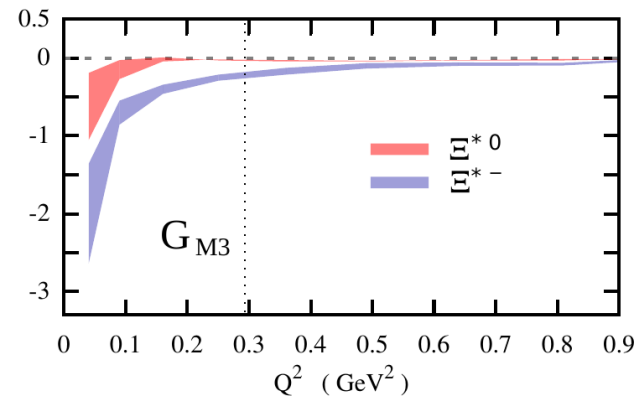
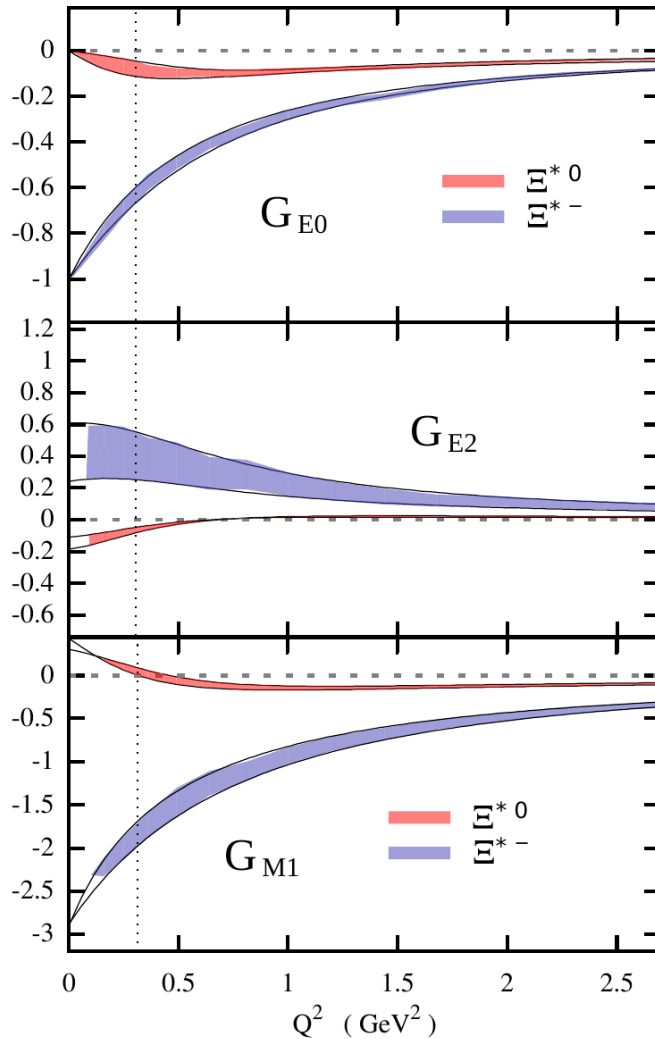
HSA, Fischer Eur.Phys.J. A52 (2016) no.2, 34



- No data at all, lattice or experiment.
- Claim: our calculation gives a **qualitative description of Hyperon FFs at low  $Q^2$  that becomes a quantitative prediction at high  $Q^2$ .**
- Some things to note:
  - FFs for  $\Sigma^{*0}$  not vanishing (they are for  $\Delta^0$ )
  - Zero-crossing for  $G_{M1}$  in  $\Sigma^{*0}$ : oblate  $\rightarrow$  prolate

# Decuplet electromagnetic FFs. $\Xi^*$

HSA, Fischer Eur.Phys.J. A52 (2016) no.2, 34



- Some things to note:
  - FFs for  $\Xi^{*0}$  not vanishing (they are for  $\Delta^0$ )
  - Zero-crossing for  $G_{M1}$  in  $\Xi^{*0}$ : oblate prolate

**Decuplet-Octet transition FFs:**

HSA, Alkofer, Fischer arXiv:1707.08463 [hep-ph]

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# Selected RL results. Baryon structure

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Take-away message

- At the present stage, gives a **qualitative description of baryon FFs at low  $Q^2$  that becomes a quantitative prediction at high  $Q^2$ .**
- Qualitative features can (most probably) be taken seriously, even at the present level of truncation (more on this later)
- **For quantitative predictions**, we have to wait at least until **pion effects** have been included (technically hard, but possible: **HSA, Fischer Phys.Lett.B733 (2014)**; **Eichmann,Fischer,Kubrak,Williams in preparation**)

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# Future

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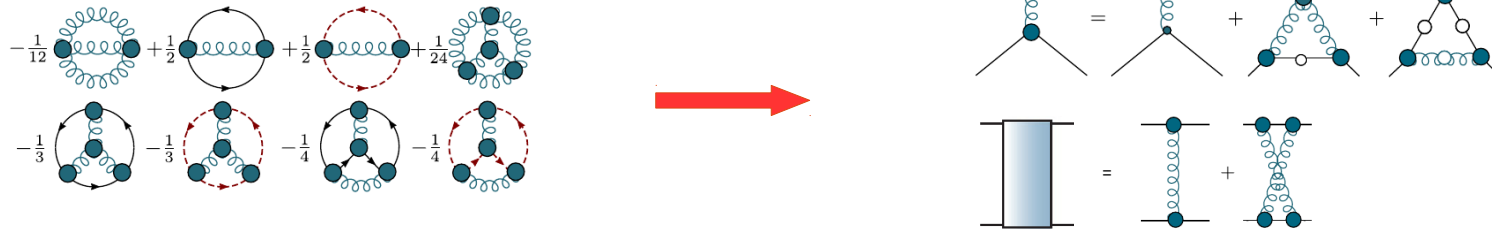
## A Glimpse into the Future

# Truncations

- A more systematic way of defining truncations is using effective action or **nPI techniques** (see, e.g. Berges et al. Phys. Rep. 363 (2002) 223–386)
- $\Gamma_{nPI}(\Phi, D, V, \dots)$  is a generating functional for all the Green's functions of the theory (QCD), where the Green's functions up to order n are considered independent.

$$\left. \frac{\delta \Gamma_{nPI}}{\delta \tilde{\phi}} \right|_{\tilde{\phi}=0} = \left. \frac{\delta \Gamma_{nPI}}{\delta D} \right|_{D=\bar{D}} = \left. \frac{\delta \Gamma_{nPI}}{\delta U} \right|_{U=\bar{U}} = \dots = 0 \quad \sim > \quad \text{DSEs} \quad ;$$

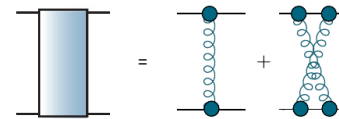
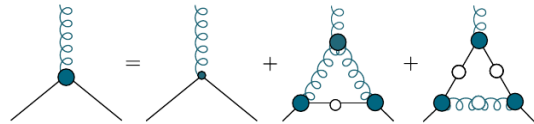
- A **loop expansion** of  $\Gamma_{nPI}$  is possible (it is an expansion in Planck's const.)
  - Interacting part to three-loop order



- The **expansion of the nPI action is systematic**, and it induces a **well-defined (truncated) BSE kernel** (Fukuda 1987 Prog.Theor.Phys. 78 | HSA, Williams, J.Phys.Conf.Ser.631(1) (2015) 012064 )
- Such a scheme also preserves chiral symmetry and its breaking patterns



# Truncations. 3PI masses



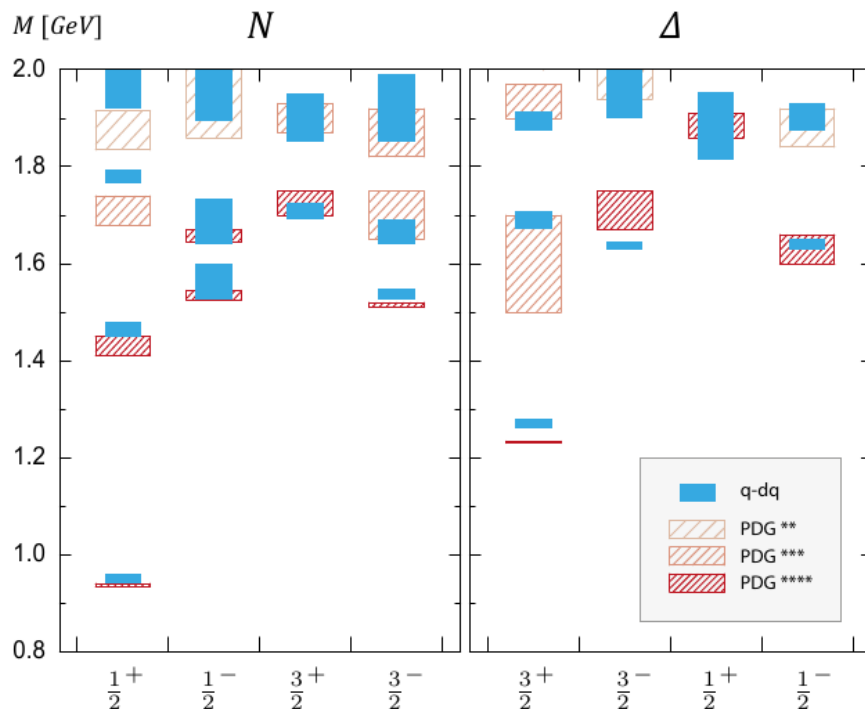
Williams, Fischer, Heupel,  
Phys.Rev. D93 (2016)

	RL	2PI-3L	3PI-3L	PDG
$0^{-+}$ ( $\pi$ )	0.14 <sup>†</sup>	0.14 <sup>†</sup>	0.14 <sup>†</sup>	0.14
$0^{++}$ ( $\sigma$ )	0.64	0.52	1.1(1)	0.48(8)
$1^{--}$ ( $\rho$ )	0.74	0.77	0.74	0.78
$1^{++}$ ( $a_1$ )	0.97	0.96	1.3(1)	1.23(4)
$1^{+-}$ ( $b_1$ )	0.85	1.1	1.3(1)	1.23

- Calculation done **without modelling!!** Propagators and vertices solved from their DSEs
- Meson spectrum in excellent agreement with experiment (scalar is expected to be heavy)
- **Baryons** in same truncation: **WIP**

# Truncations. 3PI masses

Eichmann, Fischer, HSA PRD94 (2016)



- Without a full 3PI baryon calculation, we can «mimic» the result as follows:

H. L. L. Roberts, L. Chang, I. C. Cloet, and C. D. Roberts, *Few Body Syst.* 51, 1 (2011), arXiv:1101.4244 [nucl-th].

C. D. Roberts, I. C. Cloet, L. Chang, and H. L. L. Roberts, *AIP Conf. Proc.* 1432, 309 (2012), arXiv:1108.1327 [nucl-th]

- Simplify the three-body problem to a quark-diquark problem
  - Artificially, make the pseudoscalar and vector diquarks heavier (analogous to scalar and axial-vector mesons being heavy in the 3PI truncation)
- **Baryons** spectrum shows very good agreement with experiment now, also in negative-parity channels and excited

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# Future

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- The combined DSE/BSE framework is a powerful tool to calculate hadron properties. (spacelike) Electromagnetic form factors of all baryon octet and decuplet members are calculated or underway.
- We have quantitative predictions in some  $Q^2$  regions and can make qualitative ones (e.g. shape, signs, etc.) for the rest
- Several technical and physical issues have to be tackled:

## SHORT TERM / WIP

- Baryon masses without modelling (that is, 3PI kernel)
- Other FFs (e.g. axial)

## MID TERM / MANPOWER-DEPENDENT

- Baryon FFs without modelling (that is, 3PI kernel)
- Inclusion of meson cloud (Doable. )

## LONG TERM. EXPLORATORY

- Timelike FFs