Gluon condensate and the Polyakov loop

ISMD2017, Tlaxcala

J.C. Rojas, UCN, Antofagasta, Chile and
J.P.Carlomagno, UNLP, La Plata, Argentina.

Thermodynamics

The partition function is given by

$$Z_{QCD} = \lim_{N\to\infty} \int \mathcal{D}A_\mu \mathcal{D}\psi \mathcal{D}\bar{\psi}\delta\left(G\left[A\right]\right)\det\left(D^a_\mu \frac{\partial G}{\partial A^a_\mu}\right)\mathrm{e}^{-S_G - S_F[A,\psi,\bar{\psi}]}.$$

$$S_G = \int^\beta d^4x G^a_{\mu\nu} G^{\mu\nu a},\ S_F = \int^\beta d^4x \bar{\psi}\left(i\not{D} - m\right)\psi.$$

$$\int^\beta d^4x = \int_0^\beta dx_4 \int d^3x.$$

The Euclidean gamma matrices obey

$$\gamma_4^E = i\gamma^0,\ \gamma_i^E = \gamma^i,\ \{\gamma_\mu^E, \gamma_\nu^E\} = -\delta_{\mu,\nu}$$

The free energy when a static quark is placed in a point **r**

$$e^{-\beta F_q(\mathbf{r})} = \langle \mathrm{Tr}_c L_3(\mathbf{r}) \rangle$$

$$L_3(\mathbf{r}) = \mathcal{P} \exp \left[\int_0^\beta dx_4 A_4(\mathbf{r}, x_4) \right]$$

The free energy for a static quark at point x=0 and an antiquark in **x=r** and

$$e^{-\beta F_{q\bar{q}}(\mathbf{r})} = \langle \mathrm{Tr}_c L_3^\dagger(\mathbf{r}) \mathrm{Tr}_c L_3(\mathbf{r}) \rangle$$

$$A_\mu \to A'_\mu = U(x)\left(A_\mu - \frac{1}{ig}\partial_\mu\right)U^\dagger(x),\ A'_\mu(x_4=\beta) = A'_\mu(x_4=0).$$

$$U(x_4=\beta) = z_k U(x=0),\ z_k = e^{i2\pi k/N_c},\ k = 0, 1, \cdots N_c - 1.$$

We have

$$L_3 \to L_3' = z_k L_3$$

$$\Phi_3 = \langle L_3 \rangle \ \Phi_3 = 0 \to F_q = \infty \ (\text{quark confinement})$$

In lattice field gauge theory, the partition function is formulated by link variables $U = e^{igaA_\mu(x)}$

$$Z(U_4) = \int \mathcal{D}U_i \exp\left[\frac{1}{g^2} \sum_{\mathbf{x},\mu>\nu} \mathrm{tr_c}\left(U_{\mu\nu} + U_{\mu\nu}^\dagger\right)\right],\ U_{\mu\nu} = U_\mu(x)U_\nu(x+\mu)U_\mu^\dagger(x+\mu)U_\nu^\dagger(x),$$

In order to compute an effective potential

Operator Product Expansion (OPE)

K.G. Wilson, Phys. Rev. 179, 1499 (1969)

Assuming that OPE is valid for small distances

$$i\int dx e^{iqx}\langle 0 \mid j_\Gamma(x) j_\Gamma(0) \mid 0\rangle = C_I^\Gamma + \sum_n C_n^\Gamma \langle O_n\rangle,$$

C_I^Γ, C_n^Γ Are the Wilson coefficients and O_n are local gauge invarint operators, constructed from the quark and gauge fields.

Operators

The first operators are

$$
\begin{aligned}
I & & & d = 0 \\
O_m &= m\bar{q}q & & d = 4 \\
O_G &= G^a_{\mu\nu}G^{\mu\nu a} & & d = 4 \\
O_\Gamma &= \bar{q}\Gamma_i q\bar{q}\Gamma_j q & & d = 6
\end{aligned}
$$

Gell-Man-Oakes-Renner formula

GMOR:

$$m_\pi^2 = (m_u + m_d) \mid \langle 0 \mid \bar{u}u \mid 0 \rangle \mid \frac{1}{F_\pi^2}.$$

$$\begin{aligned} \langle 0 \mid \bar{u}(x)\gamma^\mu\gamma_5 d(x) \mid \pi^- \rangle &= i\sqrt{2}F_\pi p^\mu \mathrm{e}^{-ip\cdot x} \\ \langle 0 \mid \bar{u}(x)\gamma_5 d(x) \mid \pi^- \rangle &= \sqrt{2}G_\pi \mathrm{e}^{-ip\cdot x} \end{aligned}$$

Gluon condensate

gluonic condensate is proportional to the vacuum mean value of the trace of the energy-momentum tensor $\theta_{\mu\nu}$.

$$\left.\begin{array}{ccc} \mathrm{x} & \to & \mathrm{e}^{\lambda} x \\ \mathrm{s}^{\mu} & = & \mathrm{x}_{\nu}\theta^{\mu\nu} \\ \partial_{\mu} s^{\mu} & = & \theta^{\mu}_{\mu} \end{array}\right\} \quad \theta^{\mu}_{\mu} = \frac{\tilde{\beta}(g)}{2g} G^{a}_{\mu\nu} G^{\mu\nu a} + \sum_{l=u,d,s} m_l \bar{q}_l q_l .$$

$$\beta(g) = -b\frac{g^3}{16\pi^2} + \cdots \, , \; b = 9 - \frac{2}{3} n_h .$$

Shifman, Vainshtein, and Zakharov determined its value from the sum rule for Charmonium:

$$\langle \frac{\alpha_s}{\pi} G^a_{\mu\nu} G^{\mu\nu a} \rangle = 0.012 \; GeV^4$$

There are also computations in the Lattice

D'Elia, M.; di Giacomo, A.; Meggiolaro, E. Phys Rev D67, 114504(2003)

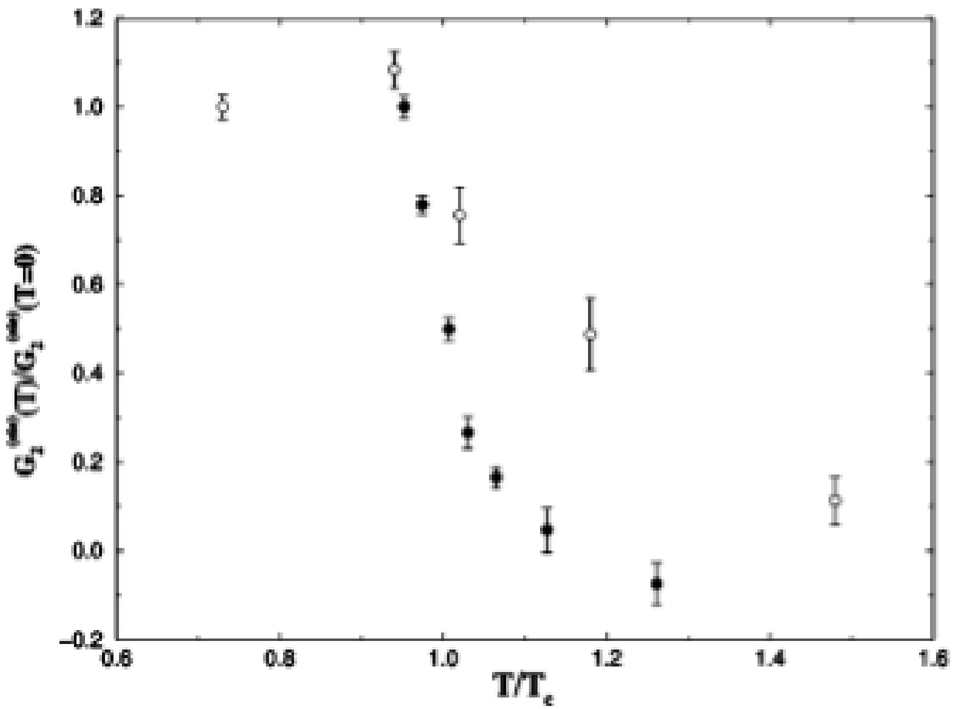

FIG. 10. The electric gluon condensate $G_2^{(ele)}(T)$ [see Eqs. (4.6) and (4.7)], in units of $G_2^{(ele)}(T=0)$, versus T/T_c. The notation is the same as in Fig. 9.

Magnetic contribution

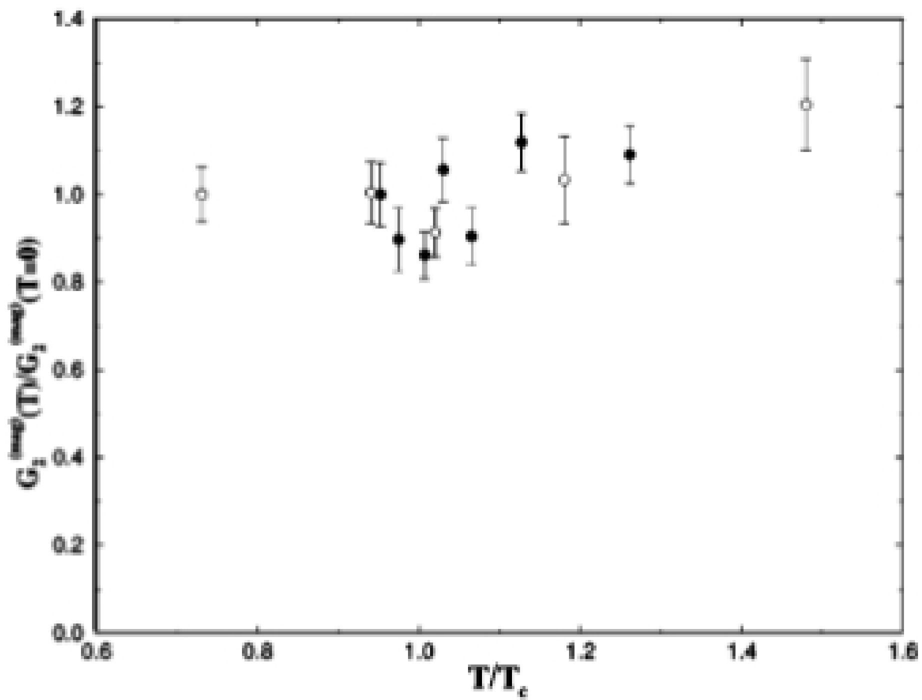

FIG. 9. The magnetic gluon condensate $G_2^{(mag)}(T)$ [see Eqs. (4.6) and (4.7)], in units of $G_2^{(mag)}(T=0)$, versus T/T_c. The black circles refer to the *quenched* case, while the white circles refer to the full-QCD case.

Gluon condensate and the Polyakov loop

In the pure Yang-Mills theory the center symmetry plays a crucial part in the description of the confinement-deconfinement phase transition. The order parameter for the latter is the trace of the Polyakov line.

$$\Phi = \frac{1}{N_c} \mathcal{P} \exp\left(i \int dx_4 A_4 \right) .$$

The Polyakov line is not invariant under gauge transformations belonging to the gauge group center. Then, if $\Phi = 0$, the Z(N) symmetry is manifest, this situation describes confinement.

For our explicit calculations we shall use the freedom to rotate the field to a diagonal A_4 form and consider it to be static.

We will work in the so-called Polyakov gauge, in which the matrix $\Phi = \phi_3\lambda_3 + \phi_8\lambda_8$ is diagonal.

Assuming that $\phi_3 \text{ and } \phi_8$ are real, we have

$$\Phi = \frac{1}{3}\left[1 + 2\cos\left(\phi_3/T\right)\right]$$

Potentials

Parametrization from lattice

$$\frac{\mathcal{U}_{\log}(\Phi,T)}{T^4} = \frac{1}{2}\,a(T)\,\Phi^2 \;+\; b(T)\,\log\big(1-6\,\Phi^2+8\,\Phi^3-3\,\Phi^4\big)$$

$$a(T) = a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2 \quad ,$$

$$b(T) = b_3\left(\frac{T_0}{T}\right)^3 \quad .$$

$$a_0 = 3.51\ , \quad a_1 = -2.47\ , \quad a_2 = 15.2\ , \quad b_3 = -1.75\ .$$

S. Roessner, C. Ratti and W. Weise, Phys. Rev. D75, 034007 (2007)

Polynomial potential

$$\frac{\mathcal{U}_{\rm poly}(\Phi,T)}{T^4} \;=\; -\,\frac{b_2(T)}{2}\,\Phi^2 - \,\frac{b_3}{3}\,\Phi^3 + \,\frac{b_4}{4}\,\Phi^4\ ,$$

$$b_2(T) = a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2 + a_3\left(\frac{T_0}{T}\right)^3\ ,$$

$$a_0 = 6.75\ , \quad a_1 = -1.95\ , \quad a_2 = 2.625\ , \quad a_3 = -7.44\ ,$$
$$b_3 = 0.75\ , \quad b_4 = 7.5\ .$$

- Based on a Ginzburg-Landau Ansatz
  O. Scavenius, A. Dumitru and J.T. Lenaghan, Phys.Rev. C66, 034903 (2002)

- Fukushima Potential

$$\mathcal{U}_{\rm Fuku}(\Phi, T) \quad = \quad -\, b\, T \left[\, 54 \, \exp(-a/T)\, \Phi^2 + \right.$$
$$\left. \log\left(1 - 6\, \Phi^2 + 8\, \Phi^3 - 3\, \Phi^4\right) \,\right] \ .$$

Inspired by strong coupling analysis of QCD.

The parameter a determines the deconfinement phase transition, And b parametrizes the relative strength of mixing between the chiral and deconfinement phase transition.

If we consider a pure gauge system

$$\begin{aligned} \mathcal{L} &= -\frac{1}{4g_s^2} G^a_{\mu\nu} G^{a\ \mu\nu} \\ G^a_{\mu\nu} &= \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - f^a_{bc} A^b_\mu A^c_\nu \end{aligned}$$

The idea is to estimate the temperature dependence of the gluon condensate throuh the thermodynamical potential. Following Agasian

$$\begin{aligned} \mathcal{Z}_T &= \int \mathcal{D}\tilde{A} \exp\left\{ -\sum\!\!\!\!\!\!\int \mathcal{L}_{eff} \right\} \\ &\approx \int \mathcal{D}\tilde{A} \exp\left\{ \frac{-1}{16\pi\alpha_s(T)} \sum\!\!\!\!\!\!\int \; G^a_{\mu\nu} G^{a\ \mu\nu} \right\} . \end{aligned}$$

$$\mathcal{U}_T = -T \ln \mathcal{Z}_T$$

- We have: $$\langle G^a_{\mu\nu} G^{a\ \mu\nu} \rangle_T = 16\pi \frac{\partial\, \mathcal{U}_T}{\partial \alpha_s^{-1}}\ .$$

Where the temperature dependent part comes from the effective potential.

$$\langle G^a_{\mu\nu} G^{a\ \mu\nu} \rangle = \langle G^a_{\mu\nu} G^{a\ \mu\nu} \rangle_0 + \langle G^a_{\mu\nu} G^{a\ \mu\nu} \rangle_T,$$

From the standard decomposition

$$\langle G^a_{\mu\nu} G^{a\ \mu\nu} \rangle = \langle G^a_{\mu\nu} G^{a\ \mu\nu} \rangle_E + \langle G^a_{\mu\nu} G^{a\ \mu\nu} \rangle_M,$$

Lattice suggests

$$\langle G^a_{\mu\nu} G^{a\ \mu\nu} \rangle_{T,M} \approx 0.$$

Massimo D'Elia, A. Di Giacomo, E. Meggiolaro, Phys.Rev. **D**67 (2003) 114504.

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where

$$E_i^a = G_{i4}^a,\ B_k^a = \frac{1}{2}\varepsilon_{ijk}G_{jk}^a,$$

We conclude that

$$\langle\frac{\alpha_s}{\pi}G^a_{\mu\nu}G^{a\ \mu\nu}\rangle_E \equiv \langle\frac{\alpha_s}{\pi}G^2\rangle_E = \langle\frac{\alpha_s}{\pi}G^2\rangle_{0,E} + \langle\frac{\alpha_s}{\pi}G^2\rangle_{T,E},$$

$$\langle\frac{\alpha_s}{\pi}G^2\rangle_{T,E} = -\frac{4}{\pi}\alpha_s(T)^2\left(\frac{\partial\alpha_s(T)}{\partial T}\right)^{-1}\frac{\partial\,\mathcal{U}(\Phi,T)}{\partial T},$$

$$\langle\frac{\alpha_s}{\pi}G^2\rangle_{T,E} = \frac{\alpha(T)_s}{\pi}\langle G^2\rangle_{T,E}.$$

We assume that the temperature is the energy scale of the system.

Coupling behaviour

A. Deur, S. J. Brodsky and G. F. de Teramond, Prog. Part. Nucl. Phys. 90, 1 (2016)

We make the guess

$$\alpha_s^{(1)}(T) = \frac{1}{1.16 + T^4/\Lambda_1^4}\left[3.49\left(1.16 - 0.07\left(\frac{T^2}{\Lambda_1^2}\right)^{\frac{2}{3}}\right) + \left(\frac{T^2}{\Lambda_1^2} + 2\right)\frac{T^4}{\Lambda_1^4}\alpha_s^{\beta_1}\right] ,$$

$$\alpha_s^{\beta_1} = \frac{4\pi}{11\ln(T^2/\Lambda_1^2)}\left[1 - \frac{102}{11^2}\frac{\ln(T^2/\Lambda_1^2)}{T^2/\Lambda_1^2}\right]$$

Where $\Lambda_1 = 0.856$ GeV.

This behavior, originally came from Dyson Schwinger approach.

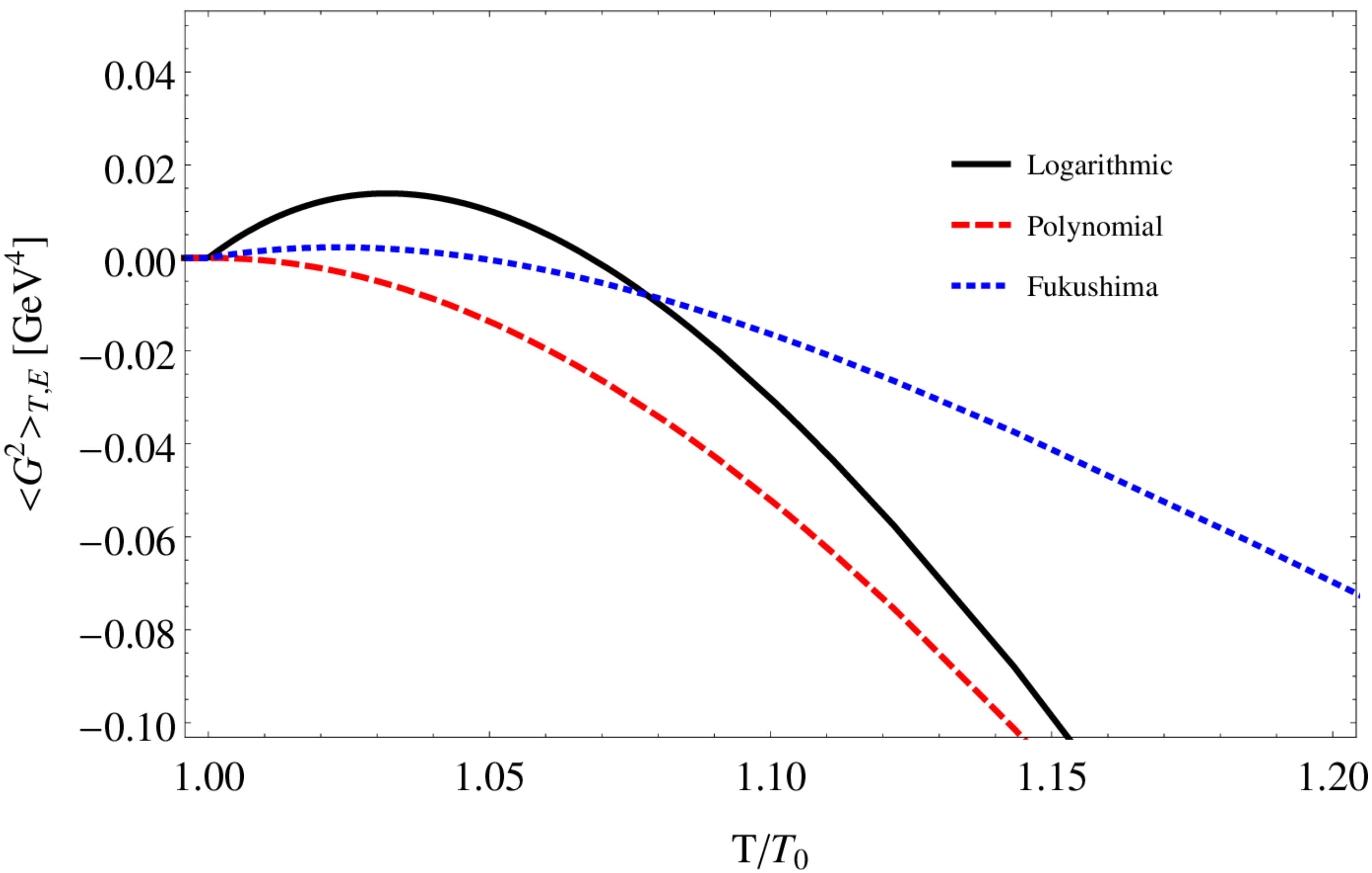
- We try with another coupling

$$\alpha_s^{(2)}(T) \;= \frac{1}{15+T^2/\Lambda_2^2}\left[15\times 2.6\;+\frac{4\pi}{11}\left(\frac{1}{\ln(T^2/\Lambda_2^2)}-\frac{1}{T^2/\Lambda_2^2-1}\right)\frac{T^2}{\Lambda_2^2}\right].$$

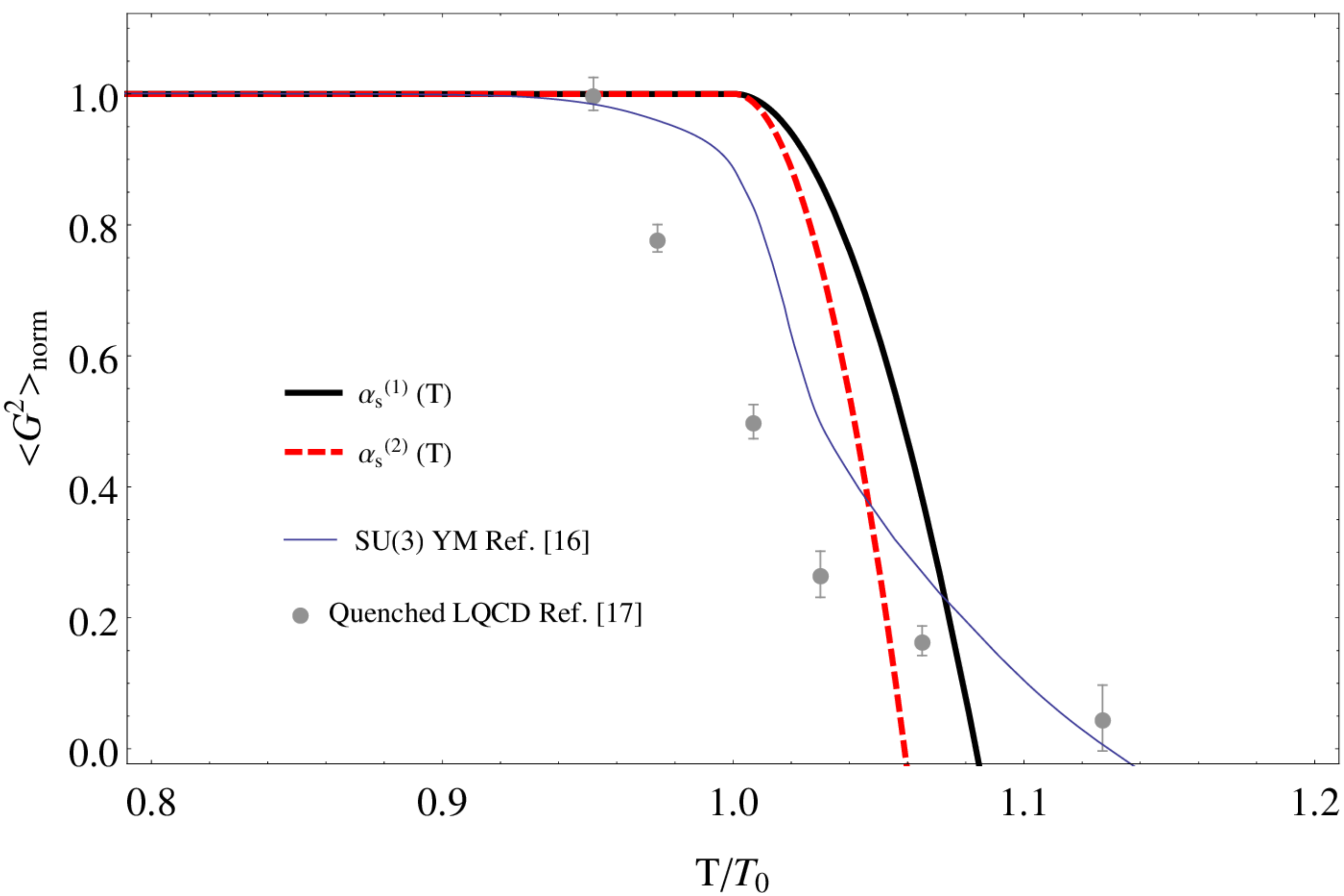
With $\Lambda_2 = 0.33$ GeV.

This behavior, originally came from another Improved Dyson Schwinger approach.

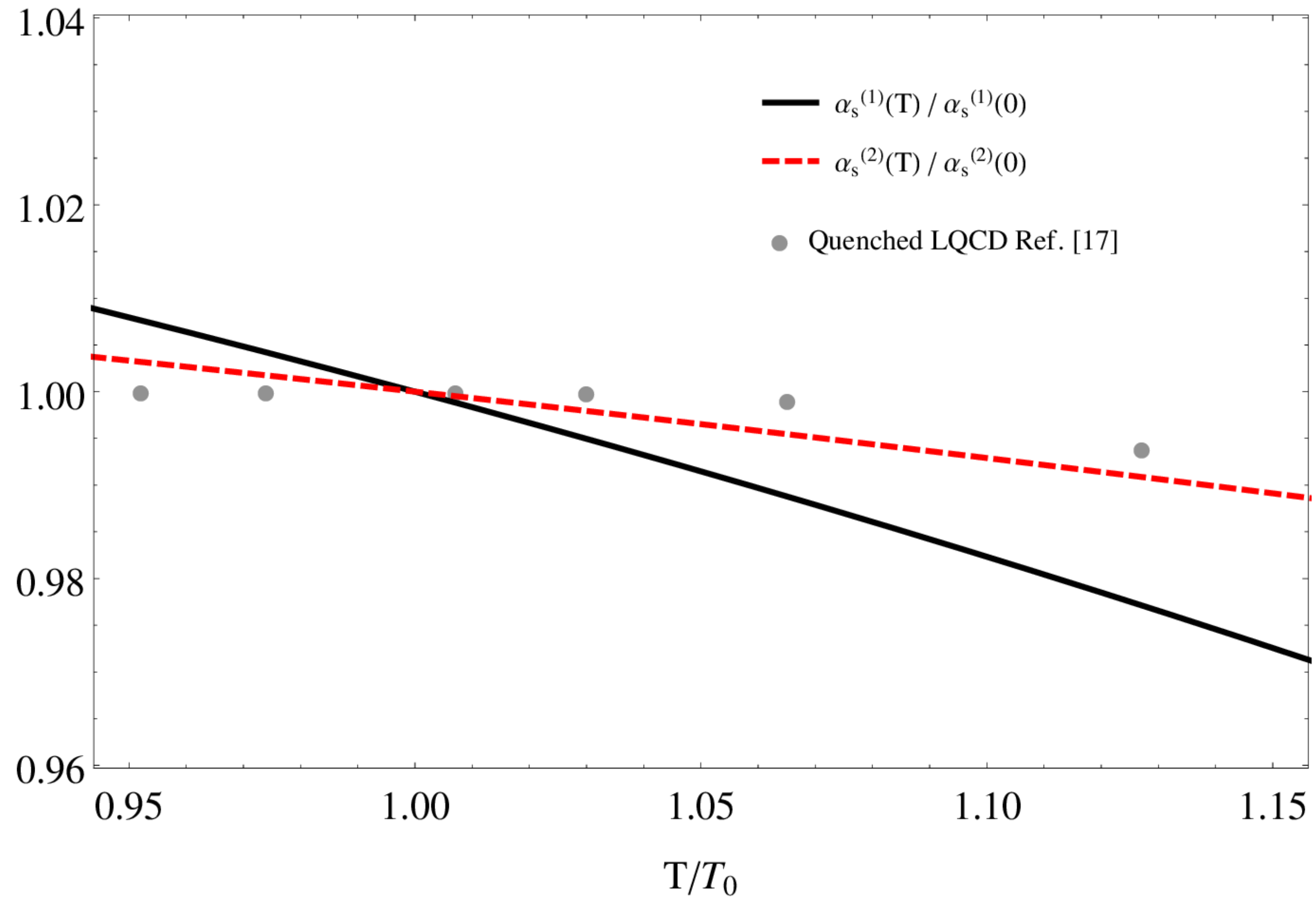
Results



0.04
0.02
0.00
−0.02
−0.04
−0.06
−0.08
−0.10
$<G^2>_{T,E}$ [GeV^4]
1.00
1.05
1.10
1.15
1.20
T/T_0
Logarithmic
Polynomial
Fukushima



1.0
0.8
0.6
0.4
0.2
0.0
0.8
0.9
1.0
1.1
1.2
$\langle G^2 \rangle_{\mathrm{norm}}$
T/T_0
$\alpha_s^{(1)}$ (T)
$\alpha_s^{(2)}$ (T)
SU(3) YM Ref. [16]
Quenched LQCD Ref. [17]



1.04
1.02
1.00
0.98
0.96
0.95
1.00
1.05
1.10
1.15
T/T_0
$\alpha_s^{(1)}$(T) / $\alpha_s^{(1)}$(0)
$\alpha_s^{(2)}$(T) / $\alpha_s^{(2)}$(0)
Quenched LQCD Ref. [17]

Conclusions

- The Polyakov Loop effective potential depends consistently with the gluon condensate.
- We showed that the polynomial effective potential has the best thermal behavior, while the qualitative dependence for both of the strong coupling parametrizations is equivalent.
- We have an accurate and simple picture of the electric gluon condensate around deconfinement critical temperature.

And Thanks!