Non-trivial thermomagnetic behavior of coupling, masses and correlation lengths in the NJL model

Alfredo Raya
(IFM-UMSNH, México)

A. Ayala, C.A. Dominguez, L.A. Hernández, M. Loewe, J.C. Rojas, C. Villavicencio, R. Zamora

XLVII ISMD, Tlaxcala 2017

Contents

- Motivation

Contents

- Motivation
- Gap equation in the NJL model

Contents

- Motivation
- Gap equation in the NJL model
- Dynamical mass and coupling

Contents

- Motivation
- Gap equation in the NJL model
- Dynamical mass and coupling
- Pressure

Contents

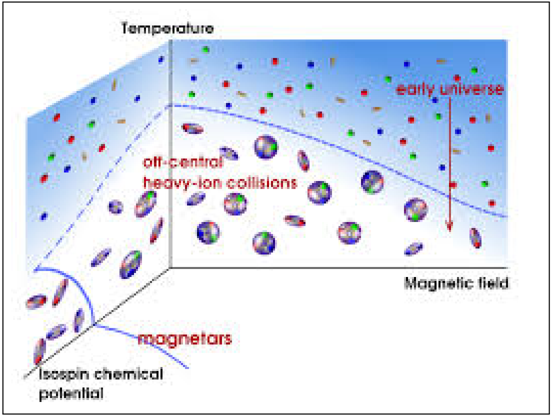
- Motivation
- Gap equation in the NJL model
- Dynamical mass and coupling
- Pressure
- Correlation lengths

Contents

- Motivation
- Gap equation in the NJL model
- Dynamical mass and coupling
- Pressure
- Correlation lengths
- Final remarks

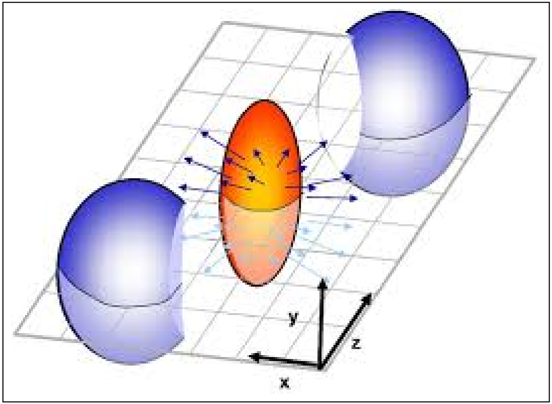
Motivation

- (Magnetized) QCD phase diagram



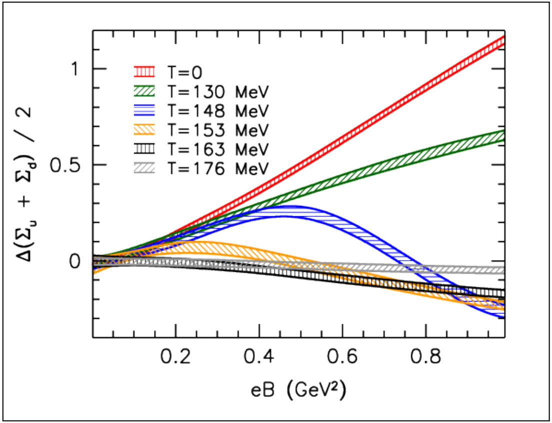
Motivation

- Peripheral collisions



Motivation

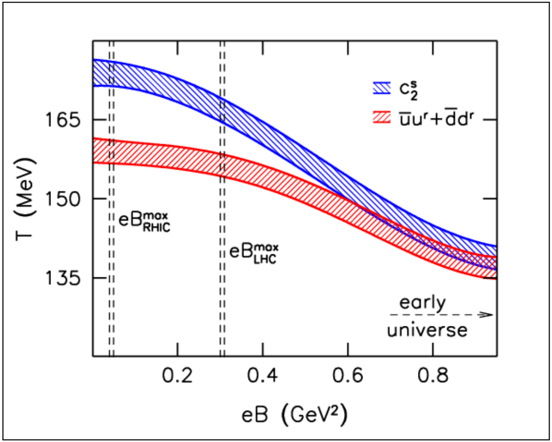
- Inverse magnetic catalysis[1]



[1] Adapted from PRD**86** 071502 (2012)

Motivation

- Inverse magnetic catalysis[2]

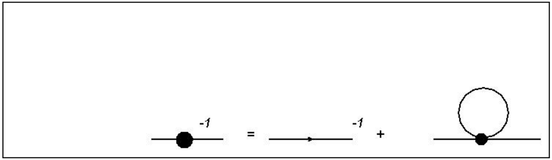


[2] Adapted from PRD**86** 071502 (2012)

Gap equation in the NJL model

- Lagrangian

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m_0)\psi + G\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\tau\psi)^2\right]$$



- Gap equation

$$M = m - 2\langle\bar{\psi}\psi\rangle$$

Gap equation in the NJL model

- Chiral condensate

$$-\langle\bar{\psi}\psi\rangle = \int \frac{d^4k}{(2\pi)^4}\mathrm{Tr}[iS(k)].$$

- Critical coupling $G > G_c$

- Need for a regulator

Dynamical mass and coupling

- We include the magnetic field effects from the Schwinger representation of the fermion propagator

$$iS(k) = \int_{s_0}^{\infty} \frac{ds}{\cos(qBs)} e^{is\left(k_{\parallel}^2 - k_{\perp}^2 \frac{\tan(qBs)}{qBs} - M^2 + i\epsilon\right)} \times \left[(\cos(qBs) + \gamma_1\gamma_2 \sin(qBs))(M + \not{k}_{\parallel}) - \frac{\not{k}_{\perp}}{\cos(qBs)} \right]$$

- Thermal effects are considered within the Matsubara formalism

$$\int \frac{d^2 k_{\parallel}}{(2\pi)^2} \to T \sum_{n=-\infty}^{\infty} \int \frac{dk_3}{2\pi}$$

Dynamical mass and coupling

- A straightforward calculation reveals

$$-\langle\bar{\psi}\psi\rangle = \frac{N_c M}{4\pi^2}\frac{1}{2}\sum_f q_f B \int_{\tau_0}^{\infty} \frac{d\tau}{\tau \tanh(q_f B\tau)} e^{-\tau M^2} \vartheta_3\left(\frac{1}{2}, \frac{i}{4\pi\tau T^2}\right)$$

- To isolate vacuum contribution, we write

$$\vartheta_3\left(\frac{1}{2}, \frac{i}{4\pi\tau T^2}\right) = 1 + 2\sum_{n=1}^{\infty}(-1)^n \exp\left(-\frac{n^2}{4\tau T^2}\right)$$

- Thus

$$-\langle\bar{\psi}\psi\rangle = -\langle\bar{\psi}\psi\rangle_0 - \langle\bar{\psi}\psi\rangle_{B,T};$$

Dynamical mass and coupling

- Vacuum part

$$-\langle\bar{\psi}\psi\rangle_0 = \frac{N_c M_0}{4\pi^2}\int_{\tau_0}^{\infty}\frac{d\tau}{\tau^2}e^{-\tau M_0^2}$$

- Thermomagnetic contribution

$$\begin{aligned}-\langle\bar{\psi}\psi\rangle_{B,T} \quad &= \quad \frac{N_c M}{4\pi^2}\frac{1}{2}\sum_f\left\{\int_0^{\infty}\frac{d\tau}{\tau^2}e^{-\tau M^2}\left[\frac{q_f B\tau}{\tanh(q_f B\tau)}-1\right]\right. \\ &\left. +2q_f B\sum_{n=1}^{\infty}\int_0^{\infty}d\tau\frac{e^{-\tau M^2}e^{-\frac{n^2}{4\tau T^2}}}{\tau\tanh(q_f B\tau)}\right\}\end{aligned}$$

Dynamical mass and coupling

- At $B = 0$,

$$-\langle\bar{\psi}\psi\rangle_{0,T} = \frac{2N_c M}{\pi^2}\int_0^\infty dk \frac{k^2}{\sqrt{k^2+M^2}}\frac{1}{e^{\sqrt{k^2+M^2}}+1}$$

- Critical Temperature

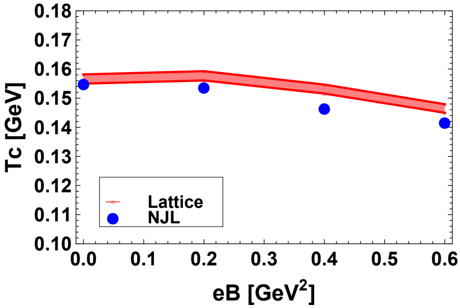
$$-\frac{\partial}{\partial T}\langle\bar{\psi}\psi\rangle_{0,T} = 0$$

τ_0	$-\langle\bar{\psi}\psi\rangle_0^{1/3}$	M_0	G_0	m_0	T_c^{NJL}
1.27	0.220	0.224	5.08	0.00758	0.267
0.74	0.260	0.192	2.66	0.00465	0.228

Dynamical mass and coupling

Phase diagram [3]

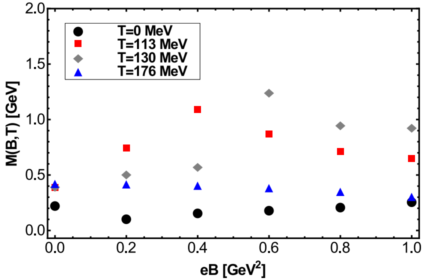
$$T^{NJL} = \left(\frac{T_c^{NJL}}{T_c} \right) T$$

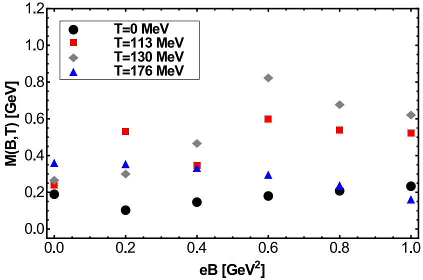


[3] A. Ayala *et al.* PRD**96** 034007 (2017)

Dynamical mass and coupling

Dynamical mass [4]

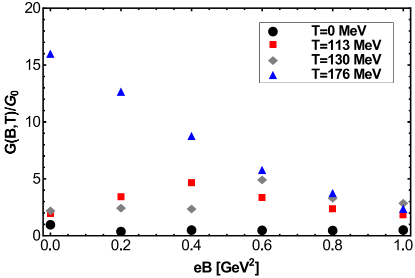


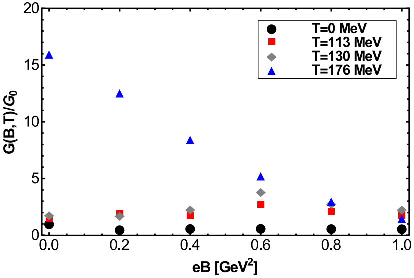


[4] A. Ayala *et al.* PRD**94** 054019 (2016)

Dynamical mass and coupling

Coupling [5]





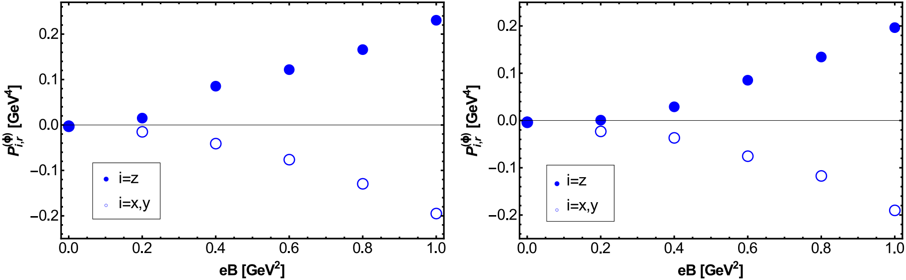
[5] A. Ayala *et al.* PRD**94** 054019 (2016)

Pressure

$$
\begin{aligned}
P_z &= -V^{\mathrm{eff}} \\
&= -\frac{(M-m)}{4G} - \frac{i}{2}\sum_f \mathrm{Tr}\int \frac{d^4p}{(2\pi)^4}\ln(S_f^{-1}(p)) \\
P_{x,y} &= P_z + e\vec{B}\cdot\vec{\mathcal{M}} \\
\vec{\mathcal{M}} &= -\frac{\partial^{\mathrm{eff}}}{\partial(eB)}\hat{z}
\end{aligned}
$$

Pressure

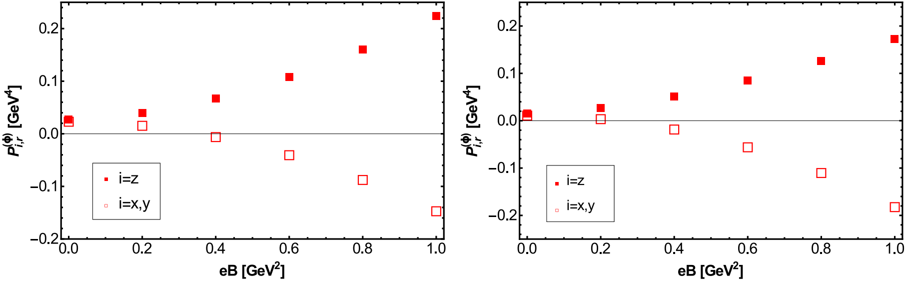
Longitudinal and transverse pressures [6]



[6] A. Ayala *et al.* PRD**94** 054019 (2016)

Pressure

Longitudinal and transverse pressures [7]



[7] A. Ayala *et al.* PRD**94** 054019 (2016)

Correlation Lengths

The probability amplitude to place a test quark $(b,\ q_f,\ \alpha)$ in position $\vec{x}'$ in a state that already contains a particle with the same $(b,\ q_F,\ \alpha)$ in $\vec{x}$ [8]

$$\begin{aligned}\vec{A}(\vec{x},\vec{x}') &= \langle\psi_a(\vec{x})\psi_b(\vec{x}')\bar{\psi}_b(\vec{x}')\bar{\psi}_a(\vec{x})\rangle \\ &= \frac{1}{2}\Big[\langle\psi_a(\vec{x})\bar{\psi}_a(\vec{x})\rangle\langle\psi_b(\vec{x}')\bar{\psi}_b(\vec{x}')\rangle \\ &\quad -\langle\psi_a(\vec{x})\bar{\psi}_b(\vec{x}')\rangle\langle\psi_b(\vec{x}')\bar{\psi}_a(\vec{x})\rangle\Big] \\ &= \frac{1}{2}\Big[(\mathrm{Tr}[S(0)])^2-(\mathrm{Tr}[S(\vec{x}'-\vec{x})])\Big]\end{aligned}$$

Correlation function

$$C(\vec{x}-\vec{x}')=1-\frac{(\mathrm{Tr}[S(\vec{x}'-\vec{x})])^2}{(\mathrm{Tr}[S(0)])^2}$$

[8] A. Ayala *et al.* PRD**94** 034007 (2017)

Correlation Lengths

In coordinate space

$$
iS(\vec{x}-\vec{x}') = \int \frac{d^4k}{(2\pi)^4} \int_{s_0}^{\infty} \frac{ds}{\cos(q_f Bs)} e^{is\left(k_{\parallel}^2 - k_{\perp}^2 \frac{\tan(q_f Bs)}{q_f Bs} - M^2 + i\epsilon\right)}
$$

$$
\times \left[(\cos(q_f Bs) + \gamma_1\gamma_2 \sin(q_f Bs))(M + \not{k}_{\parallel}) - \frac{\not{k}_{\perp}}{cos(q_f Bs)} \right] \Bigg|_{x_0 = x_0'}
$$

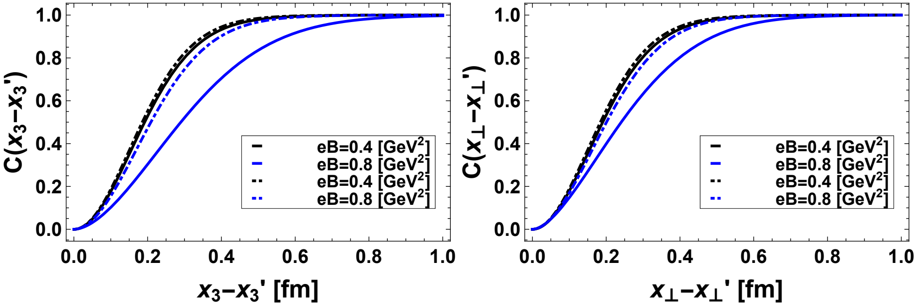
Correlation Lengths

Thus, with $X_3 = x_3 - x_3'$ and $X_\perp = x_\perp - x_\perp'$,

$$
\begin{aligned}
\mathrm{Tr}[S(\vec{x} - \vec{x}')]_{B,T} = \; & \frac{N_c M}{4\pi^2} \frac{1}{2} \sum_f \Bigg\{ \int_0^\infty \frac{d\tau}{\tau^2} \left[\frac{q_f B\tau}{\tanh(q_f B\tau)} - 1 \right] \\
& e^{-\tau M^2 - \frac{X_3^2}{4\tau} - \frac{q_f B X_\perp^2}{4\tanh(q_f B\tau)}} \\
& + \int_0^\infty \frac{d\tau}{\tau^2} e^{-\tau M^2} \left[e^{-\frac{X_3^2}{4\tau} - \frac{q_f B X_\perp^2}{4\tanh(q_f B\tau)}} - e^{-\frac{X_3^2}{4\tau} - \frac{X_\perp^2}{4\tau}} \right] \\
& + 2 q_f B \int_0^\infty \frac{d\tau}{\tau \tanh(q_f B\tau)} e^{-\tau M^2 - \frac{X_3^2}{4\tau} - \frac{q_f B X_\perp^2}{4\tanh(q_f B\tau)}} \\
& \times \sum_{n=1}^{\infty} (-1)^n e^{-\frac{n^2}{4\tau T^2}} \Bigg\}
\end{aligned}
$$

Correlation Lengths

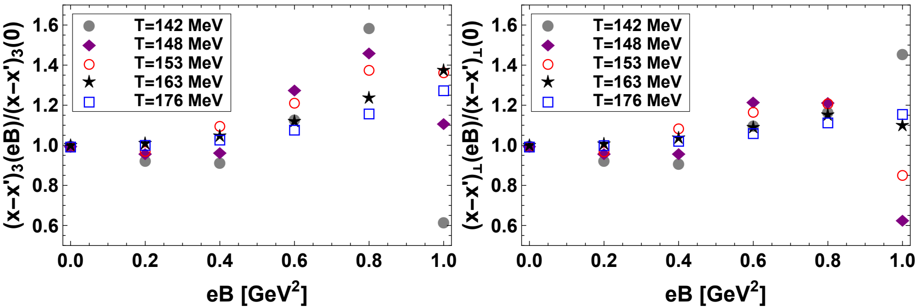
Correlation functions [9]



[9] A. Ayala *et al.* PRD**96** 054007 (2017)

Correlation Lengths

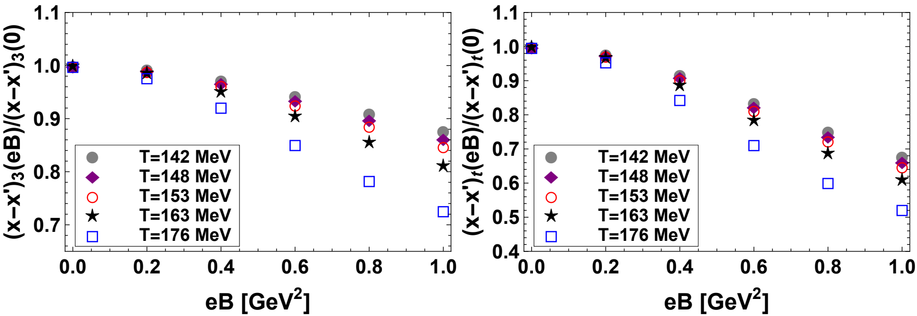
Correlation distances [10]



[10] A. Ayala *et al.* PRD**96** 054007 (2017)

Correlation Lengths

Correlation distances at fixed $M = 224\mathrm{MeV}$ [11]



[11] A. Ayala *et al.* PRD**96** 054007 (2017)

Final Remarks

- For $T < T_c$, the couplings are monotonically decreasing functions of B
- For $T \lessapprox T_c$, the couplings cease to increase and start decreasing
- Similar turn-over behavior of the dynamical masses are found
- The behavior of the transverse pressure is such that fot $T > T_c$, particles are pulled together
- In order to be consistent with lattice, from the behavior of P_z we observe that the thermomagnetic medium behaves diamagnetically
- Correlation lenghts also exhibit this behavior: grow ($T < T_c$) and decrease ($T > T_c$) as eB increases

GRACIAS