Nonperturbative Approach to Equation of State and Collective Modes of the QGP

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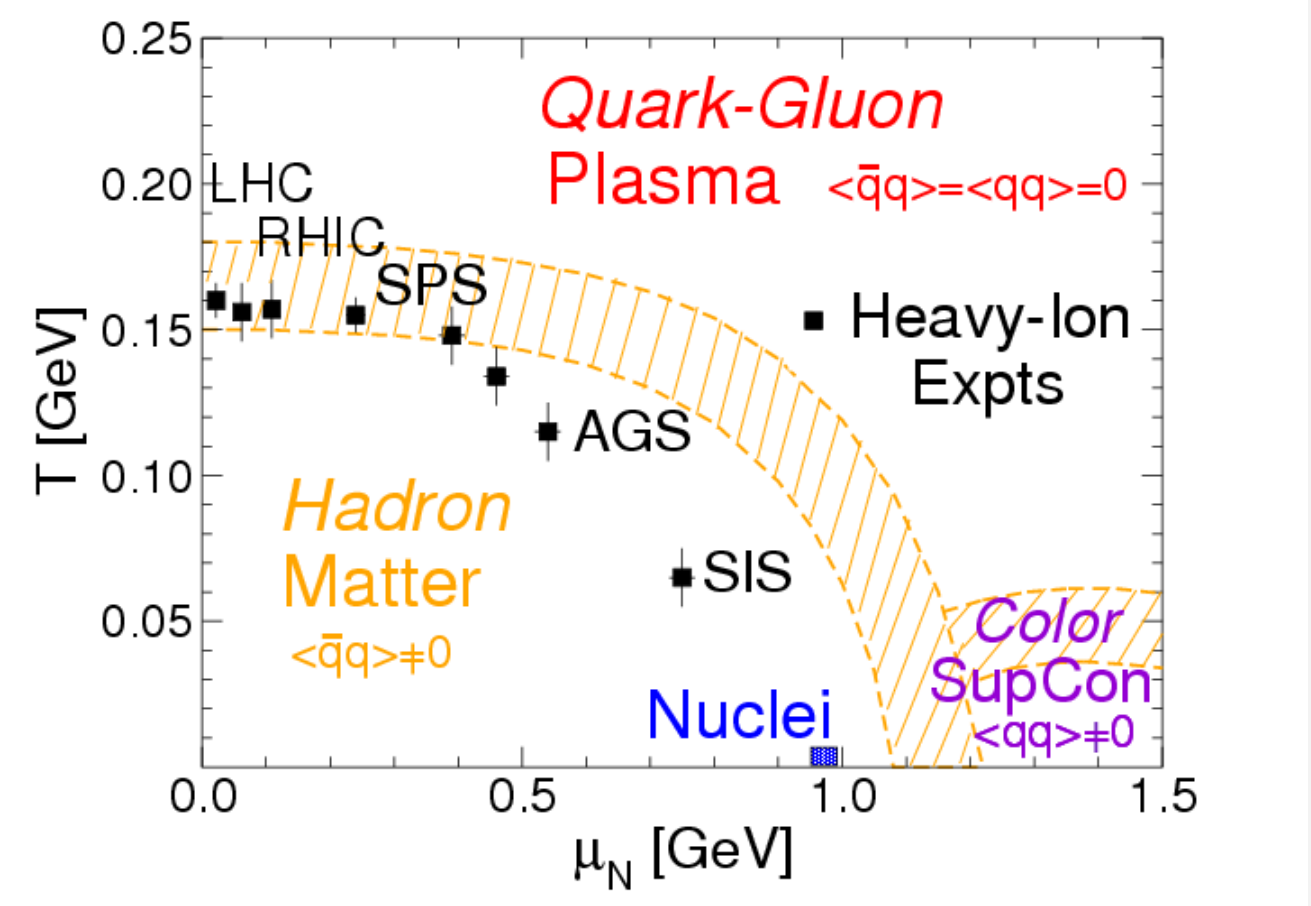
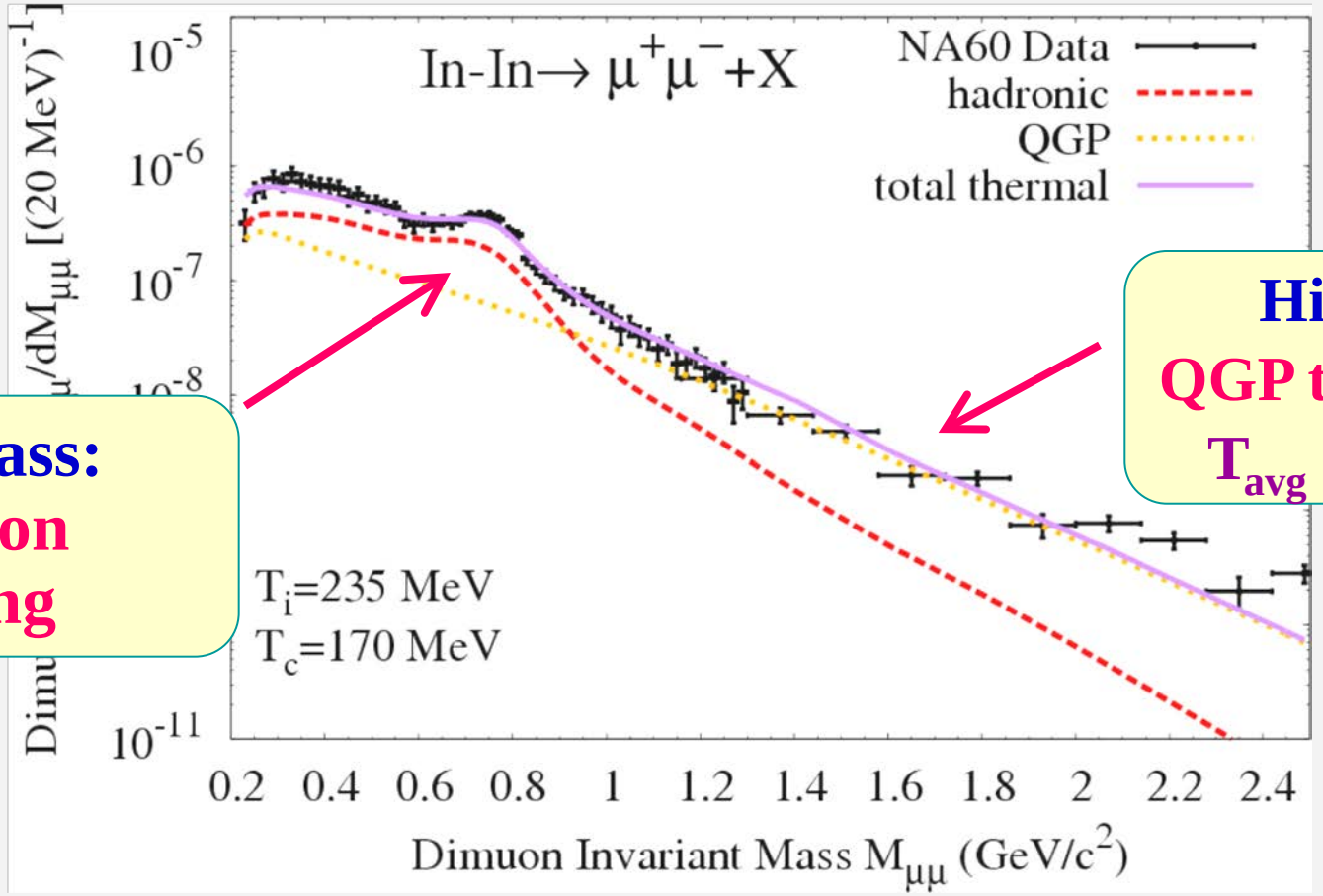
1.) Exploring QCD Matter



- **Bulk Properties:** **Equation of State, Transport Coefficients**
- **Microscopic Properties:** **Degrees of Freedom, Spectral Functions**
- **Phase Transitions:** **Condensate Structure**

1.2 Dilepton Radiation at SPS ($\sqrt{s}$=17.3 GeV)

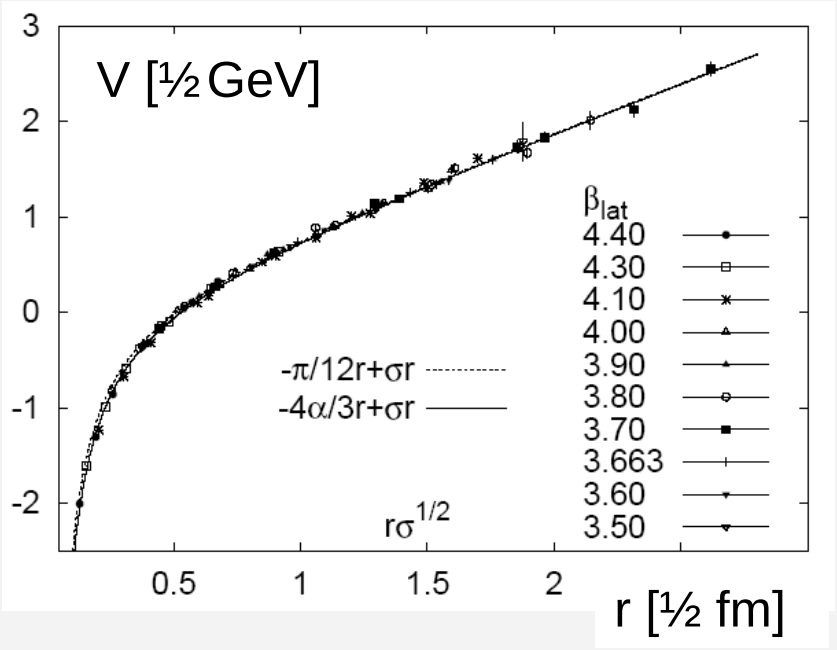


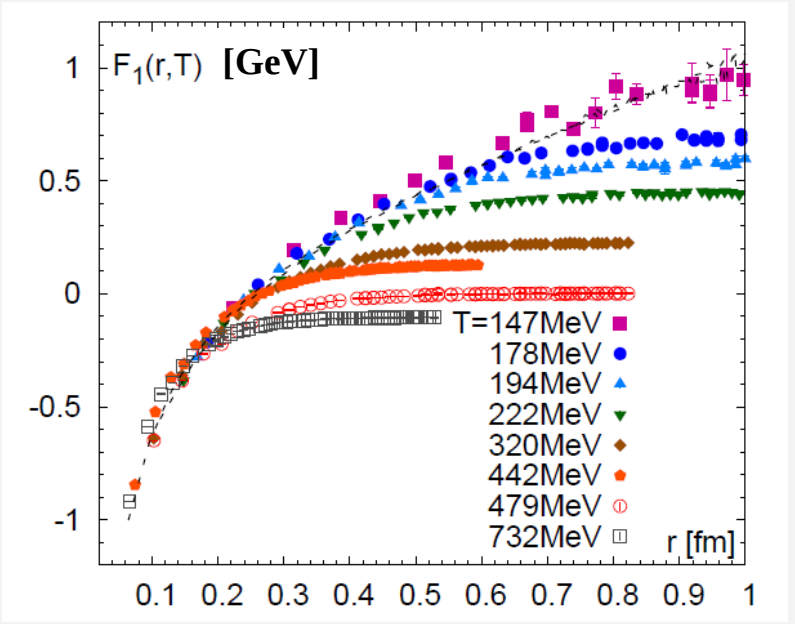
Low mass:
ρ-meson melting

High mass:
QGP thermometer
T_{avg} ~ 200 MeV

- Strong (resonance) coupling ⇒ **hadrons melt** approaching T_{pc}
- How do hadrons emerge when approaching T_c from above?
- Origin of strongly coupled QGP?

1.3 Introduction: A "Calibrated" QCD Force





[Bazavov et al '13]

- Vacuum quarkonium spectroscopy well described
- Free energy encodes medium modified potential

Objective: **Determine in-medium of QCD force + infer emerging spectral + transport properties probing QGP at varying resolution**

Outline

1.) Introduction

2.) Thermodynamic T-Matrix Approach

- Hamiltonian, In-Medium Potential + Selfconsistency

3.) Equation of State + Spectral Functions

4.) Transport Properties

5.) Conclusions

2.1 Hamiltonian Approach

- **In-Medium Hamiltonian** with ``bare" 2-body interactions

$$H = \sum \varepsilon_i(\mathbf{p})\psi_i^\dagger(\mathbf{p})\psi_i(\mathbf{p}) + \psi_i^\dagger(\frac{\mathbf{P}}{2}-\mathbf{p})\psi_j^\dagger(\frac{\mathbf{P}}{2}+\mathbf{p})V_{ij}^a\psi_j(\frac{\mathbf{P}}{2}+\mathbf{p}')\psi_i(\frac{\mathbf{P}}{2}-\mathbf{p}')$$

  - effective in-medium mass $\varepsilon_i(\mathbf{p}) = \sqrt{M_i^2+\mathbf{p}^2}$

- Interaction ansatz: **Cornell potential** with relativistic corrections

$$V_{ij}^a(\mathbf{p},\mathbf{p}') = \mathcal{R}_{ij}^{\mathcal{C}}\mathcal{F}_a^{\mathcal{C}}V_{\mathcal{C}}(\mathbf{p}-\mathbf{p}') + \mathcal{R}_{ij}^{\mathcal{S}}\mathcal{F}_a^{\mathcal{S}}V_{\mathcal{S}}(\mathbf{p}-\mathbf{p}')$$

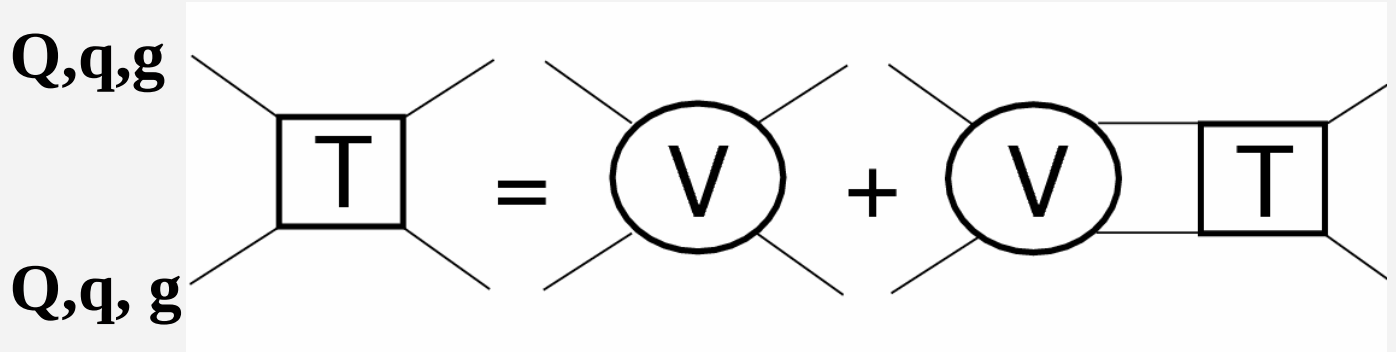
  - **color-Coulomb** and **string** (“**confining**”) interaction
  - decent spectroscopy in vacuum

[Liu+RR ‘16]

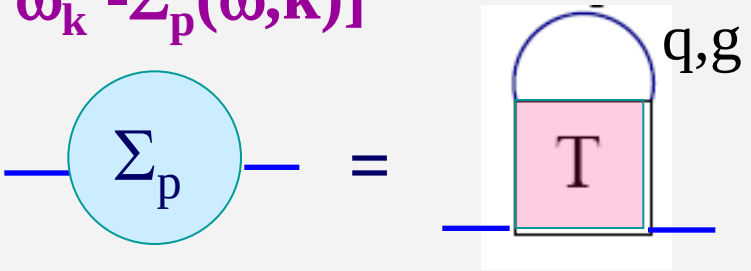
- **Implement into Brueckner / Luttinger-Ward-Baym approach**

2.2 Thermodynamic T-Matrix in QGP

- **Scattering equation**



- Perturbative approximation (**weak** coupling): $\mathbf{T_{ij} \approx V_{ij}}$
- **Strong** coupling → resummation: $\mathbf{T_{ij} = V_{ij} + \int V_{ij}\, D_1\, D_p\, T_{ij}}$
- Thermal parton propagators: $\mathbf{D_p = 1 / [\omega - \omega_k - \Sigma_p(\omega,k)]}$
- Parton self-energies → **self-consistency**:

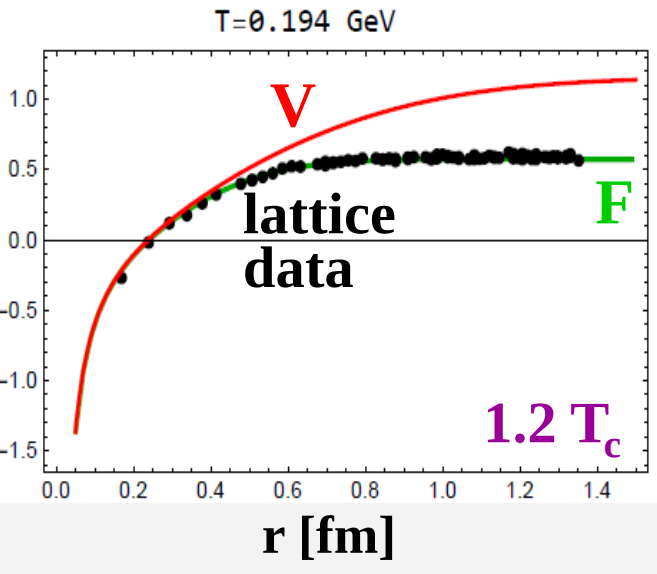


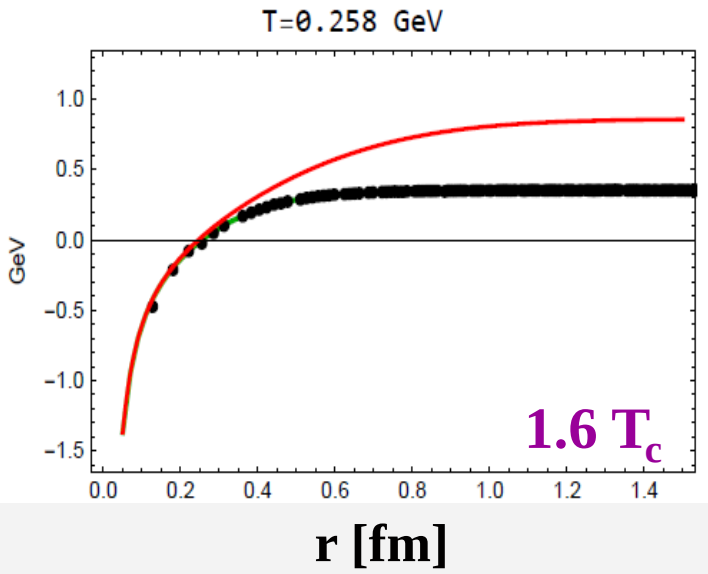
- **In-medium potential V?**

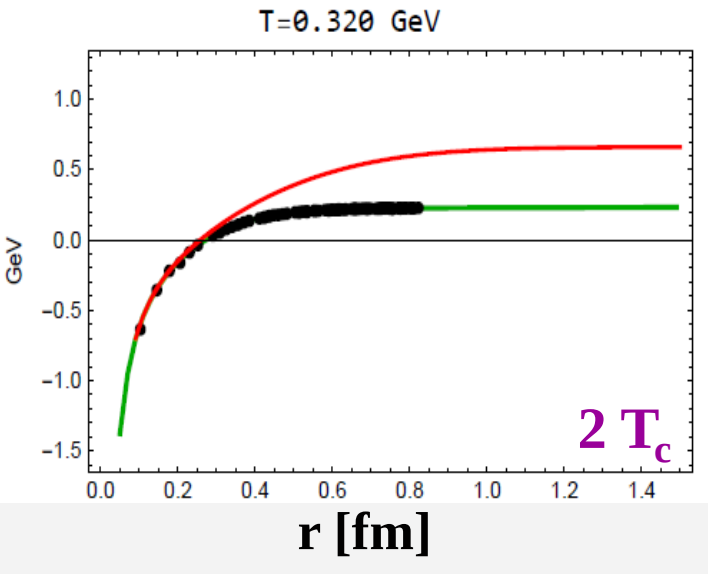
[Mannarelli+RR '05, Cabrera+RR '06, Riek+RR '10, Liu+RR '15, '16]

2.3 Potential from Heavy-Quark Free Energy

- $Q\bar{Q}$ Free Energy: $$F_{Q\bar{Q}}(r_1 - r_2) = -\frac{1}{\beta}\ln\left(\int_{-\infty}^{\infty} d\omega\sigma(\omega, r_1 - r_2)\, e^{-\beta\omega}\right)$$
- Spectral Function: $$\sigma(\omega, r) = \frac{1}{\pi}\frac{(V+\Sigma)_I(\omega)}{(\omega - (V+\Sigma)_R)^2 + (V+\Sigma)_I^2(\omega)}$$
- Potential ansatz: $$V_R(E, r) = -\frac{4}{3}\alpha_s\frac{e^{-m_D r}}{r} - \sigma\frac{e^{-m_s r}}{m_s} \quad -\frac{4}{3}\alpha_s m_D + \sigma\frac{1}{m_s}$$







- **remnant of long-range "confining" force in QGP**

[S.Liu+RR '15]

Outline

1.) Introduction

2.) Thermodynamic T-Matrix Approach

- Hamiltonian, In-Medium Potential + Selfconsistency

3.) Equation of State + Spectral Functions

4.) Transport Properties

5.) Conclusions

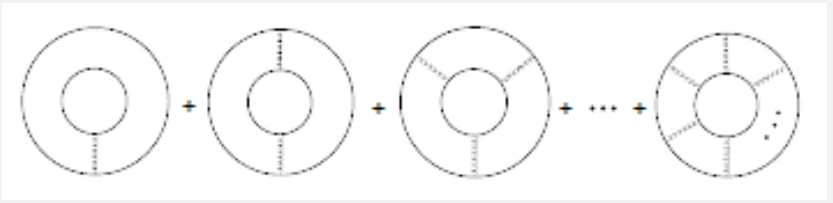
3.1 Self-Consistent Equation of State for QGP

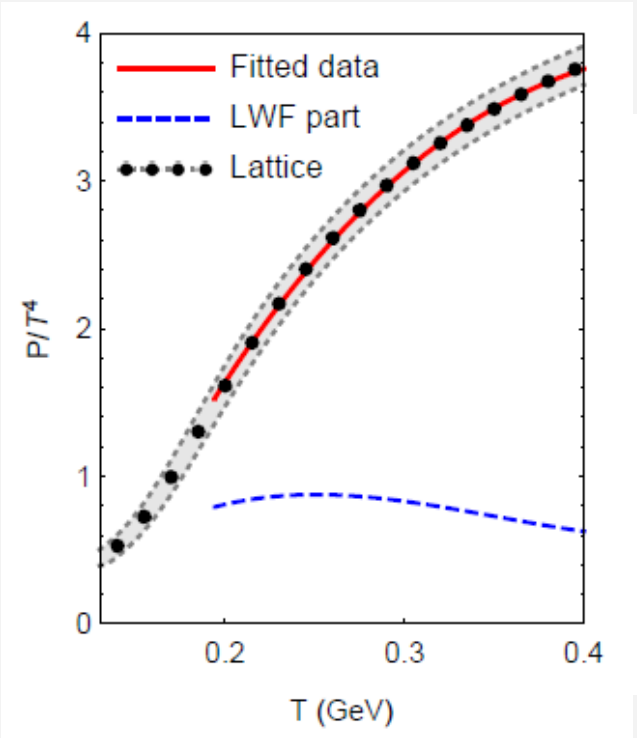
Thermodynamic Potential

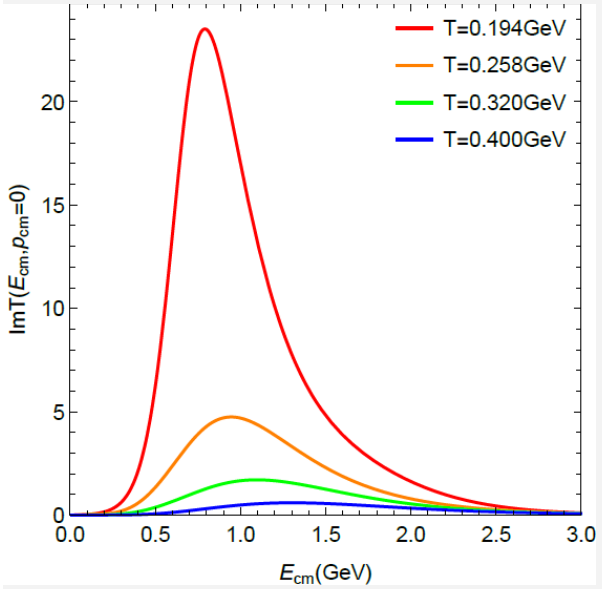
$$\Omega = \mp \frac{-1}{\beta} \sum_n \mathrm{Tr}\{\ln(-G^{-1}) + (G_0^{-1} - G^{-1})G\} \pm \Phi$$

"Skeleton" Diagrams

$$\Phi = \frac{-1}{\beta} \sum_{n,\nu} \mathrm{Tr}\{\frac{1}{2\nu}(\frac{-1}{\beta})^\nu [(-\beta)^\nu \Sigma_\nu(G)]G\}$$





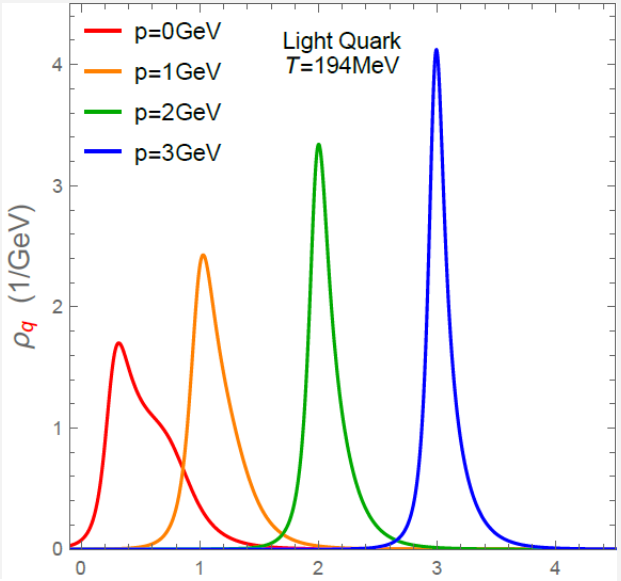


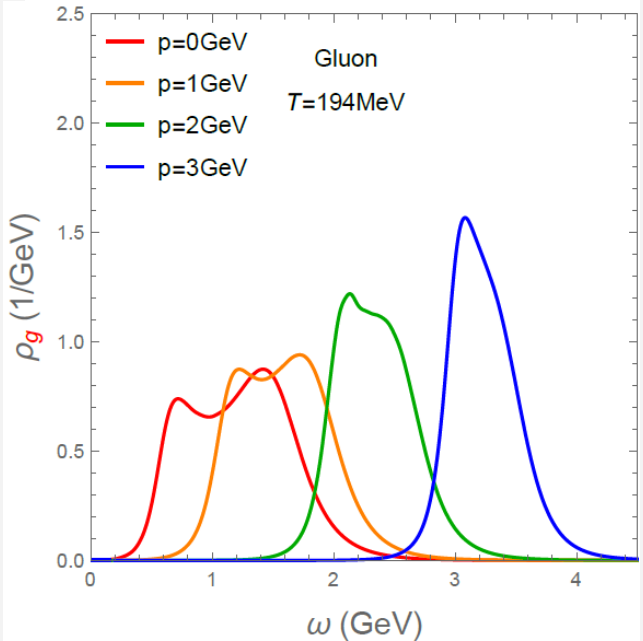
- **Resum Φ with Matrix-Log technique**
- **Broad hadronic resonances emerge near T_{pc}, dominate EoS**

[S.Liu+RR '16]

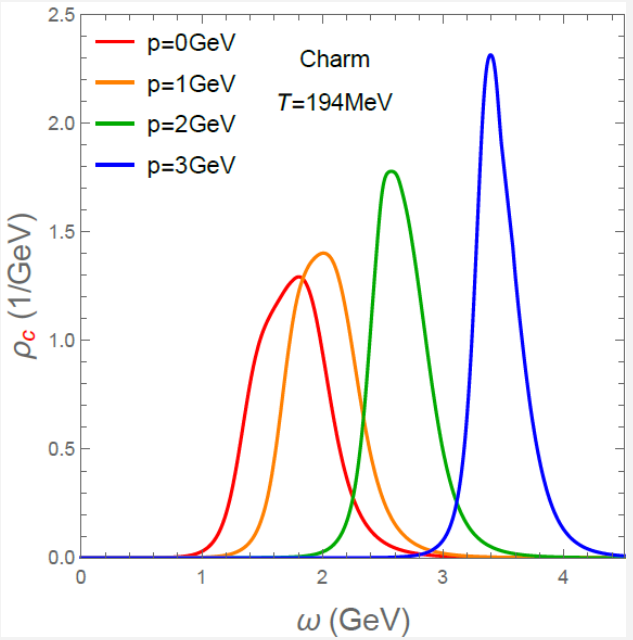
3.2 Parton Spectral Functions in QGP

Up/Down + Gluon

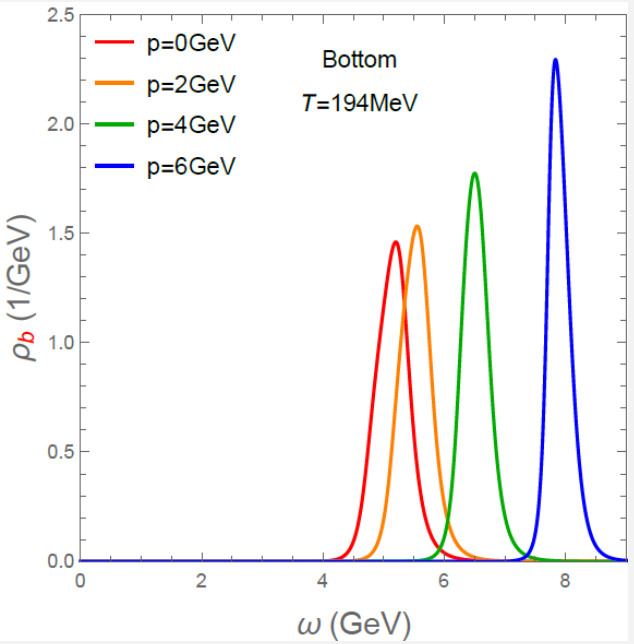




Charm



Bottom



- QGP structure changes with resolution scale
- Soft parton quasi-particles dissolve at low **T**, re-emerge with increasing **M_q**, **p**, **T**

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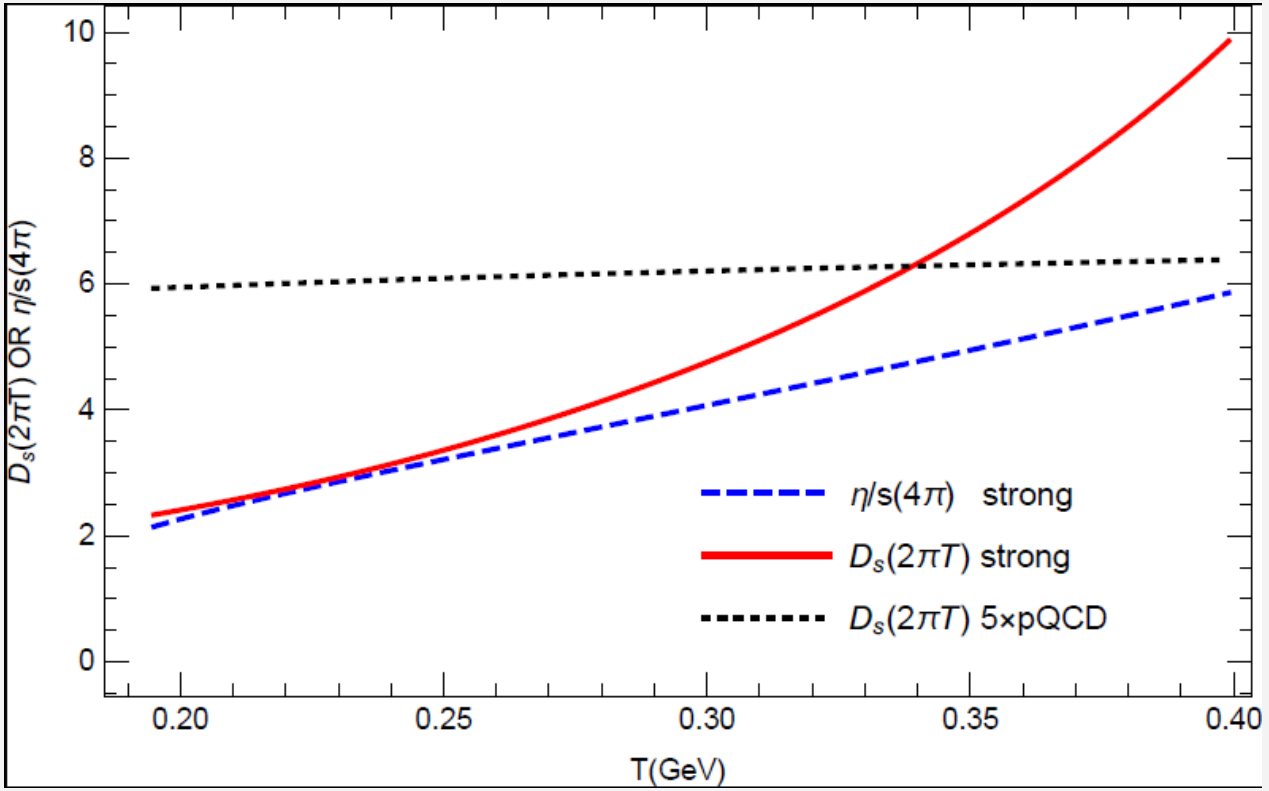
4.1 Transport Coefficients

Viscosity and Heavy-Quark Diffusion

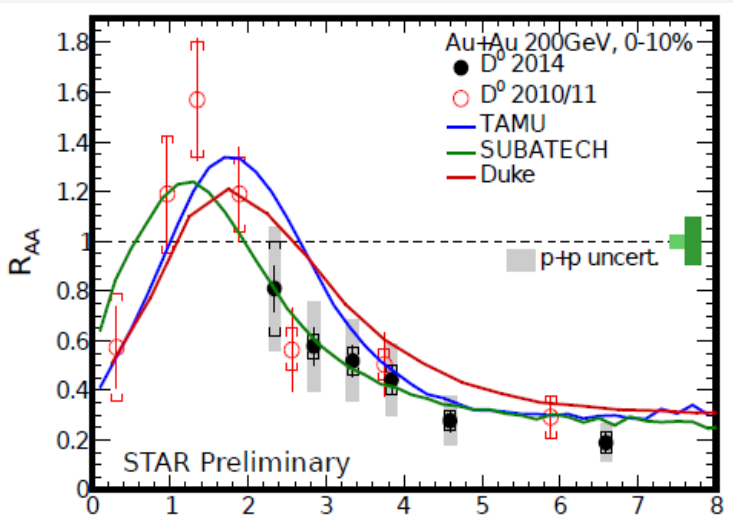


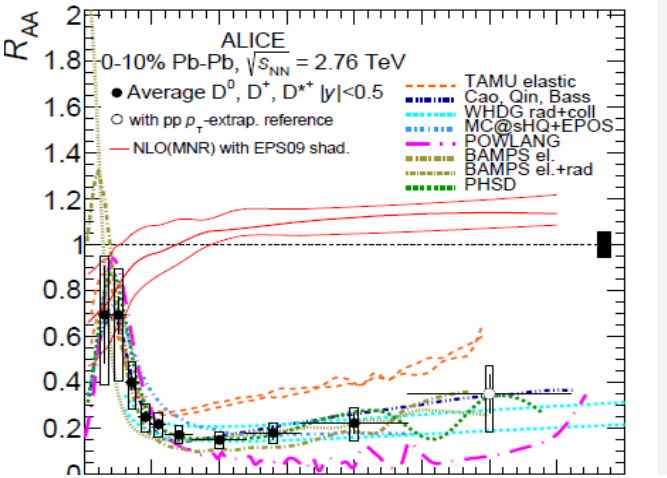
- Strongly coupled: **$(4\pi)\ \eta/s \sim (2\pi T)\ D_s$**
- Perturbative: **$(4\pi)\ \eta/s \sim 2/5\ (2\pi T)\ D_s$**
- Transition as **T** increases

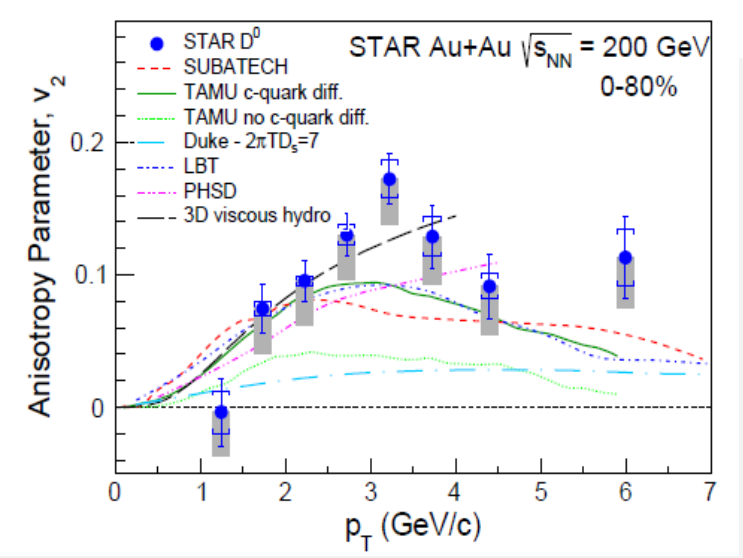
5.) Summary

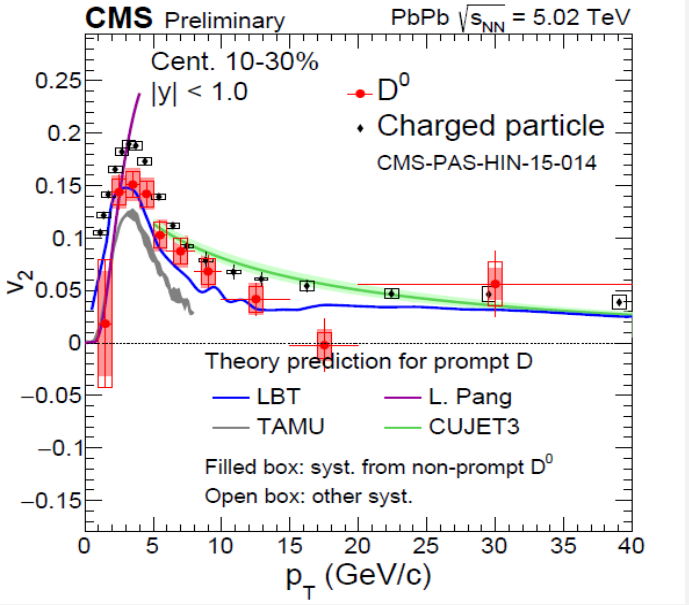
- Self-consistent quantum many-body approach to QGP
- Interaction kernel rooted in lattice QCD (fit HQ free energy)
- Remnants of long-range confining force survive well above $\mathbf{T_{pc}}$
  $\Rightarrow$ **large scattering rates $\rightarrow$ broad spectral functions**
- Strong-coupling description of QGP equation of state
  - no long-wavelength quasi-partons near $\mathbf{T_{pc}}$ (re-emerge at high $\mathbf{p}$,$\mathbf{T}$,$\mathbf{m_Q}$)
  - partons convert into broad hadronic states near $\mathbf{T_{pc}}$
  $\Rightarrow$ **intimate relation between strong coupling and hadronization**
- Transport coefficients show minimum structure + transition to perturbative behavior

4.1 Heavy-Flavor Transport at RHIC + LHC





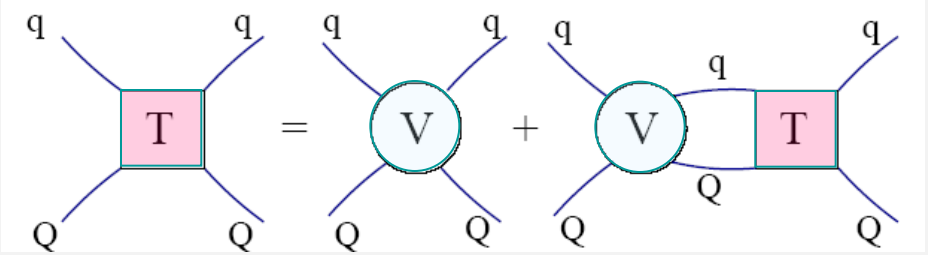


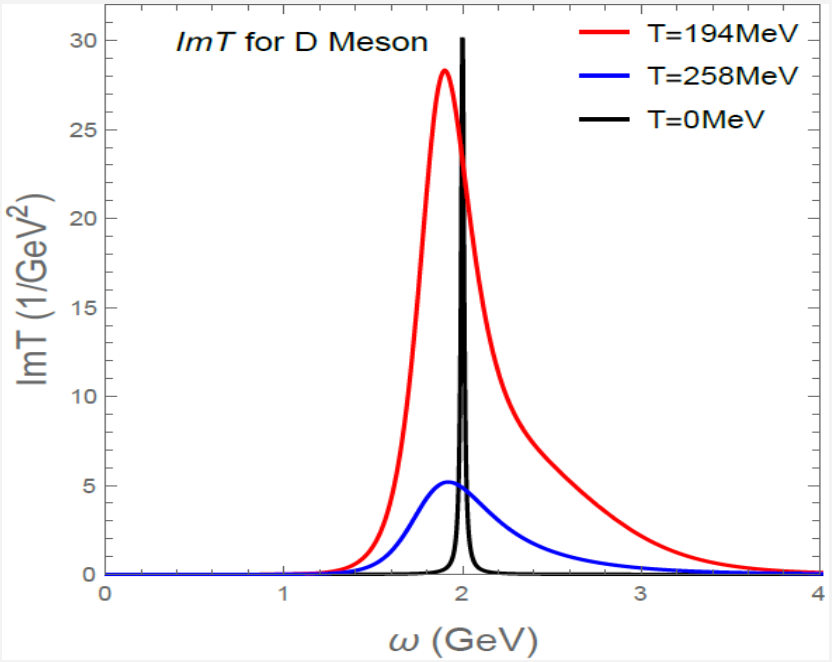


- flow bump in R_{AA} + large v_2 ↔ strong coupling near T_{pc} (**recombination**)
- high-precision v_2: transition from elastic to radiative regime?

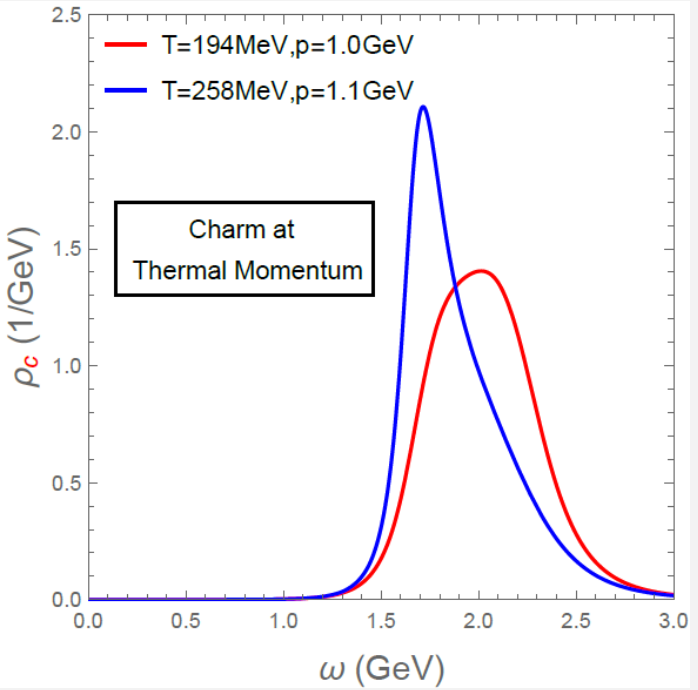
2.2 D-Meson + c-Quark Spectral Functions in QGP

T-matrix w/ "lattice potential" V





In-Medium c-Quark Selfenergy



D-meson resonances near T_{pc} → **c**-quark quasi-particles at high T

2.4.2 Free Energy from T-Matrix

- **Free Energy** $F_{Q\bar{Q}}(r_1 - r_2) = -\frac{1}{\beta}\ln\left(G^{>}(-i\beta, r_1 - r_2)\right) = -\frac{1}{\beta}\ln\left(\int_{-\infty}^{\infty} d\omega\sigma(\omega, r_1 - r_2)\, e^{-\beta\omega}\right)$

[Beraudo et al '08]

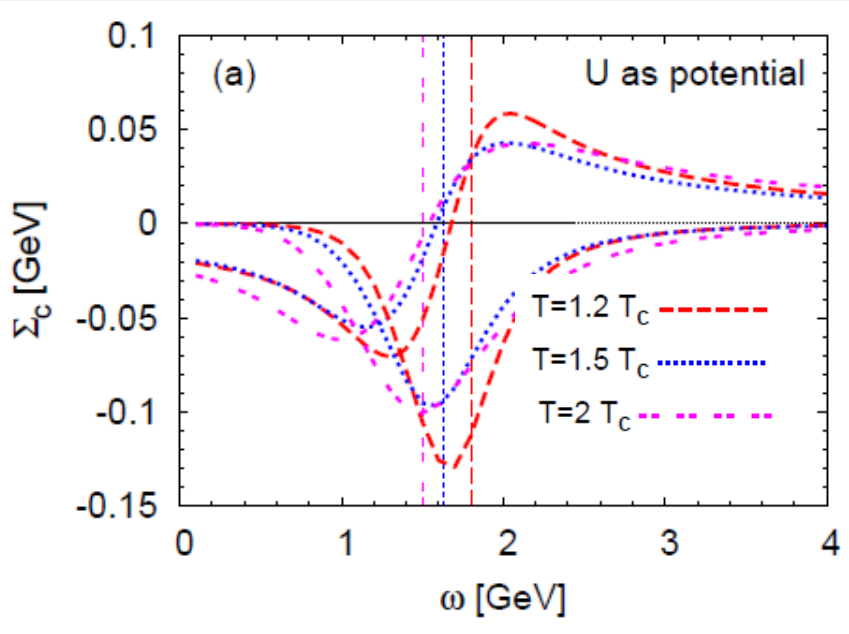
- Euclidean **T-matrix** in static limit

$$\tilde{T}(z_t|r) = V(z_t, r) + V(z_t, r)\,\tilde{G}_0^{(2)}(z_t - v^a, v^a)\;\tilde{T}(z_t|r) = \frac{V(z_t, r)}{1 - V(z_t, r)\,\tilde{G}_0^{(2)}(z_t)}$$

- **Spectral Function** $\sigma(\omega, r) = \frac{1}{\pi}\frac{(V+\Sigma)_I(\omega)}{(\omega - (V+\Sigma)_R)^2 + (V+\Sigma)_I^2(\omega)}$

[S.Liu+RR '15]

- Key ingredients:
  imaginary parts + their ω dependence
- heavy-quark self-energies calculated self-consistently from **Qq T-matrix**

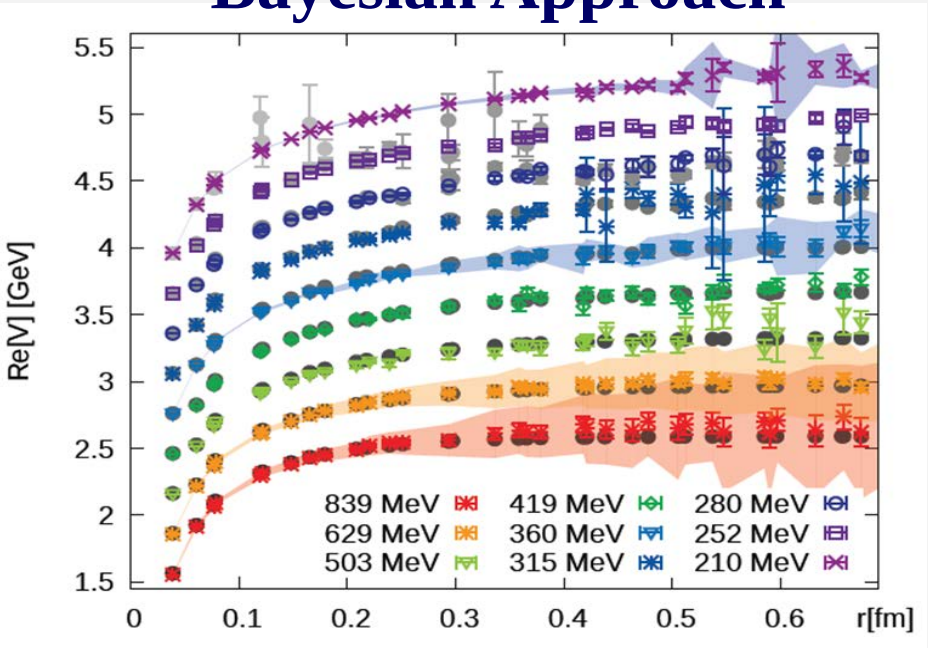


2.4 Potential Extraction from Lattice Data

- **Free Energy** $F_{Q\bar{Q}}(r_1 - r_2) = -\frac{1}{\beta}\ln\left(G^{>}(-i\beta, r_1 - r_2)\right) = -\frac{1}{\beta}\ln\left(\int_{-\infty}^{\infty} d\omega\sigma(\omega, r_1 - r_2)\, e^{-\beta\omega}\right)$

- **$Q\bar{Q}$ Spectral Function** $\sigma(\omega, r) = \frac{1}{\pi}\frac{(V+\Sigma)_I(\omega)}{(\omega - (V+\Sigma)_R)^2 + (V+\Sigma)_I^2(\omega)}$

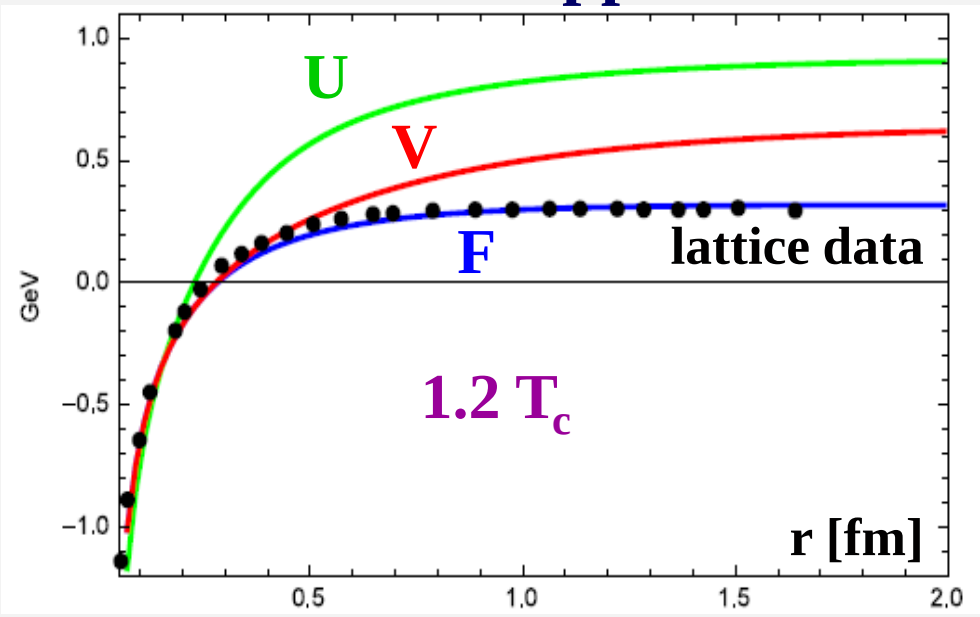
Bayesian Approach



- **Potential close to free energy**

[Burnier et al '14]

T-Matrix Approach



- Account for large imaginary parts
- **Remnant of confining force!**

[S.Liu+RR '15]