(Inverse) Magnetic Catalysis from the Properties of the QCD Coupling in a Magnetic Field.

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International workshop
QCD challenges at the LHC: from pp to AA.
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Outline

- Motivation.
- Magnetic fields ⇔ QCD phase transition.
- Magnetic fields ⇔ Vacuum and finite temperature systems.
- Magnetic fields ⇔ Coupling constants.
- Conclusions.
Nature in the presence of Magnetic fields

- The earth’s magnetic field → 0.6 Gauss.
- A common hand-held magnet → 100 Gauss.
- The strongest steady magnetic fields achieved so far in the laboratory → $4.5 \times 10^5$ Gauss.
- Surface field of magnetars → $10^{15}$ Gauss.
- Heavy Ion Collisions, the strongest magnetic field ever achieved in the laboratory → $10^{18}$ Gauss ≈ $m_{\pi}^2$ ($m_{\pi} = 135$ MeV).
Magnetic fields generated in HIC.

Estimation of magnetic fields strength


K. Tuchin, arXiv:1508.06925
Charged particles in presence of magnetic fields.

- The physical system loses isotropy.
- Reduction from 3D to 2D.
QCD phase transition & $eB \neq 0$.

Is the $T_c$ modified by strong magnetic fields?
Does the CEP move due to the strength of the magnetic field?
Magnetic catalysis & Inverse Magnetic catalysis

\( \frac{\Sigma_u + \Sigma_d}{2} \) vs. \( T \) (MeV)

- \( eB = 1.0 \) GeV\(^2\)
- \( eB = 0.6 \) GeV\(^2\)
- \( eB = 0 \)

G. S. Bali et al., PoS ConfinementX (2012) 197
Magnetic catalysis & Inverse Magnetic catalysis

\[ T_c \text{ (MeV)} \]
\[ \Sigma_u + \Sigma_d \]
\[ eB \text{ (GeV}^2\) ]

G. S. Bali et al., PoS ConfinementX (2012) 197
Some theoretical models.

Linear Sigma Model Coupled to quarks.


Beyond mean field approximation.

- More than one-loop correction.
- First correction to coupling constants.
Some theoretical models.

First order correction to coupling constants.


Magnetized Effective QCD Phase Diagram.

Inverse Magnetic Catalysis

CEP(eB)

Physics behind (inverse) magnetic catalysis.

Is the coupling constant the key to understand the (inverse) magnetic catalysis?

- Working within QCD.
- In the framework of perturbation theory.
  - High temperature \((T^2 > eB)\).
  - High virtuallity \((q^2 > eB)\).
The thermo-magnetic correction to the quark-gluon vertex in the presence of a weak magnetic field was computed.

The vacuum one-loop quark-gluon vertex correction at zero temperature in the presence of magnetic field was computed.

(a) QED-like contribution.

(b) Pure QCD contribution.
Weak field approximation.

Magnetic fields modify the behaviour of Quarks.

\[
S_B(K, m_f) = \frac{(m_f - K)}{K^2 + m_f^2} - i \frac{\gamma_1 \gamma_2 (qB)(m_f - K^\parallel)}{K^2 + m_f^2}^2 \\
+ \frac{2(qB)^2 K^2_\perp}{(K^2 + m_f^2)^4} \left[ (m_f - K^\parallel) + \frac{K^\perp (m_f^2 + K^2_\parallel)}{K^2_\perp} \right].
\]
thermo-magnetic correction of $g$.

$$
\delta \Gamma^{(a)}_{\mu} = ig^2(C_F - \frac{C_A}{2})(qB)T \sum_n \int \frac{d^3k}{(2\pi)^3} \gamma_\nu \left[ \gamma_1 \gamma_2 K_\parallel \gamma_\mu K \tilde{\Delta}(P_2 - K) \right.
$$

$$
+ K \gamma_\mu \gamma_1 \gamma_2 K_\parallel \tilde{\Delta}(P_1 - K) \left. \right] \gamma_\nu \Delta(K) \tilde{\Delta}(P_2 - K) \tilde{\Delta}(P_1 - K)
$$

$$
\delta \Gamma^{(b)}_{\mu} = 2ig^2 \frac{C_A}{2}(qB)T \sum_n \int \frac{d^3k}{(2\pi)^3} \left[ K \gamma_1 \gamma_2 K_\parallel \gamma_\mu - 2\gamma_\nu \gamma_1 \gamma_2 K_\parallel \gamma_\nu K_\mu \right.
$$

$$
+ \gamma_\mu \gamma_1 \gamma_2 K_\parallel K \left. \right] \tilde{\Delta}(K)^2 \Delta(P_1 - K) \Delta(P_2 - K) \tilde{\Delta}(P_1 - K)
$$

with

$$
\Delta(K) = \frac{1}{\omega_n^2 + k^2}
$$

$$
\tilde{\Delta}(K) = \frac{1}{\tilde{\omega}_n^2 + k^2 + m^2}
$$
Result

$$\delta \Gamma_{\mu} = \delta \Gamma_{\mu}^{(a)} + \delta \Gamma_{\mu}^{(b)}$$

$$= g^2 C_F \left( \frac{qB}{6\pi^2 T^2} \right) \left[ \log(2) - \frac{\pi T}{2m} \right] \vec{\gamma} \parallel \Sigma_3$$

Then

$$g_{eff} = g \left[ 1 - \frac{m^2}{T^2} + \delta \Gamma \right]$$
Vacuum

Magnetic correction of $g$.

$$\delta \Gamma^\mu_{(a)} = g^2 (qB) \left( C_F - \frac{C_A}{2} \right) \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2} \times \left[ \gamma^\nu \frac{p_2 - k}{(p_2 - k)^2} \gamma^\mu \gamma_1 \gamma_2 (\gamma_1 p_1 - \gamma_2 k) \| \gamma_\nu + \gamma^\nu \frac{\gamma_1 \gamma_2 (\gamma_1 p_2 - \gamma_2 k) \|}{(p_2 - k)^4} \gamma^\mu \frac{p_1 - k}{(p_1 - k)^2} \gamma_\nu \right]$$

$$\delta \Gamma^\mu_{(b)} = -2g^2 (qB) \frac{C_A}{2} \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^4} \left[ g^\mu\nu (2p_2 - p_1 - k)^\rho \right.$$  

$$\left. + g^{\nu\rho}(2k - p_2 - p_1)^\mu + g^{\rho\mu}(2p_1 - k - p_2)^\nu \right] \gamma_\rho \frac{\gamma_1 \gamma_2 (k \|}{(p_2 - k)^2 (p_1 - k)^2} \gamma_\nu$$
Vacuum

Result

$$\delta \Gamma^\mu = \delta \Gamma^\mu_{(a)} + \delta \Gamma^\mu_{(b)}$$

$$= -g^2 \frac{1}{3\pi^2} \left( \frac{q \sum \cdot \vec{B}}{Q^2} \right) \left[ [1 - \log(4)] C_F - \frac{[7 + 3 \log(4)]}{10} C_A \right]$$

Then

$$g_{eff} = g [1 + \delta \Gamma]$$
Our results show that the geometrical effect produced by the magnetic field at high temperature, whereby quarks and anti-quarks get closer on average, is accompanied by the decrease of their effective interaction due to the asymptotic freedom.

In contrast, at $T = 0$ such geometrical effect does not take place. This because the color charge associated to gluons produces a kind of screening of the color charge associated to quarks.
Many Thanks!!!